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CONTROL POLICIES IN CSMA LOCAL AREA NETWORKS: ETHERNET CONTROLS

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Abstract

An analysis of the random carrier sense multiple access channel is presented in terms of the behaviour of each participating station. A detailed model of the station protocol, including the control policy used in case collisions, is used to derive the traffic and throughput of each station. The channel traffic characteristics are derived from this model and used, in turn, to derive the traffic parameters entering into the station model. This provides a solution method for complete system characteristics for a finite prespecified set of stations. The approach is then used to analyse control policies of the type used in ETHERNET. We show, in particular, that as the propagation delay becomes small, the specific form of the control policy tends to have a marginal effect on network performance. The approach also applies to the DANUBE and XANTHOS networks.

Résumé

Nous présentons une nouvelle méthode d'analyse des protocoles d'accès de type CSMA (accès aléatoire à un canal unique de type "ETHERNET") dans le contexte des réseaux locaux. Cette méthode permet de calculer le débit du canal et le temps de réponse aux paquets émis. La classe de politiques de contrôle proposée par METCALFE et BOGGS pour ETHERNET est analysée en détail et l'équivalence du point de vue des performances de toutes ces politiques est indiqué pour des délais de propagation faibles devant le délai d'émission d'un paquet. Ce modèle est validé par un ensemble de simulations et s'applique également aux réseaux DANUBE et XANTHOS.

I. - INTRODUCTION

To say that local computer networks have received much attention in the last few years would be an understatement. A recent annotated bibliography (Shoch, [8]) whose coverage of the subject is by no means exhaustive, contains over 300 references ; most of these are less than five years old. This surge of interest is, of course, easy to explain : organisations which previously concentrated their computing facilities, tend now to distribute them. There are many reasons in favour of distribution, including reliability, flexibility, convenience, sometimes even efficiency.

The defining characteristic of local computer networks is that the propagation delay - the time necessary for a unit of information to traverse the network - is small compared to the packet transmission time. This implies among other things that even if packet collisions are possible, they can be detected quickly and the transmissions involved can be aborted without wasting too much channel time. Hence, one can reasonably argue against centralised control algorithms that prevent collisions and for distributed ones that maintain performance despite collisions.

Perhaps the best-known local computer network with distributed control is Ethernet (Metcalfe and Boggs, [6]). The Ethernet control policy is of the CSMA (Carrier sense multiple access) type [5] : a station wishing to transmit "listens" to the channel first ; if it senses a transmission in progress then it waits, otherwise it begins transmission immediately. Collisions occur when the interval between two consecutive starts of transmission is less than the propagation delay : then the second station is not aware that a transmission is in progress. To minimise the likelihood of repeated collisions, each colliding station waits for a random period before trying again ; moreover, the durations of these periods depend on the number of previous collisions suffered while attempting to transmit a packet.

Another existing network of this type is the XANTHOS system (Gelenbe et al. [2]) which uses a fibre optics transmission medium.

An exact mathematical analysis of the Ethernet control policy has not, so far, been accomplished. Existing works in that general area have either assumed constant Poisson traffic [2] , [3] , [5], or a discrete (slotted) timing with fixed retransmission probability, [1][3]. The principal obstacle to obtaining exact results is the dependency between stations : the behaviour of one node in the network influences that of all other nodes and vice versa.

We shall present an approach that will allow an approximate analysis of the Ethernet protocol with arbitrary but finite number of stations. The idea is to divide the analysis into two stages : first, a single station is examined in isolation, taking certain global network parameters (probability of collision, probability of detecting a transmission in progress, etc ...) as given. Characteristics of the submitted traffic are then obtained which, in turn, yield equations for the unknown global parameters. Thus the evaluation of the dependency between stations is reduced to the determination of a small number of parameters.

The model and its solution are described in section 2. Section 3 contains a first-order approximation to the solution when the propagation delay is very small. Some validation of the approach is presented in section 4 together with numerical examples.

II. - THE MODEL AND ITS SOLUTION

The behaviour of a station in an Ethernet network is described by a set of possible states that the station can be in, the average periods of time that it remains in those states and the probabilities of transition from state to state. A diagram illustrating that behaviour is shown in Figure 1.

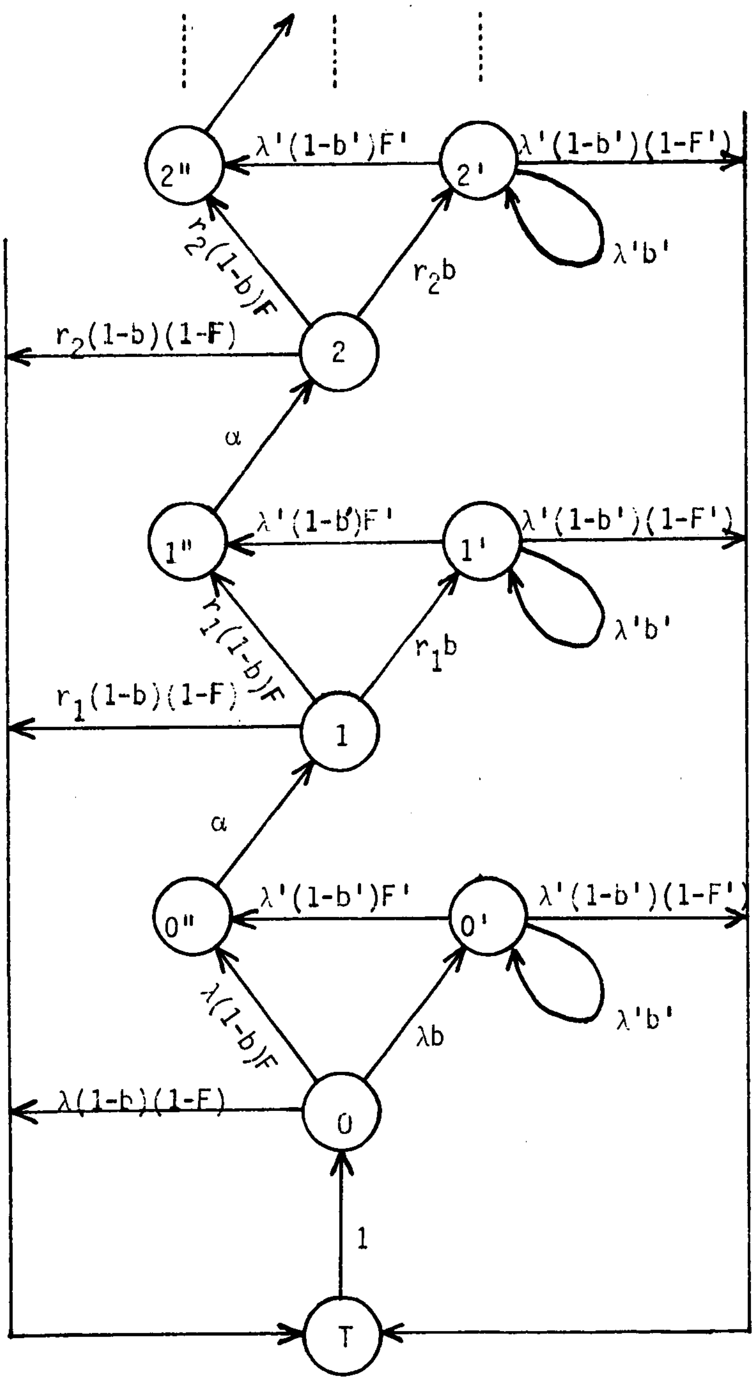

Figure 1

One of the possible states in the "successful transmission" state, T. The average duration of a transmission is taken to be 1 ; on leaving state T, the station goes with probability 1 to the "idle state", 0. It is convenient to divide the rest of the states into groups of three : i, i', i" for i = 0, 1, 2, ... In state i, the station is preparing to attempt a transmission, having already attempted it i times before and suffered i collisions. The average residence time in state i is $1/\lambda$ when i = 0 (this is the average interval between completing the transmission of one packet and attempting that of the next) ; it is $1/r_i$ when i > 0. The constants r_i (i = 1,2,...) are control parameters of the Ethernet policy ; intuitively, the more collisions one has suffered in the course of sending a packet, the longer one should wait before attempting again. In Metcalfe's algorithm, [6], the r_i take a special form which will be examined in section 4.

Upon leaving state i, one of three things may happen : either the channel is sensed busy (in which case state i' is entered), or the channel is sensed idle but a collision occurs (state i" is entered), or the channel is sensed idle and a collision does not occur (state T is entered). The probabilities of these three outcomes are b, (1-b) F and (1-b)(1-F) respectively. The probability of a busy channel b, and that of a collision, F, are for the moment assumed given.

Having sensed a busy channel, the station remains in state i' for an average period $1/\lambda'$; it then experiences again a busy channel (reenters state i'), a collision (state i"), or a successful transmission (state T) with probabilities b' , (1-b) F' and (1-b')(1-F') respectively. Note that b and b' and F and F', are not, in general, equal. λ' is also a control parameter of the policy. When $\lambda' \to \infty$, the policy is said to be "persistent".

In the i-th collision state, i", the station remains for an average period $1/\alpha$ (for the moment assumed given). On leaving state i" it goes to state i+1 with probability 1, there to await the next retransmission attempt.

We thus model the bahaviour of each station in the network by a semi-Markov process. The distributions of the various residence times are not important, only their means. The parameters of the process are λ (a known characteristic of the station), λ', r_1, r_2, ... (known control parameters), plus b,b',F,F' and α (global network parameters yet to be determined). All stations are assumed to be statistically identical. The whole network is assumed to be in statistical equilibrium.

A few words concerning the values of some of the parameters are in order. We have tacitly taken it for granted that the averages $1/\lambda$ and $1/r_i$ $(i = 1,2,\dots)$ are large compared to the packet transmission time, so that every attempt at transmission can be considered as a random observation point. That is why the probabilities of finding a busy channel or a collision (b and F) were assumed constant. On the other hand, $1/\lambda'$ may be quite small, hence b' is different from b and F' is different from F.

We shall solve the above model and obtain the contribution of an individual station to the channel traffic. A superposition of the N stations will then yield the global traffic characteristics. These, plus an assumption that the global traffic is (non-homogeneous) Poisson, will in turn determine the unknown parameters b, b', F, F' and α.

Let us now go ahead with this procedure.

II.1. - Solution of the single station model

The stationary probabilities of states T, i, i' and i" $(i = 0, 1,\dots)$ will be denoted π_T, π_i, $\pi_{i'}$ and $\pi_{i''}$ respectively. These probabilities satisfy the following system of balance equations :

$$(1) \quad \begin{cases} \lambda\, \pi_0 = \pi_T \\ r_i \pi_i = \alpha\, \pi''_{i-1}\,, \quad i = 1,2,\dots \\ \alpha\, \pi_i'' = \gamma_i\,(1-b)\, F\, \pi_i + \lambda'(1-b')\, F'\, \pi_i'\,, \quad i = 0, 1,\dots \\ \lambda'(1-b')\pi_i' = \gamma_i\, b\, \pi_i\,, \quad i = 0,1,\dots\,, \end{cases}$$

where $\gamma_i = \begin{cases} \lambda & \text{if } i = 0 \\ r_i & \text{if } i > 0\,. \end{cases}$

To these we must add the normalising equation

$$(2) \qquad \pi_T + \sum_{i=0}^{\infty} (\pi_i + \pi_{i'} + \pi_{i''}) = 1.$$

Substituting the fourth of the equations in (1) into the third and the third into the second, and remembering the definition of γ_i, we get :

$$\gamma_i \, \pi_i = \gamma_{i-1} \, \delta \, \pi_{i-1}, \quad i = 1,2,\ldots \qquad \text{where } \delta = (1-b)\, F + b\, F'.$$

This last equation, together with (1), yields

$$(3) \qquad \begin{cases} \pi_i = (\delta^i/\gamma_i)\, \pi_T \,, \; i = 0, 1, \ldots, \\ \pi_i' = [b\, \delta^i/(\lambda' - \lambda' b')]\, \pi_T \,, \; i = 0, 1, \ldots, \\ \pi_i'' = (\delta^{i+1}/\alpha)\, \pi_T \,, \; i = 0, 1, \ldots . \end{cases}$$

The probability π_T can now be determined from (2) :

$$(4) \qquad \pi_T = [1 + \frac{\delta}{\alpha\,(1-\delta)} + \frac{b}{\lambda'(1-b')(1-\delta)} + \sum_{i=0}^{\infty} (\delta^i/\gamma_i)]^{-1}.$$

II.2. - Traffic and response time characteristics

Let s be the throughput of the station, i.e. the average number of packets that it transmits successfully per unit time. Since the average duration of a transmission is 1, s is equal to the fraction of time that the station spends transmitting :

$$(5) \qquad s = \pi_T .$$

Next, let c be the average number of transmission attempts that result in collisions per unit time. That is the average number of times that states i" (i = 0, 1,...) are entered (or left) per unit time :

$$(6) \qquad c = \alpha \sum_{i=0}^{\infty} \pi_i'' = \frac{\delta}{1-\delta}\, \pi_T .$$

Hence, the total rate of traffic, g, contributed by one station is given by

(7) $$g = s + c = \frac{1}{1-\delta}\,\pi_T = \frac{s}{1-\delta} \;.$$

Let us denote by W the average response time of a packet, which is the total delay incurred by a packet between the instant at which the station decides to transmit it and the instant when the transmission is successfully completed (passage from state 0 to state 0). Since at any moment there is at most one packet undergoing attempted or actual transmission, the average number of such packets is $1 - \pi_0$, or $1 - (s/\lambda)$. According to Little's theorem, the average response time can be expressed as :

(8) $$W = (1-\pi_0)/s = (1/s) - (1/\lambda) = [1/(g(1-\delta))] - (1/\lambda) \;.$$

In fact, the throughput-versus-traffic formula (7) is valid for the global N-station network. Let the global N-station traffic and throughput be G and S, respectively. Then :

(9) $$G = Ng \quad , \quad S = Ns$$

and therefore :

$$\frac{S}{G} = 1 - \delta$$

The average response time can also be expressed in terms of the global traffic by rewriting (8) :

(10) $$W = \frac{N}{G(1-\delta)} - \frac{1}{\lambda} \;.$$

One cannot help noticing that (10) is very similar to a well-known expression for the response time in closed queueing networks, (see, for example, [9]). This similarity is rather remarkable because the behaviour of packets in Ethernet is not at all like that of customers in a queueing network.

In what follows, we shall also need the traffic offered by a station, given that it is not in a collision or a transmission state. Denoting that traffic rate by $\tilde{g}$ we have :

(11) $$\tilde{g} = \sum_{i=0}^{\infty} (\gamma_i\pi_i + \lambda'\pi_i') \Big/ \sum_{i=0}^{\infty} (\pi_i + \pi_i') = \left[\frac{\pi_T(1 + b - b')}{(1-\delta)(1-b')}\right] \Big/ \left[1 - \pi_T\,\frac{\alpha(1-\delta)+\delta}{\alpha(1-\delta)}\right]$$

II.3. - Total traffic dependent parameters

Throughout the preceding analysis, the parameters F, F', α, b and b' appear. These depend on the total traffic characteristics and have not yet been determined. We shall proceed to do so here.

Let us start with F ; it is the probability that a collision occurs (after the transmitting station has sensed an idle channel) due to the round-trip propagation delay D which we assume to be fixed for all pairs of stations. As shown on Figure 2, station A transmits a packet at time t when it believes that the channel is idle.

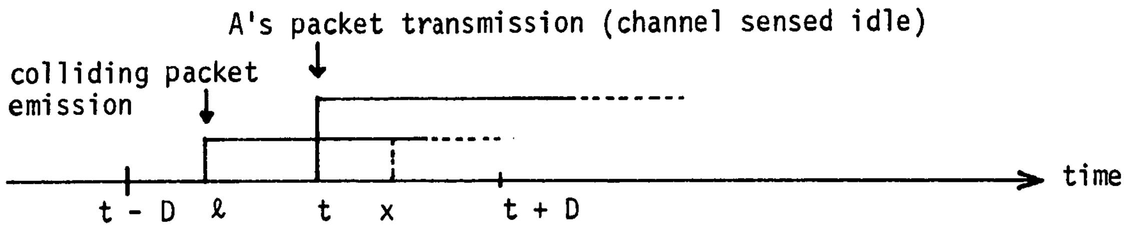

Figure 2 : Collision between packets in the carrier-sense protocol.

Yet at some instant ℓ, $t-D < \ell \leq t$, some other station has already started a packet transmission which A will not be aware of at time t ; A will only become aware of the collision at time $x = \ell + D$. Similarly, a collision may occur if a station transmits a packet at time $t \leq \ell < t+D$, since this station connot be aware of A's transmission at time t. Thus F is the probability that another station will transmit in the interval $]t-D, t+D[$. Assuming Poisson traffic and statistically identical stations and remembering that at time T-D the channel was idle (i-e no station was in a collision state) we have :

(12) $F = 1 - \exp(-2 D\tilde{g} (N-1))$

If D is very small, which will be the case in local area networks, we have :

(13) $F \approx 2 D\tilde{g} (N-1)$

Let us now turn our attention to b, the probability that when a station senses the channel it discovers that it is busy. In fact it is **simpler** to obtain (1-b), the probability that the channel is sensed idle, first.

A necessary and sufficient condition for the channel to be sensed idle at time t, is that it is indeed idle at time (t-D). Since the instant t is associated with the observation of a "random observer" or with the steady state, so is the instant (t-D). Thus (1-b) will be the steady state probability that the channel is idle.

The channel's behaviour, as observed by the transmitting station, will be modelled by a three state process whose states are I (idle), C (collision), T (normal collision free transmission) ; this behaviour is represented in Figure 3. Here τ_I, τ_C, τ_T, are

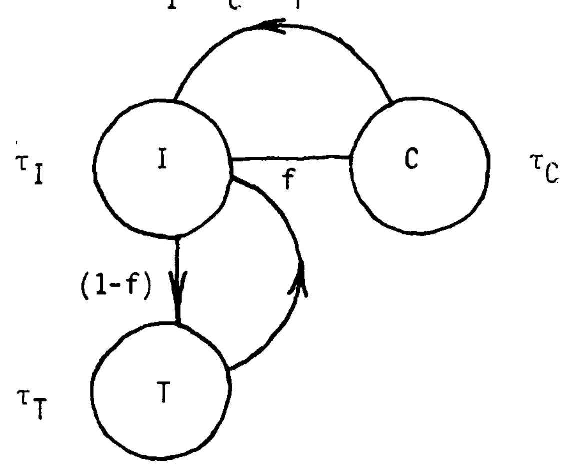

Figure 3 : Channel behaviour, as viewed from the transmitting station.

the average residence times in the three states, and the model is a semi-Markov chain, where f and (1-f) are the transition probabilities from state I to states C and T, respectively. This behaviour concerns (N-1) stations ; i.e. all but the "observer" station.

Obviously, since we have assumed that the average length of packets is unity,

$$\tau_T = 1 .$$

Furthermore, since we assume that total traffic entering the channel from the (N-1) other stations is Poisson , and since the channel is idle there are no stations in a collision state,

$$\tau_I = 1 / \tilde{g}(N-1).$$

τ_C is the average time that the channel will be occupied by a collision. To compute τ_C consider the first of the (N-1) stations which starts transmitting at time t' forcing the channel out of the I state and leaving (N-2) stations behind in a non-transmitting state ; if the channel then enters the C-state it is because at least one other station starts transmitting. If the first such station transmits at time $(t' + x)$ then the total time the channel will spend in state C will be $D + x$, and its average value will then be

$$\tau_C = \int_0^D (D + x)\, \tilde{g}\,(N-2)\; e^{-x\tilde{g}\,(N-2)}\, dx \,/\, (1 - e^{-D\tilde{g}\,(N-2)})$$

$$= D + \frac{1}{\tilde{g}(N-2)} - \frac{D\, e^{-\tilde{g}\,(N-2)D}}{1-e^{-\tilde{g}\,(N-2)D}} \cong \frac{3D}{2} \qquad \text{if D is very small.}$$

Finally, f is the probability of at least one transmission from the remaining (N-2) stations at some time t'', $t' < t'' < t' + D$;

therefore

$$f = 1 - \exp\,(- D\tilde{g}\,(N-2))$$

$$\cong D\tilde{g}\,(N-2) \qquad \text{if D is very small.}$$

Thus :

$$(14) \qquad (1-b) \cong \frac{1 / \tilde{g}(N-1)}{\frac{1}{\tilde{g}(N-1)} + (1-f) + \frac{3f}{2} D}$$

$$\cong \frac{1}{1 + \tilde{g}(N-1) + D\tilde{g}\,(N-2)(\frac{3}{2} D-1)\, \tilde{g}\,(N-1)}$$

$$\cong \frac{1}{1 + \tilde{g}(N-1)(1-D\, \tilde{g}\,(N-2) + \frac{3}{2} D^2\, \tilde{g}\,(N-2))}$$

$$\cong \frac{1}{1 + \tilde{g}(N-1)\,(1 - D\, \tilde{g}\,(N-2))}$$

$$\cong \frac{1 + D\tilde{g}\,(N-2)}{1 + \tilde{g}\,(N-1)} \qquad \text{if D is very small.}$$

Next we have to compute α, or its inverse $1/\alpha$ which is the average time spent by the station in the channel in case it collides during packet transmission. Rather than consider the interval $]t-D, t+D[$ as shown on Figure 2, it is more convenient to shift the time scale by $(t-D)$ so that the interval is now $]0, 2D[$. Let ℓ be still be the instant at which the colliding extra packet arrives, and D be now the instant at which our observed station starts transmitting its packet.

Whether $0 < \ell \leq D$ or $D < \ell < 2D$, the total time the observed station continues transmitting is $\ell + D - D = \ell$; thus

$$\frac{1}{\alpha} = \int_0^{2D} y(1 - e^{-y\tilde{g}(N-1)})\, dy \,/\, (1 - e^{-2D\tilde{g}(N-1)})$$

or :

$$(15) \qquad \frac{1}{\alpha} = \frac{1}{\tilde{g}(N-1)} - 2D \frac{e^{-2D\tilde{g}(N-1)}}{1 - e^{-2D\tilde{g}(N-1)}}$$

$$\cong D \quad \text{if } D \text{ is very small.}$$

It remains to determine the probabilities F' and b', that the next attempt to transmit will result in a collision or find the channel busy, respectively, given that at the last attempt the channel was busy.

II.4. - Computation of F' and b'

The successful transmission of a packet causes in its aftermath a "turbulence zone" due to the stations which have sensed the channel busy during the transmission.

This effect will perturb the traffic because of the long duration, relative with respect to D, of successful transmissions. Hence, in periods after a station senses that the channel is busy, total traffic related quantities cannot have their average values.

While there is a transmission in progress, each of the remaining N - 1 stations offers traffic at rate $\tilde{g}$. Assuming that the intervals between attempts are exponentially distributed (as is the transmission itself), the probability that a given station will make an attempt before the transmission is over is $\tilde{g} / (\tilde{g} + 1)$. Hence, with probability $q_i = \binom{N-1}{i} \tilde{g}^i / (\tilde{g} + 1)^{N-1}$, thereafter i of the stations will be offering traffic at rate λ' and the others at rate $\tilde{g}$.

Repeating the argument which led to expression (12) we now obtain

$$F' = 1 - \sum_{i=0}^{N-1} q_i \exp\left[-2 D (i\lambda' + (N - 1 - i)\tilde{g})\right]$$

(16)

$$= 1 - \left[\tilde{g} \exp(-2 D \lambda') + \exp(-2 D \tilde{g})\right]^{N-1} / (\tilde{g} + 1)^{N-1}$$

The behaviour of this expression when D is very small depends on whether λ' is very large or not. If $D\lambda'$ is very large, the stations follow a persistant retransmission policy. The value of F' is then, approximately.

(17) $F' \simeq 1 - 1/(\tilde{g} + 1)^{N-1} + 2D\tilde{g}(N-1)/(\tilde{g} + 1)^{N-1}$.

This last result has an intuitive appeal since, under a persistant retransmission policy a collision is certain if at least one other station makes an attempt to transmit. If, on the other hand, $D\lambda'$ is very small (non-persistent policy), then (16) yields

(18) $F' \simeq 2D\tilde{g} (N - 1) (\lambda' + 1) / (\tilde{g} + 1)$

The computation of b' should take into account the fact that the "observer" station has already sensed the channel busy at least once.
Thus

$$(19) \qquad b' = \frac{\lambda'}{1+\lambda'} \cdot 1 + \frac{1}{1+\lambda'} B$$

where the first term indicates that if the new attempt to access the channel occurs before the current transmission ends (with probability $\lambda'/(1+\lambda')$) then the channel will again be sensed busy. The second term covers the case where the new attempt takes place after the end of the current transmission, and B is the corresponding probability that the channel is busy. Denoting by Q the rate of traffic offered by the other stations (excluding the tagged one) following a transmission, we have :

$$(20) \quad Q = \frac{(N-1)\tilde{g}}{\tilde{g}+1}\lambda' + \frac{(N-1)\tilde{g}}{\tilde{g}+1} = \frac{(N-1)\tilde{g}}{\tilde{g}+1}(\lambda'+1).$$

The probability B is now given by an expression similar to (14), but with $\tilde{g}$ replaced by Q / (N - 1) :

$$1 - B \simeq \frac{1+\tilde{g}+D\tilde{g}\,(\lambda'+1)\,(N-2)}{1+\tilde{g}+\tilde{g}\,(\lambda'+1)\,(N-1)} \quad \text{for small D.}$$

Substituting this in (19) we get

$$(21) \quad b' = \frac{\lambda'}{\lambda'+1} + \frac{\tilde{g}\,(N-1) - D\tilde{g}\,(N-2)}{1+\tilde{g}+\tilde{g}\,(\lambda'+1)\,(N-1)}$$

II.5. - Numerical solution

The system performance measures can now be determined by solving a system of non-linear equations. The easiest way to proceed is perhaps by iteration on $\tilde{g}$. For a given value of $\tilde{g}$, equations (13), (14), (15), (18), (21) and (4) provide values for F, b, α, F' (and hence δ), b' and π_T, respectively. Substituting these in the right-hand side of (11), that equation can be written in the form

$$(22) \qquad \tilde{g} = \varphi(\tilde{g})$$

(assuming that D is very small). There are several ways of solving iteratively equations of the type (22) . An algorithm that is guaranteed to

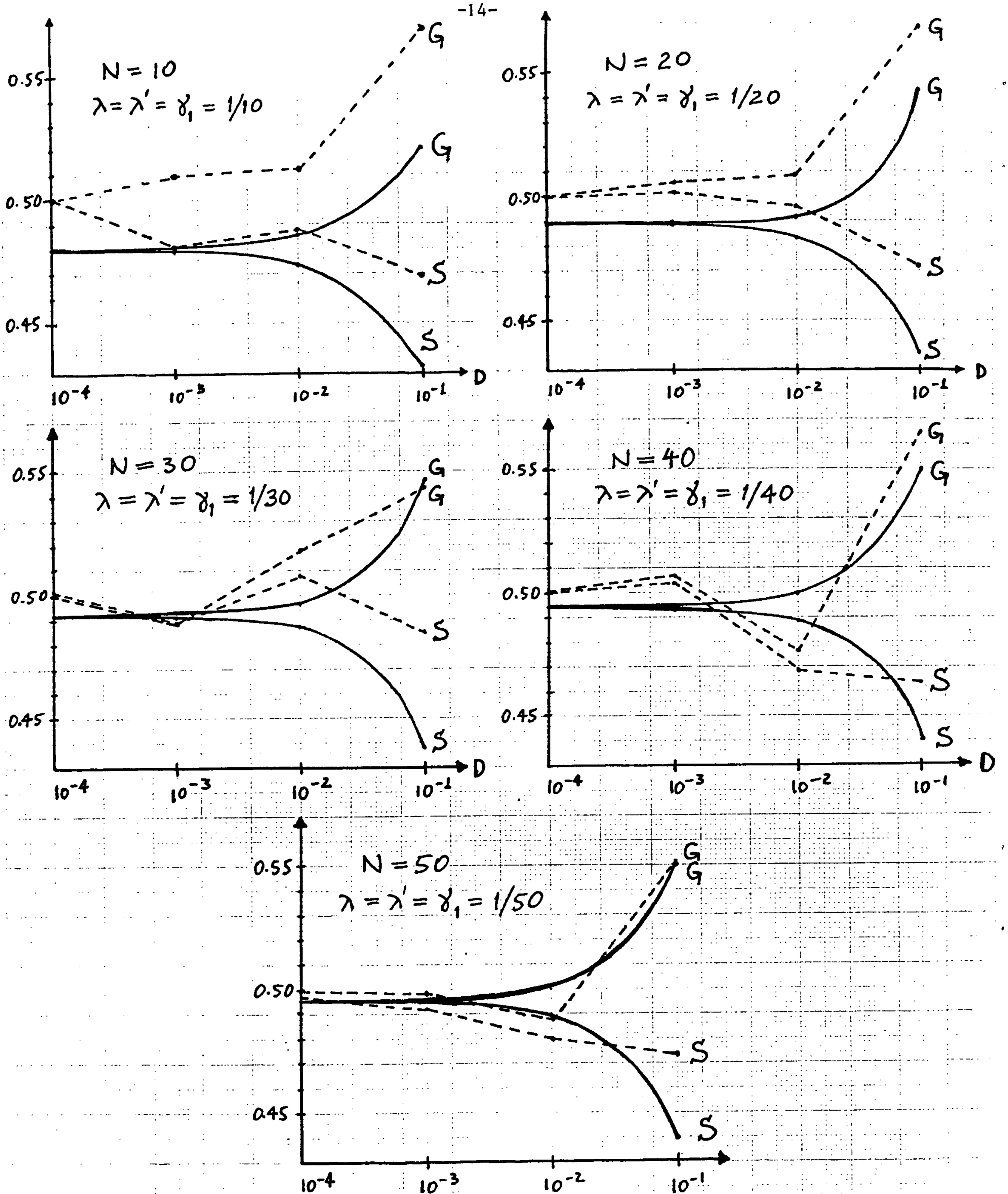

Figure 4. Throughput, S, and total traffic, G, as functions of D.
Solid lines: derived from the model; dotted lines: simulations.
(confidence intervals ~ 5% of estimated values)

converge is the 'binary chop' : first a value $\tilde{g}_0$ is found such that $\tilde{g}_0 > \varphi(\tilde{g}_0)$; since $\varphi(o) > 0$, the desired solution is in the interval $(0,\tilde{g}_0)$ (under a non-persistant retry policy, say with λ and λ' on the order of 1/N, $\varphi(x)$ is also on the order of 1/N, so that $\tilde{g}_0$ can be quite small) ; then at each iteration the interval containing the solution is halved, choosing the left or the right half depending on the value of φ at the mid-point. It is conceivable that equation (22) has more than one solutions on the interval $(0,\tilde{g}_0)$; the iterations could then converge to any of them. However, in our experiments the solution has always been unique.

Figure 4 shows the throughput and the total traffic as functions of D, for different values of tbe parameters. For comparison, curves obtained from simulation are superimposed on those provided by the numerical solution. The agreement is seen to be very acceptable.

III. - THE EFFECT OF SPECIFIC CONTROL LAWS WHEN D IS SMALL

For a given number of stations, N, and average intertransmission time, $1/\lambda$ (demand characteristic), the parameters through which system performance is controlled are λ' and r_i (i=1,2,...). The 'persistency' of a station is measured by the rate λ' of retrying a transmission after finding the channel busy. Let us assume that it, too, is fixed and is finite (i.e. the stations are non-persistent). There are many possible choices for the retransmission rates after a collision ; we shall mention three :

CL1. $r_i = 1/(i\Gamma)$, where Γ is a positive constant. Here the average time between successive attempts to transmit a packet which has collided increases linearly with the number of collisions experimented.

CL2. $r_i = 1/(\Gamma_1 a^i)$, where $\Gamma_1 > 0$ and $a > 1$ are constants. This is the form suggested by Metcalfe [4]; the average retransmission time increases exponentially with the number of collisions.

CL3. $r_i = r$, where r is a positive constant. This 'control law' ignores the number of collisions completely.

It is not difficult to see that, up to first order accuracy in D, these control laws are equivalent. That is, if $\Gamma = a\Gamma_1 = 1/r$, then the system performance measures under the three laws differ by not more than a quantity o(D), where $[o(D)/D] \to 0$ as $D \to 0$. More generally, the following proposition holds :

Proposition : For small D, the performance measures s, g and W are determined, up to a quantity o(D), by the retransmission rate after the first collision, r_1 ; they are independent of r_i $(i \geq 2)$.

To establish the proposition, consider the expression for π_T, (4), which (remembering that $\gamma_0 = \lambda$ and $\gamma_i = r_i$, $i \geq 1$) can be rewritten as

$$(23) \qquad \pi_T = [A + \frac{1}{\lambda} + \frac{\delta}{r_1} + \sum_{i=2}^{\infty} (\delta^i/r_i)]^{-1} ,$$

denoting by A the first three terms in the right-hand side of (4). From (13), (14) and (18) it follows that δ, which is equal to $(1-b)F + bF'$, is of the form

$$\delta = xD + o(D),$$

where $x = 2(N-1)\tilde{g}[1-b+b(1+\lambda')/(1+\tilde{g})]$. Hence, the last term in (23) is o(D) and that equation becomes

$$(24) \qquad \pi_T = [A + \frac{1}{\lambda} + \frac{xD}{r_1}]^{-1} + o(D)$$

Thus everything can be determined, up to o(D), knowing only r_1 and ignoring r_i $(i \geq 2)$, q.e.d. Intuitively, when D is small a packet is unlikely to suffer more than one collision, so the actions taken after the second, third, etc. consecutive collisions do not really matter.

IV. - ZERO-ORDER AND FIRST-ORDER APPROXIMATIONS

The performance of an Ethernet with a small propagation delay can be approximated by assuming a zero propagation delay. This last case is much easier to analyse since collisions are now impossible. One does not have to consider one station in isolation and model the iteraction between stations by extra parameters.

The state of the full system (N stations) can be described by a pair of integers (i,n), where i=0,1 is the number of transmissions in progress and n=0,1,...,N-1 is the number of stations which, having attempted to transmit, have found the channel busy and have backed off. Let $\pi_{i,n}$ be the steady-state probability of state (i,n). These probabilities satisfy the following system of balance equations :

$$\pi_{o,n}[(N-n)\lambda+n\lambda'] = \pi_{1,n}, \quad n=0,1,\ldots,N-1, \tag{25}$$

$$\pi_{1,n}[1+(N-n-1)\lambda] = (N-n)\lambda\pi_{o,n} + (n+1)\lambda'\pi_{o,n+1} + (N-n)\lambda\pi_{1,n-1}, \quad n=0,1,\ldots,N-1$$

where $\pi_{o,N} = \pi_{1,-1} = 0$ by definition.

It is not very diffixult to verify that the solution to (25) is given by

$$\pi_{1,n} = \left[\binom{N-1}{n}(\lambda/\lambda')^n \prod_{j=1}^{n}\sigma_j\right]\pi_{1,o}, \quad n=1,2,\ldots,N-1 \tag{26}$$

$$\pi_{o,n} = (1/\sigma_n)\pi_{1,n}, \quad n=0,1,\ldots N-1,$$

where $\sigma_j = (N-j)\lambda + j\lambda'$, j=0,1,...,N-1. The probability $\pi_{1,o}$ is determined from the condition that all probabilities must sum up to 1 :

$$\pi_{1,o} = \left\{\frac{\sigma_o+1}{\sigma_o} + \sum_{n=1}^{N-1}\left[\binom{N-1}{n}(\lambda/\lambda')^n \left(\prod_{j=1}^{n-1}\sigma_j\right)(\sigma_n+1)\right]\right\}^{-1} \tag{27}$$

The total throughput of packets is equal to the probability that there is a transmission in progress :

$$(28) \qquad S = G = \sum_{n=0}^{N-1} \pi_{1,n}$$

The throughput per station is, of course, $s = g = S/N$. The average response time for a packet is, as before,

$$(29) \qquad W = \frac{1}{s} - \frac{1}{\lambda} = \frac{N}{G} - \frac{1}{\lambda} .$$

In the special case when $\lambda=\lambda'$, relations (26) - (29) simplify considerably. We have then

$$S = G = \frac{N\lambda}{N\lambda+1}$$

and

$$W = N .$$

It is rather remarkable that in that case W is independent of λ.

Let us return now to our model with non-zero transmission delay D. Treating $s = \pi_T$ as a function of D, a first order approximation would be of the form

$$(30) \qquad \pi_T(D) \approx \pi_T(0) + D \frac{d\pi_T(o)}{dD} .$$

Here, $\pi_T(o)$ is obtained from the above analysis : $\pi_T(o) = S/N$, where S is given by (28). To find $d\pi_T(o)/dD$, one can take derivatives in (4), (13), (14), (15), (18) and (21) with respect to D at D=0. This gives another system of equations for $d\pi_T(o)/dD$, $d\tilde{g}(o)/dD$, $dF(o)/dD$, $dF'(o)/dD$, $db(o)/dD$ and $db'(o)/dD$.

The approximation (30) has the advantage that the first term, $\pi_T(o)$, is exact. The decoupling of stations is only applied in order to obtain the second term.

V. - CONCLUSION

An exact analysis of a general Ethernet network with finite number of stations appears to be intractable. What we have presented here is an approximate method which reduces the complexity of the problem by considering the behaviour of one station in isolation ; the effect of all other stations is represented by a few parameters. The method can be applied in the general case ; when the transmission delay is small, zero and first order approximations are available.

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