The fluid-dynamic limit of a model Boltzmann equation in the presence of a shock

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RESUME :

L'équation de Broadwell est le modèle le plus simple de l'équation de Boltzmann pour lequel les équations de la dynamique des fluides sont non triviales.

Et il a une théorie complète pour l'existence de solution de cette équation avec valeur initiale donnée. Ici nous montrons formellement que si les équations de la dynamique des fluides a une solution . Alors la solution de Broadwell converge vers celle-ci quand le libre parcours moyen tend vers 0. Cette limit est valable même s'il y a un choc dans le flux, bien que dans une fine couche limite le long du choc la convergence n'a pas lieu. Une donnée régulière conduit à l'existence d'une couche initiale qui n'entre cependant pas en interaction avec la couche limite de choc.

ABSTRACT :

The Broadwell equation is the simplest model of the Boltzmann equation of kinetic theory for which the corresponding model fluid dynamic equations are non trivial. For this equation there is a complete existence theory for the initial value problem. Here we show formally that if the model fluid dynamic equations can be solved, the the Broadwell solution asymptotically converges to the fluid dynamic solution as the mean free path goes to zero. This limit is valid even if there is a shock in the fluid flow, although there is a thin shock layer in which the convergence does not hold. Arbitrary smooth initial data is allowed, which leads to a short initial layer of non-convergence, but the initial and shock layers do not interact due to the assumed initial smoothness.

I. INTRODUCTION

The Broadwell model of gas kinetics [1] describes a gas as composed of particles of only six speeds with a binary collision law and spatial variation in only one direction. It leads to a relatively simple model Boltzmann equation (called the Broadwell equation) and to non-trivial fluid dynamic (Euler) equations. There is an asymptotic equivalence between these two equations at small mean free path, which was proved by Caflisch and Papanicolaou [2] when the fluid flow is smooth. The purpose of this paper is to demonstrate formally that even if the fluid flow includes a shock this fluid-dynamic approximation is still valid, although there is a thin shock layer in which the convergence is non-uniform.

From the Broadwell equation a smooth profile for a steady plane shock was found explicitly by Gatignol [6], Broadwell [1], and Caflisch [3]. The key to the present analysis of a time dependent shock is that to leading order the time dependent shock profile has exactly the shape of a steady shock profile with paremeters that change in time.

There is a complete existence theory for the initial value problem for the Broadwell equation developed by Inoue and Nishida [9] and Tartar [15]. The classical Boltzmann equation and its fluid-dynamic limit has been investigated by Grad [7, 8], Ellis and Pinsky [5], Nishida [14], Kawashima , Matsumura and Nishida [10] and Caflisch [4] and the corresponding steady plane shock solution was found by Nicolaenko [13]. The fluid-dynamic limit of the Carleman model was analyzed by Kurtz [11] and McKean [12].

After the Broadwell and model Euler equations are introduced in section 2 and 3, the fluid dynamic limit is described in section 4. The Hilbert expansion (or outer expansion) is developed in section 5 and the shock layer expansion in section 6, but the proof that the corresponding equations can be solved is in section 7. The initial layer expansion is presented in section 8.

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II. THE BROADWELL EQUATION

Broadwell [1] proposed a simplified Boltzmann equation which describes the evolution of a gas composed of particles all of speed c and each moving in one of the six directions parallel to one of the three spatial axes . If the gas is assumed to have spatial variation in only the x direction and to be constant in the y and z directions, particles moving in the latter two directions have effective velocity o and the model describes a gas consisting of 3 species of particles with velocities +c,o and -c. The density function for each species is written respectively as f_+, f_o, f_-, and the equations for the evolution of the svstem are :

$$(2.1)\quad \begin{cases} \left(\frac{\partial}{\partial t} + c\,\frac{\partial}{\partial x}\right) f_+ = f_o^2 - f_+ f_- \\ \frac{\partial}{\partial t} f_o = -\frac{1}{2}\left(f_o^2 - f_+ f_-\right) \\ \left(\frac{\partial}{\partial t} - c\,\frac{\partial}{\partial x}\right) f_- = f_o^2 - f_+ f_- \end{cases}$$

The terms on the left side of this system represent the streaming of particles ; those on the right represent binary collisions between particles.

We introduce the following vector notation :

$$(2.2)\quad \begin{cases} f = \begin{pmatrix} f_+ \\ f_o \\ f_- \end{pmatrix} \qquad V = \begin{pmatrix} c & o & o \\ o & o & o \\ o & o & -c \end{pmatrix} \\ \bar{Q}(f,g) = f_o g_o - \frac{1}{2}(f_+ g_- + f_- g_+) \\ Q(f,g) = \bar{Q}(f,g)\begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix} \\ D = \frac{\partial}{\partial t} + V\,\frac{\partial}{\partial x}\,, \end{cases}$$

and rewrite (2.1) as :

$$(2.3) \qquad Df = Q(f,f).$$

We will now describe the basic properties of (2.3) which can be found in [2] and all of which are analogous to properties of the classical Boltzmann equation.

A density function f describes a gas only if $f_i \geq o$, for $i = +, o, -$. In fact we will assume throughout that each $f_i > o$. Equation (2.3) preserves positivity, i.e. if $f_i(t = o, x) > o$ for all i and x and if $f(t,x)$ solves (2.3), then $f_i(t,x) > o$ for all i and x.

The equilibrium states $\omega = \begin{pmatrix} \omega_+ \\ \omega_o \\ \omega_- \end{pmatrix}$ for the collision process satisfy $Q(\omega, \omega) = o$, i.e.

$$(2.4) \qquad \omega_o^2 = \omega_+ \omega_- .$$

They are called local Maxwellians and will plays a basic role in the following theory. Each local Maxwellian can be represented in a unique way as

$$(2.5) \qquad \begin{cases} \omega_+ = \dfrac{\rho}{2c^2} (F(u) + cu) \\ \omega_o = \dfrac{\rho}{4c^2} (c^2 - F(u)) \\ \omega_- = \dfrac{\rho}{2c^2} (F(u) - cu) \end{cases}$$

in which ρ and u are any functions of x and t satisfying $\rho > o$, $|u| < c$ (in order to insure positivity) and

$$(2.6) \qquad F(u) = \frac{1}{3} c^2 \{2(1 + 3u^2/c^2)^{1/2} - 1\}.$$

During a collision the number and total momentum of the particles is conserved. If we denote

$$(2.7) \qquad \begin{cases} \psi_1 = (1,\ 4,\ 1) \\ \psi_2 = (c,\ o,\ -c) \end{cases}$$

and use $<,>$ for the usual vector inner product, these conservation law are written respectively as

$$(2.8) \qquad <\psi_i,\ Q(f,h)> \ = o, \text{ for } i = 1,\ 2,$$

for any density functions f,h. Define corresponding moments of f by

$$(2.9) \qquad \begin{cases} \rho_f = f_+ + 4f_o + f_- = <\psi_1,\ f> \\ \rho_f u_f = m_f = c(f_+ - f_-) = <\psi_2,\ f> . \end{cases}$$

If (2.8) is used in (2.3) we see that a solution of (2.3) must satisfy

$$(2.10) \qquad \begin{cases} \dfrac{\partial}{\partial t} \rho_f + \dfrac{\partial}{\partial x} m_f = o \\ \dfrac{\partial}{\partial t} m_f + \dfrac{\partial}{\partial x} c^2(f_+ + f_-) = o. \end{cases}$$

Now let f be a fixed positive distribution function. The linearized collision operator L_f is defined by

$$(2.11) \qquad L_f = 2\ Q(f,.) ,$$

and has the matrix representation

$$(2.12) \qquad L_f = \begin{pmatrix} -f_- & 2f_o & -f_+ \\ \frac{1}{2}f_- & -f_o & \frac{1}{2}f_+ \\ -f_- & 2f_o & -f_+ \end{pmatrix}$$

Zero is a double eigenvalue of L_f and $-(f_+ + f_o + f_-)$ is the third one. Corresponding to (2.8) we choose for left eigenvectors ψ_1, ψ_2 and

$$(2.13) \qquad \psi_3 = \psi_3^f = (f_+ + f_o + f_-)^{-1} (f_-, -2f_o, f_+).$$

The dual basis of right eigenvectors is

$$(2.14) \begin{cases} \phi_1^f = (f_+ + f_o + f_-)^{-1} \begin{pmatrix} \frac{1}{2}f_o \\ \frac{1}{4}(f_+ + f_-) \\ \frac{1}{2}f_o \end{pmatrix} \\ \\ \phi_2^f = (f_+ + f_o + f_-)^{-1} \begin{pmatrix} \frac{1}{2c}(2f_+ + f_o) \\ \frac{1}{4c}(f_- - f_+) \\ -\frac{1}{2c}(2f_- + f_o) \end{pmatrix} \\ \\ \phi_3 = \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix} \end{cases}$$

We denote by P_f the projection operator into the null space of L_f. It has the explicit form

$$(2.15) \qquad P_f = \phi_1^f \, \psi_1 + \phi_2^f \, \psi_2,$$

in which the products on the right are tensor products between vectors resulting in 3 X 3 matrices. From (2.9) we see that

$$(2.16) \qquad P_f h = \rho_h \, \phi_1^f + m_h \, \phi_2^f$$

Given a vector h, consider the linear system

$$(2.17) \qquad L_f u = h.$$

From the Fredholm alternative, (2.17) can be solved if and only if $P_f h = o$, and then the solution is unique up to addition of an element in the nullspace. We denote the inverse of L_f in the range of $I - P_f$ by K_f, such that $P_f K_f = o$. Explicitly if

$$(2.18) \qquad h = (\psi_3^f, h)\ \phi_3 = \tilde{h} \begin{pmatrix} 1 \\ -\ 1/2 \\ 1 \end{pmatrix}$$

it follows that

$$(2.19) \qquad K_f h = L_f^{-1}(I - P_f)h = \frac{-\tilde{h}}{(f_+ + f_o + f_-)} \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix}.$$

III. THE MODEL EULER EQUATIONS

The Euler equations of fluid dynamics for the model gas are derived by assuming that the gas is in a state of equilibrium for the collision process, i.e. that f is a local Maxwellian. Then $c^2(f_+ + f_-) = \rho_f\, F(u_f)$ and the conservation laws (2.10) become

$$(3.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}\,\rho + \dfrac{\partial}{\partial x}\, m = o \\[2ex] \dfrac{\partial}{\partial t}\, m + \dfrac{\partial}{\partial x}\, \rho F(\dfrac{m}{\rho}) = o \end{cases}$$

These model Euler equations give an alternate description of the gas in terms of its macroscopic density ρ and momentum m.

These equations have been studied in [3] and [6], and we will only list some of their properties. Let ρ, m be a solution of (3.1) with $\rho(t{=}o)>o$ and $|u(t{=}o)| = |m/\rho\,(t{=}o)| < c$, to be consistent with (2.9), then

$$(3.2)\qquad \rho(t) > o, \quad |u(t)| < c, \text{ for } t \geq o.$$

The system (3.1) is strictly hyperbolic and genuinely nonlinear with characteristic speeds λ and ν given by

$$(3.3)\qquad \lambda = 2\,\frac{u + F(u)^{1/2}}{\frac{3}{c^2}\,F(u) + 1} \qquad\qquad \nu = 2\,\frac{u - F(u)^{1/2}}{\frac{3}{c^2}\,F(u) + 1}$$

which satisfy the bounds

$$(3.4)\qquad - c < \nu < o < \lambda < c, \text{ if } |u| < c.$$

Shock solutions of these equations satisfy the Rankine-Hugoniot jump conditionns

$$(3.5)\qquad \begin{cases} \rho^{\ell}(u^{\ell} - s) = \rho^{r}(u^{r} - s) = a \\ \rho^{\ell}(F(u^{\ell}) - u^{\ell} s) = \rho^{r}(F(u^{r}) - u^{r}s) = b \end{cases}$$

in which (ρ^{ℓ}, u^{ℓ}) and (ρ^{r}, u^{r}) denote the limits from the left and right respectively of (ρ,u) at the shock and s is the shock speed. There is also an entropy condition

$$(3.6)\qquad u^{r} < u^{\ell} \quad \text{or } s(\rho^{\ell} - \rho^{r}) > o$$

which is equivalent to the condition that either

$$(3.7)\qquad \lambda^{r} < s < \lambda^{\ell} \quad \text{or} \quad \nu^{r} < s < \nu^{\ell}$$

We will assume that (3.1) has a solution ρ_o, m_o satisfying the following :

Condition S

For $o \le t \le T$, the fluid state $(\rho_o(x,t), m_o(x,t))$ has a single shock which starts at $t = t_1 > o$ and moves along the curve $x = p(t)$, with $p'(t) = s(t)$. The fluid state is smooth away from the shock and tangentially to the shock ; across the shock it satisfies (3.5). The shock curve $x = p(t)$ is smooth and starts off in a characteristic direction; to be specific we assume that

$$(3.8)\qquad s(t_1) = \lambda(t_1)$$

It follows that the shock has zero strength at $t = t_1$, i.e. $u_o^{r}(t_1) = u_o^{\ell}(t_1)$ and $\rho_o^{r}(t_1) = \rho_o^{\ell}(t_1)$, and that

$$(3.9)\qquad o < \lambda^{r}(t) \le s(t) \le \lambda^{\ell}(t) \text{ for } t > t_1.$$

Next we study the linearized inhomogeneous Euler equations :

$$(3.10)\quad \begin{cases} \dfrac{\partial}{\partial t}\rho_1 + \dfrac{\partial}{\partial x} m_1 = o \\[2ex] \dfrac{\partial}{\partial t} m_1 + \dfrac{\partial}{\partial x}\left((F(u_o) - u_o F'(u_o))\,\rho_1 + F'(u_o) m_1\right) = \dfrac{\partial}{\partial x} w_1 \end{cases}$$

in which the inhomogeneity w_1 will be a function of ρ_o, $u_o = m_o/\rho_o$ and their derivatives. The functions ρ_1 and u_1 can be discontinuous along $x = p(t)$ and must satisfy the following linearized inhomogeneous Rankin-Hugoniot conditions

$$(3.11)\quad \begin{cases} [-s\,\rho_1 + m_1] = \alpha_1 \\[2ex] [-s\,m_1 + (F(u_o) - u_o\,F'(u_o))\,\rho_1 + F'(u_o)\,m_1 + w_1] = \beta_1 \end{cases}$$

in which the brackets [ ] denote the jump across the shock, and α_1 and β_1 will come from the inner expansion at the shock.

The system (3.10) can be diagonalized as

$$(3.12)\quad \frac{\partial}{\partial t}\begin{pmatrix}\theta_1\\ \eta_1\end{pmatrix} + N\frac{\partial}{\partial x}\begin{pmatrix}\theta_1\\ \eta_1\end{pmatrix} = A^{-1}\begin{pmatrix}o\\ w_1\end{pmatrix} - A^{-1}\left(\frac{\partial}{\partial t}A + \frac{\partial}{\partial x}(A\,N)\right)\begin{pmatrix}\theta_1\\ \eta_1\end{pmatrix}$$

in which

$$N = \begin{pmatrix}\lambda & o\\ o & \nu\end{pmatrix} \qquad A = \begin{pmatrix}1 & 1\\ \lambda & \nu\end{pmatrix}$$

$$(3.13)\quad \begin{pmatrix}\theta_1\\ \eta_1\end{pmatrix} = A\begin{pmatrix}\rho_1\\ m_1\end{pmatrix}$$

The jump condition (3.11) becomes

$$(3.14)\quad (-s\,A + AN)\begin{pmatrix}\theta_1\\ \eta_1\end{pmatrix} + \begin{bmatrix}o\\ w_1\end{bmatrix} = \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}$$

The system (3.12) has the same characteristics as (3.1) with slopes λ and ν. It follows from (3.9) that the λ^r, λ^ℓ, and ν^r characteristics are incoming at the shock while the ν^ℓ characteristic is outgoing (cf. Figure 1).

Thus $\theta_1^r, \theta_1^\ell$ and η_1^r will be determined by integrating along the characteristics and η_1^ℓ can be chosen to solve (3.14) (here θ_1^r, θ_1^ℓ, η_1^r, η_1^ℓ denote the limits of θ_1 and η_1 from each side of the shock as before). Since there are two equation in (3.14) this leads to a solveability condition on α_1 and β_1 which is found after eliminating η_1^ℓ to be

$$(3.15)\quad \begin{cases} \nu^\ell \alpha_1 - \beta_1 = (\nu^\ell - \lambda^r)(s - \lambda^r)\, \theta_1^r + (\nu^\ell - \nu^r)(s - \nu^r)\, \eta_1^r - \\ \qquad - (\nu^\ell - \lambda^\ell)(s - \lambda^\ell)\, \theta_1^\ell - [w_1]. \end{cases}$$

The remaining equation for η_1^ℓ can be rewritten as

$$(3.16)\quad \begin{cases} \eta_1^\ell = (s - \nu^\ell)^{-2} \ \{(s\,\alpha_1 - \beta_1) - (s - \lambda^r)^2\, \theta_1^r - (s - \nu^r)^2\, \eta_1^r + \\ \quad + (s - \lambda^\ell)^2\, \theta_1^\ell + [w_1]\} \end{cases}$$

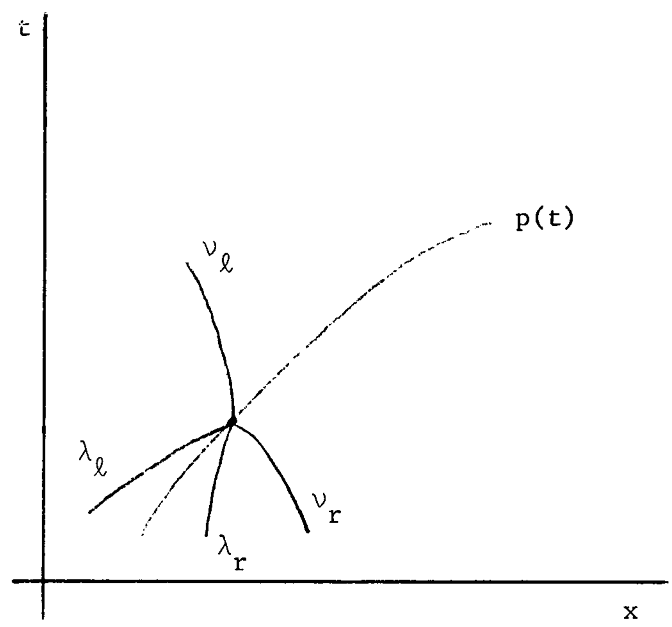

Figure 1 : Shock curve with characteristics :

$\nu_\ell < 0$, $\quad \nu_r < 0$, $\quad 0 < \lambda_r < s < \lambda_\ell$

IV. THE FLUID-DYNAMIC LIMIT

The model Boltzmann equation (2.3) can be rewritten in dimensionless form as

(4.1) $$Df^{\varepsilon} = \frac{1}{\varepsilon} Q(f^{\varepsilon}, f^{\varepsilon})$$

(4.2) $$f^{\varepsilon}(t = o, x) = \bar{f}(x)$$

in which the one parameter ε is the mean free time between collisions. The fluid dynamic limit which relates (4.1) and (3.1) is the limit $\varepsilon \to o$.

Let

(4.3) $$\bar{\rho}(x) = < \psi_1, \bar{f}(x) >, \bar{m}(x) = < \psi_2, \bar{f}(x) >$$

and suppose that $(\rho_o(x,t), m_o(x,t))$ is a solution of (3.1) satisfying condition S with

(4.4) $$\rho_o(t = o) = \bar{\rho}, m_o(t = o) = \bar{m}$$

Let $\omega(x,t)$ be the local Maxwellian density associated with ρ_o and m_o as in (2.5). Then we will construct an expansion for f^{ε} in powers of ε, so that f^{ε} is a formal solution of (4.1) and formally satisfies

(4.5) $$\sup_{\substack{o<\delta\leq t\leq T \\ |p(t)-x|\geq\delta>o}} |f^{\varepsilon}(x,t) - \omega(x,t)| \leq \varkappa \varepsilon ,$$

in which the constant $\varkappa$ will depend on δ.

The solution f^{ε} will be the sum of three terms -an outer expansion (the Hilbert expansion) f^H, a shock layer expansion f^s, and an initial layer expansion f^I- written as

$$(4.6) \qquad f^{\varepsilon}(x,t) = f^{H}(x,t) + f^{S}\left(y = \frac{x - p(t)}{\varepsilon}\, t\right) + f^{I}\left(x,\ \tau = \frac{t}{t}\right).$$

in which $\lim\limits_{|y|\to\infty} f^{S} = \lim\limits_{\tau\to\infty} f^{I} = 0$, i.e. f^{S} is confined to a neighborhood of the shock and f^{I} to a neighborhood of $t = 0$.

The restrictions to $\delta \le t$ and $|p(t) - x| \ge \delta$ in (4.5) are due to the initial and shock layers.

We shall obtain asymptotic expansions for f^{H}, f^{S} and f^{I}, although we are unable to show that these expansions are asymptotic to an actual solution.

V. HILBERT EXPANSION WITH A SHOCK

The Hilbert expansion is

$$f^H = \omega + \varepsilon f_1 + \varepsilon^2 f_2 + \dots \tag{5.1}$$

Substituting this into (4.1) and equating the coefficients of power of ε leads to

$$o = Q(\omega,\omega) \tag{5.2}$$

$$D\omega = L_\omega f_1 \tag{5.3}$$

$$Df_1 = L_\omega f_2 + Q(f_1,f_1) \tag{5.4}$$

From (5.2) we conclude that ω is a local Maxwellian as suggested by the notation. A solution f_1 of (5.3) exists if and only if

$$< \psi_1, D\omega > = < \psi_2, D\omega > = o \tag{5.5}$$

These are exactly the model Euler equations (3.1) for

$$\rho_o = < \psi_1,\omega > , \; m_o = < \psi_2, \omega > \tag{5.6}$$

and their solution is taken to be the fluid state described in section 4, with a single shock at $x = p(t)$ and satisfying condition S.

Now from (2.19) we have

$$f_1 = \tilde{f}_1 + K_g D\omega \tag{5.7}$$

with

$$P_\omega \tilde{f}_1 = \tilde{f}_1, \tag{5.8}$$

i.e. $\tilde{f}_1$ is a purely fluid-dynamic vector and is determined by its moments, which are

(5.9) $$\begin{cases} \rho_1 = \langle\psi_1, \tilde{f}_1\rangle = \langle \psi_1, f_1 \rangle \\ m_1 = \langle\psi_2, \tilde{f}_1\rangle = \langle \psi_2, f_1 \rangle . \end{cases}$$

These two quantities are determined by the solveability condition for (5.4), which is

(5.10) $$p_\omega D f_1 = p_\omega D \tilde{f}_1 + p_\omega D K_\omega (I - p_\omega) D\omega = o$$

It was shown in [2] that

(5.11) $$c^2(f_{1+} + f_{1-}) = (F(u_o) - u_o F'(u_o))\, \rho_1 + F'(u_o)\, m_1 - \mu(u_o) \frac{\partial}{\partial x} u_o$$

and that (5.10) are exactly the following inhomogeneous linearized Euler equations

(5.12) $$\begin{cases} \frac{\partial}{\partial t} \rho_1 + \frac{\partial}{\partial x} m_1 = o \\ \frac{\partial}{\partial t} m_1 + \frac{\partial}{\partial x} \{(F(u_o) - u_o F'(u_o))\, \rho_1 + F'(u_o) m_1\} = \frac{\partial}{\partial x} \left(\mu(u_o) \frac{\partial}{\partial x} u_o \right), \end{cases}$$

in which

(5.13) $$\mu(u) = 2 \frac{c^2 - F(u)}{(1 + 3u^2/c^2)^{3/2}} .$$

The jump conditions for ρ_1 and m_1 at $x = p(t)$ are taken to be (3.11) with $w_1 = \mu(u_o) \frac{\partial}{\partial x} u_o$; α_1 and β_1 will be determined later. Using (5.11) the jump conditions can be rewritten as

$$(5.14)\quad \begin{cases} [-\, s(f_{1+} + 4f_{1o} + f_{1-}) + c(f_{1+} - f_{1-})] = \alpha_1 \\ [-\, sc(f_{1+} - f_{1-}) + c^2(f_{1+} + f_{1-})] = \beta_1 \end{cases}$$

This procedure is continued to find f_i for $i \geq 2$ in the obvious way.

VI. THE SHOCK EXPANSION

The shock expansion is

$$(6.1) \qquad f^S(x,t) = g_o(y,t) + \varepsilon g_1(y,t) + \varepsilon^2 g_2(y,t) + \dots$$

with

$$(6.2) \qquad y = \frac{x - p(t)}{\varepsilon}$$

The equation for g_i are found by supposing that $f^H + f^S$ is a continuous solution of (4.1). After substituting (5.1) and (6.1) into (4.1) and matching powers of ε, the terms containing only ω or f_i can be eliminated using the Hilbert expansion equations. To simplify the calculations we replace ω and f_i by their Taylor series expansion in x around $x = p(t)$ which must be done separately for $x < p(t)$ and $x > p(t)$, and we denote

$$(6.3) \qquad \begin{cases} \omega^r = \omega(x = p(t) +) & \omega^\ell = \omega(x = p(t) -) \\ \dfrac{\partial \omega^r}{\partial x} = \dfrac{\partial \omega}{\partial x}(x = p(t) +) & \dfrac{\partial \omega^\ell}{\partial x} = \dfrac{\partial \omega}{\partial x}(x = p(t) -) \\ \text{etc. for } \dfrac{\partial^n \omega}{\partial x^n} \text{ and } f_i . & \end{cases}$$

The resulting equations are

$$(6.4) \qquad (V - sI)\frac{\partial}{\partial y} g_o = \begin{cases} Q(g_o + 2\omega^\ell, g_o), & y < o \\ Q(g_o + 2\omega^r, g_o), & y > o \end{cases}$$

$$(6.5) \qquad (V - sI)\frac{\partial}{\partial y} g_1 = \begin{cases} 2Q(g_o + \omega^\ell, g_1) + 2Q(y\dfrac{\partial \omega^r}{\partial x} + f_1^\ell, g_o) - \dfrac{\partial}{\partial t} g_o, & y < o \\ 2Q(g_o + \omega^r, g_1) + 2Q(y\dfrac{\partial \omega^r}{\partial x} + f_1^r, g_o) - \dfrac{\partial}{\partial t} g_o, & y > o \end{cases}$$

We look for a solution satisfying :

$(6.6.)_a$ Each sum $\omega + g_o$, $f_1 + g_1$, $f_2 + g_2$ is continuous

$(6.6)_b$ For $t \le t_1, g_o = g_1 = \dots = o$

$(6.6)_c$ As $y \to \pm\infty, g_o \to o,\ g_1 \to o, \dots$

The first condition implies that each g_i is continuous except at $x = p(t)$; the last condition is to insure that the shock layer is confined to a small region around the shock.

Denote

$$(6.7) \qquad k = \begin{cases} g_o + \omega^{\ell}, & y < o \\ g_o + \omega^{r}, & y > o \end{cases}$$

Then k is continuous and solve

$$(6.8) \qquad (V - sI)\frac{\partial}{\partial y} k = Q(k,k)$$

$$(6.9) \qquad \begin{cases} k(y = \infty) = \omega^{r} \\ k(y = -\infty) = \omega^{\ell} \end{cases}$$

This system was analyzed in [3] and [6]. Applying (2.8) and (3.5) to (6.8) and (6.9) yields the following identities for $\rho_k = \langle\psi_1, k\rangle$ and $m_k = \langle\psi_2, k\rangle$:

$$(6.10) \qquad \begin{cases} - s\ f_k + m_k = a \\ - s\ m_k + c^2(k_+ + k_-) = b, \end{cases}$$

from which it follows that

$$(6.11) \qquad k = \begin{pmatrix} k_+ \\ k_o \\ k_- \end{pmatrix} = \Lambda\, k_o + A_o$$

with

$$(6.12) \qquad \Lambda = \begin{pmatrix} \frac{2s}{c-s} \\ 1 \\ \frac{-2s}{c+s} \end{pmatrix} \qquad A_o = \begin{pmatrix} \frac{a+b/c}{2(c-s)} \\ o \\ \frac{-a+b/c}{2(c+s)} \end{pmatrix}$$

Now we have

$$(6.13) \qquad \begin{aligned} Q(k,k) &= \{\bar{Q}(\Lambda,\Lambda)\, k_o^2 + 2\bar{Q}(\Lambda, A_o)\, k_o + \bar{Q}(A_o, A_o)\} \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix} \\ &= q(k_o) \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix} \end{aligned}$$

Since $Q(\omega^r, \omega^r) = Q(\omega^\ell, \omega^\ell) = o$, it must be that

$$(6.14) \qquad q(k_o) = \bar{Q}(\Lambda,\Lambda)(k_o - \omega_o^r)(k_o - \omega_o^\ell)$$

Therefore k_o solves

$$(6.15) \qquad - s \frac{\partial}{\partial y} k_o = - \frac{1}{2} q\, (k_o),$$

which has the solution

$$(6.16) \qquad k_o(y) = - \frac{1}{2}(\omega_o^\ell - \omega_o^r) \tanh\, (\tfrac{1}{4s}\, \bar{Q}(\Lambda,\Lambda)(\omega_o^\ell - \omega_o^r)(y + y_o)) + \frac{1}{2}(\omega_o^\ell + \omega_o^r)$$

Since

$$(6.17) \qquad \bar{Q}(\Lambda,\Lambda) = \frac{c^2 + 3s^2}{c^2 - s^2} \qquad \omega_o^\ell - \omega_o^r = \frac{1}{4c^2}(c^2 - s^2)(\rho_o^\ell - \rho_o^r),$$

we see that k satisfies (6.9) if and only if the entropy condition (3.6) is true, or equivalently if and only if

(6.18) $$s(\omega_o^{\ell} - \omega_o^{r}) > o$$

The shift parameter $y_o = y_o(t)$ may depend on time and will be chosen later.

Apply (2.8) to (6.5) and integrate once using $(6.6)_c$ to obtain

$$\begin{pmatrix} -s(g_{1+} + 4g_{1o} + g_{1-}) + c(g_{1+} - g_{1-}) \\ -sc(g_{1+} - g_{1-}) + c^2(g_{1+} + g_{1-}) \end{pmatrix} =$$

(6.19)
$$= \begin{cases} -\int\limits_{-\infty}^{y} \frac{\partial}{\partial t} \begin{pmatrix} g_{0+} + 4g_{0o} + g_{0-} \\ c(g_{0+} - g_{0-}) \end{pmatrix}(z,t)\, dz, & y < o \\ \int\limits_{y}^{\infty} \frac{\partial}{\partial t} \begin{pmatrix} g_{0+} + 4g_{0o} + g_{0-} \\ c(g_{0+} - g_{0-}) \end{pmatrix}(z,t)\, dz, & y > o \end{cases}$$

This can be rewritten as in (6.11) as

(6.20) $$g_1 = \Lambda\, g_{1o} + A_1$$

with

$$A_1 = \begin{cases} \displaystyle\int_{-\infty}^{y} b_1(z)\, dz, & y < o \\ \displaystyle -\int_{y}^{\infty} b_1(z)\, dz, & y > o \end{cases}$$

(6.21)

$$b_1(z) = \begin{pmatrix} \dfrac{2c}{(c-s)^2} \\ o \\ -\dfrac{2c}{(c-s)^2} \end{pmatrix} \frac{\partial}{\partial t} g_o(t,z)$$

Then (6.5) can be reduced to a single linear first order equation for g_{1o}. There is a complicated inhomogeneity which we abbreviate as $\bar{Q}_1$, and then

$$(6.22) \qquad -s \frac{\partial}{\partial y} g_{1o} = -\bar{Q}(k, g_1) + \bar{Q}_1$$

Now

$$(6.23) \qquad \bar{Q}(k, g_1) = \bar{Q}(\Lambda k_o + A_o, \Lambda g_{1o} + A_1) = \frac{1}{2} q'(k_o) g_{1o} + \bar{Q}(A_o, A_1),$$

and (6.22) can be rewritten as

$$(6.24) \qquad \frac{\partial}{\partial y} g_{1o} = \frac{1}{2s} q'(k_o)\, g_{1o} + \bar{Q}_2$$

From (6.18), (6.9) and the quadratic nature of q, it follows that for some positive number $\bar{q}$,

$$(6.25) \qquad \begin{cases} \dfrac{1}{2s} q'(k_o) < -\bar{q} & \text{for } y \text{ large enough} \\ \dfrac{1}{2s} q'(k_o) > \bar{q} & \text{for } y \text{ large enough.} \end{cases}$$

Also the inhomogeneous term Q_2 is a linear functional of g_o and $\frac{\partial}{\partial t} g_o$

which is decaying exponentially to o at $y = \pm\infty$. Therefore g_{1o} and also g_{1+} and g_{1-} are decaying exponentially to o at $y = \pm\infty$. This is true for any values of g^r_{1o} and g^ℓ_{1o}, the right and left limits of g_{1o} at $y = o$.

Next we check the continuity of $f_1 + g_1$. First we require that

$$(6.26) \qquad g^r_{1o} - g^\ell_{1o} = -(f^r_{1o} - f^\ell_{1o})$$

This still leaves one value, say g^r_{1o} undetermined. The continuity for the other two components is checked by first writing the following jump condition from (6.19)

$$(6.27) \qquad \begin{bmatrix} -s(g_{1+} + 4g_{1o} + g_{1-}) + c(g_{1+} - g_{1-}) \\ -sc(g_{1+} - g_{1-}) + c^2(g_{1+} + g_{1-}) \end{bmatrix} = \int_{-\infty}^{\infty} \frac{\partial}{\partial t} \begin{pmatrix} g_{0+}+4g_{0o}+g_{0-} \\ c(g_{0+}-g_{0-}) \end{pmatrix} (y,t)\,dy$$

Comparison with (5.14) shows that we must take

$$(6.28) \quad \begin{cases} \alpha_1 = -\displaystyle\int_{-\infty}^{\infty} \frac{\partial}{\partial t} (g_{0+} + 4g_{0o} + g_{0-})\ (y,t)\,dy \\[2ex] \beta_1 = -\displaystyle\int_{-\infty}^{\infty} \frac{\partial}{\partial t}\, c(g_{0+} - g_{0-})(y,t)\,dy \end{cases}$$

From (6.13) these can be calculated explicitly to be

$$(6.29) \quad \begin{cases} \alpha_1(t) = -\,y_o'(t)\ (\rho^\ell(t) - \rho^r(t)) - y_o(t)(\rho^{\ell\prime}(t) - \rho^{r\prime}(t)) \\[1ex] \beta_1(t) = s(t)\ \alpha_1(t) - s'(t)\ y_o(t)\ (\rho^\ell(t) - \rho^r(t)). \end{cases}$$

Finally from (6.26) and (6.24) we see that we can write the dependence of g_{1o} on the (so far) undetermined value g^r_{1o} as

$$g_{1o}(y) = e^{-\gamma(y)}\, g_{1o}^{r} + \tilde{g}_{1o}(y) \tag{6.30}$$

in which

$$\gamma(y) = \int_{o}^{y} \frac{1}{2s}\, q'(k_o(z))\,dz \tag{6.31}$$

and $\tilde{g}_{1o}$ is known in terms of f_1, ω and g_0 and is decaying exponentially to o at $y = \pm\infty$.

A similar procedure is used to solve for g_1. We will only display the result that the inhomogeneities in the jump condition are

$$\begin{aligned} \alpha_2 &= \frac{d}{dt}(\bar{\gamma}\, g_{1o}^{r}) + \frac{d}{dt}\left(\int_{-\infty}^{\infty} <\psi_1,\ \Lambda\tilde{g}_{1o} + A_1> \quad dy \right) \\ \beta_2 &= \frac{d}{dt}(s\ \bar{\gamma}\ g_{1o}^{r}) + \frac{d}{dt}\left(\int_{-\infty}^{\infty} <\psi_2,\ \Lambda g_{1o} + A_1> \quad dy \right) \end{aligned} \tag{6.32}$$

in which

$$\bar{\gamma}(t) = \frac{4c^2}{s^2-c^2} \int_{-\infty}^{\infty} e^{-\gamma(y)}\, dy. \tag{6.33}$$

VII. SOLUTION OF THE LINEARIZED EULER EQUATION WITH A SHOCK

In this section we solve the linearized Euler equations (3.12) with the jump conditions (3.15) and (3.16) with α_1 and β_1 given by (6.29). First we define the characteristic curves $X_1(t,\bar{X})$ and $X_2(t,\bar{X})$ starting from $\bar{X}$, which satisfy

$$(7.1)\qquad \begin{cases} \dfrac{d}{dt} X_1(t,\bar{X}) = \lambda \qquad \dfrac{d}{dt} X_2(t,\bar{X}) = \nu \\ \qquad X_1(o,\bar{X}) = X_2(o,\bar{X}) = \bar{X} \end{cases}$$

Denote by $\bar{t}(\bar{X})$ the time at which the characteristic curve $X_2(t,\bar{X})$ crosses the shock, so that $X_2(\bar{t}(\bar{X}),\bar{X}) = p(\bar{t}(\bar{X}))$. Using (6.29) the equations (3.12), (3.15), and (3.16) can be written as

$$(7.2)\qquad \begin{cases} \dfrac{d}{dt}\theta_1(X_1,t) = a_1\,\theta_1 + a_2\,\eta_1 + b_1 \\ \dfrac{d}{dt}\eta_1(X_2,t) = a_3\,\theta_1 + a_4\,\eta_1 + b_2 \\ \eta_1^{\ell} = a_5\,\theta_1^{r} + a_6\,\eta_1^{r} + a_7\,\theta_1^{\ell} + a_8\,y_o + b_3, \text{ at } t = \bar{t}(\bar{X}) \\ \dfrac{d}{dt} y_o = a_9\,\theta_1^{r} + a_{10}\,\eta_1^{r} + a_{11}\,\theta_1^{\ell} + a_{12}\,\eta_1^{\ell} + a_{13}\,y_o + b_4 \end{cases}$$

in which a_i and b_i are functions of ρ_o, m_o, and their derivatives. In particular there are constants λ_1 and γ_1 so that

$$(7.3)\qquad |a_i| \le \gamma_1 \;.\; |b_i| \le \gamma_1\, e^{\lambda_1 t}$$

Now we solve these equations by iteration. The $(K+1)^{st}$ iterate is chosen to be

$$
(7.4)\quad \begin{cases}
\theta_1^{(K+1)}(X_1,t) = \displaystyle\int_o^t (a_1\,\theta_1^{(K)} + a_2\,\eta_1^{(K)} + b_1)(X_1(s),s)ds + \theta_1(\bar{X},o) \\[2ex]
\eta_1^{(K+1)}(X_1,t) = \displaystyle\int_o^t (a_3\,\theta_1^{(K)} + a_4\,\eta_1^{(K)} + b_2)(X_2(s),s)ds + \eta_1(\bar{X},o) \text{ for} \\
\qquad\qquad\qquad o \le t \le \bar{t}(\bar{X}) \\[2ex]
\eta_1^{\ell(K+1)} = a_5\,\theta_1^{r(K+1)} + a_6\,\eta_1^{r(K+1)} + a_7\,\theta_1^{\ell(K+1)} + a_8\,y_o^{(K+1)} + b_3 \quad \text{at} \\
\qquad\qquad\qquad t = \bar{t}(\bar{X}) \\[2ex]
\eta_1^{(K+1)}(X_1,t) = \displaystyle\int_{\bar{t}}^t (a_3\,\theta_1^{(K)} + a_4\,\eta_1^{(K)} + b_2)(X_2(s),s)ds + \eta_1^{\ell(K+1)}(X_1(\bar{t}),\bar{t}) \\
\qquad\qquad\qquad \text{for } \bar{t} \le t \\[2ex]
y_o^{(K+1)}(t) = \displaystyle\int_o^t (a_9\,\theta_1^{r(K)} + a_{10}\,\eta_1^{r(K)} + a_{11}\,\theta_1^{\ell(K)} + a_{12}\,\eta_1^{\ell(K)} + \\
\qquad + a_{13}\,y_o^{(K)} + b_4)(s)ds.
\end{cases}
$$

Suppose that for some positives constants γ_2, γ_3 and λ_2

$$
(7.5)\quad \begin{cases}
|\theta_1^{(K)}(t)| \le \gamma_3\, e^{\lambda_2 t} \\
|\eta_1^{(K)}(t)| \le \gamma_3\, e^{\lambda_2 t} \qquad \text{for } o \le t \le \bar{t} \\
|\eta_1^{(K)}(t)| \le \gamma_3\, e^{\lambda_2 t} \qquad \text{for } \bar{t} \le t \\
|\, y_o^{(K)}(t)| \le \gamma_3\, e^{\lambda_2 t} \\
|\theta_1(t = o)| < \frac{1}{2}\gamma_2 \qquad |\eta_1(t = o)| < \frac{1}{2}\,\gamma_2
\end{cases}
$$

After requiring that

$$(7.6)\quad \begin{cases} \lambda_1 \le \lambda_2, \quad \gamma_1 \le \gamma_2 \le \gamma_3 \\ \lambda_2^{-1}(5\,\gamma_2\,\gamma_3 + \gamma_2) \le \frac{1}{2}\gamma_2 \\ 4\,\gamma_2^2 + \gamma_2 \le \frac{1}{2}\gamma_3 \end{cases}$$

we can make the following estimates :

$$(7.7)\quad \begin{cases} |\theta_1^{(K+1)}(t)| \le \lambda_2^{-1}(2\,\gamma_2\,\gamma_3 + \gamma_2)\, e^{\lambda_2 t} + \frac{1}{2}\gamma_2 \le \gamma_2\, e^{\lambda_2 t} \\ |\eta_1^{(K+1)}(t)| \le \lambda_2^{-1}(2\,\gamma_2\,\gamma_3 + \gamma_2)\, e^{\lambda_2 t} + \frac{1}{2}\gamma_2 \le \gamma_2\, e^{\lambda_2 t} \quad \text{for } o \le t \le \bar{t} \\ |\eta_1^{\ell(K+1)}(t)| \le (4\,\gamma_2^2 + \gamma_2)\, e^{\lambda_2 \bar{t}} \le \gamma_3\, e^{\lambda_2 t} \\ |\eta_1^{(K+1)}(t)| \le \lambda_2^{-1}(2\,\gamma_2\,\gamma_3 + \gamma_2)\, e^{\lambda_2 t} + (4\,\gamma_2^2 + \gamma_2)\, e^{\lambda_2 t} \le \gamma_3\, e^{\lambda_2 t} \quad \text{for } \bar{t} \le t \\ |y_o^{(K+1)}(t)| \le \lambda_2^{-1}(5\,\gamma_2\,\gamma_3 + \gamma_2)\, e^{\lambda_2 t} \le \gamma_2\, e^{\lambda_2 t} \end{cases}$$

If in addition we require that

$$(7.8)\qquad \lambda_2^{-1}\gamma_2 \le \mu \qquad (3 + \gamma_2)\mu < 1,$$

then we find that

$$(7.9)\quad \begin{cases} \sup\limits_{o \le t \le T} e^{-\lambda_2 t}\, |\Delta^{K+1}\,\theta_1(t)| \le \mu \sup\limits_{o \le t \le T} e^{-\lambda_2 t}\, (|\Delta^K\,\theta_1(t)| + |\Delta^K\,\eta_1(t)|) \\ \sup\limits_{o \le t \le \bar{t}} e^{-\lambda_2 t}\, |\Delta^{K+1}\,\eta_1(t)| \le \mu \sup\limits_{o \le t \le \bar{t}} e^{-\lambda_2 t}\, (|\Delta^K\,\theta_1(t)| + |\Delta^K\,\eta_1(t)|) \\ \sup\limits_{o \le t \le T} e^{-\lambda_2 t}\, |\Delta^{K+1}\,y_o(t)| \le \mu \sup\limits_{o \le t \le T} e^{-\lambda_2 t}\, (|\Delta^K\,\theta_1(t)| + |\Delta^K\,\eta_1(t)| + \\ \qquad\qquad + |\Delta^K\,y_o(t)|) \end{cases}$$

$$
\begin{cases}
e^{-\lambda_2 \bar{t}} \, |\Delta^{K+1} \, \eta_1^{\ell}(\bar{t})| \le \gamma_2 \, e^{-\lambda_2 \bar{t}} \left(|\Delta^{K+1} \theta_1^{r}(\bar{t})| + |\Delta^{K+1} \, \eta_1^{r}(\bar{t})| + |\Delta^{K+1} \, \theta_1^{\ell}(\bar{t})| + \right. \\
\qquad \left. + \, |\Delta^{K+1} \, y_o(\bar{t})| \right) \\
\sup\limits_{\bar{t} \le t \le T} e^{-\lambda_2 t} \, |\Delta^{K+1} \, \eta_1(t)| \le \mu \sup\limits_{\bar{t} \le t \le T} e^{-\lambda_2 t} \left(|\Delta^{K} \, \theta_1(t)| + |\Delta^{K} \, \eta_1(t)| \right) + \\
\qquad + \, e^{-\lambda_2 \bar{t}} \, |\Delta^{K+1} \, \eta_1^{\ell}(\bar{t})|
\end{cases}
$$

in which $\Delta^{K+1} \, \theta_1 = \theta_1^{(K+1)} - \theta_1^{(K)}$, etc.. These can be combined to get

$$
\begin{cases}
\sup\limits_{o \le t \le T} e^{-\lambda_2 t} \left(|\Delta^{K+1} \, \theta_1| + |\Delta^{K+1} \, \eta_1| + |\Delta^{K+1} \, y_o| \right) \le \\
\qquad \le (3 + \gamma_2)\mu \sup\limits_{o \le t \le T} e^{-\lambda_2 t} \left(|\Delta^{K} \, \theta_1| + |\Delta^{K} \, \eta_1| + |\Delta^{K} \, y_o| \right).
\end{cases} \tag{7.10}
$$

Using (7.8) it follows that

$$
7.11) \qquad \theta_1^{(K)} \to \theta_1 \quad \eta_1^{(K)} \to \eta_1 \quad y_o^{(K)} \to y_o
$$

which are solutions of (7.2) satisfying the bounds (7.5).

Now that θ_1, η_1 and y_o are known, we can find f_1 and g_1 from (5.7), (6.20), and (6.30) although this does not determine g_{1o}^{r}. It is found along with θ_2 and η_2 using the linearized Euler equations and their jump conditions exactly as above, in which α_2 and β_2 are given by (6.32).

VIII. INITIAL LAYER EXPANSION

The initial layer expansion is

$$f^I(x,t) = h_o(x,\tau) + \varepsilon h_1(x,\tau) + \varepsilon^2 h_2(x,\tau) + \dots \tag{8.1}$$

with

$$\tau = \frac{t}{\varepsilon} \tag{8.2}$$

As before we ask that $f^H + f^I$ is a (formal) solution of the Boltzmann equation. Using the Hilbert expansion equations we can omit all terms involving only ω or f_i, and we replace the remaining ω of f_i terms by their Taylor series expansions in t around t = o. We also require that

$$\begin{cases} \omega(x, o) + h_o(x, o) = \bar{f}(x) \\ f_i\ (x, o) + h_i(x, o) = o \end{cases} \tag{8.3.}$$

and

$$h_i \to o \text{ as } \tau \to \infty \tag{8.4}$$

The shock does not appear until $t = t_1$ or $\tau = t_1/\varepsilon$, by which time f^I will be quite small. Thus we can ignore the effect of f^s on f^I and vice-versa. The equations for the h_i's are

$$\frac{\partial}{\partial \tau} h_o = Q(2\omega + h_o, h_o) \tag{8.5}$$

$$\frac{\partial}{\partial \tau} h_1 = 2\ Q(\omega + h_o, h_1) + 2\ Q(\tau \frac{\partial}{\partial t}\omega + f_1, h_o) - V \frac{\partial}{\partial x} h_o \tag{8.6}$$

$$\begin{cases} \frac{\partial}{\partial \tau} h_2 = 2\ Q(\omega + h_o, h_2) + 2\ Q(\tau \frac{\partial}{\partial \tau}\omega + f_1, h_1) + 2\ Q(\frac{1}{2}\tau^2 \frac{\partial^2}{\partial t^2}\omega + \\ \qquad + \tau \frac{\partial}{\partial t} f_1 + f_2, h_o) - V \frac{\partial}{\partial x} h_1 \end{cases} \tag{8.7}$$

Here we denote $\omega = \omega(t=o)$, $\frac{\partial}{\partial t}\,\omega = \frac{\partial}{\partial \tau}\,\omega(t=o)$, $f_1 = f_1(t=o)$, etc...

From (5.6), (4.3) and (4.4) we have that $< \psi_i, \omega(t=o)> \; = <\psi_i, f>$. It follows from (8.3) and (8.5) that

(8.8) $$< \psi_i, h_o(\tau = o)> \; = \; < \psi_i, h_o(\tau) > \; = o \text{ for } i = 1, 2, \quad o \leq \tau \leq T$$

Thus

(8.9) $$h_o = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} h_{0_o}$$

so that (8.5) reduces to

(8.10) $$\frac{\partial}{\partial \tau}\; h_{0o} = -\frac{1}{2}\,\{2(\,\omega_o + \omega_+ + \omega_-) - 3\, h_{0o}\}\;\; h_{0o}$$

which has the solution

(8.11) $$\left\{ \begin{aligned} h_{0o}(x,\tau) = -\frac{1}{3}\,(\,\omega_o + \omega_+ + \omega_-)\, \tanh\,(\tfrac{1}{2}(\omega_o + \omega_+ + \omega_-)(\tau + \tau_o(x)) \, + \\ + \frac{1}{3}\,(\omega_o + \omega_+ + \omega_-) \end{aligned} \right.$$

This goes exponentially to 0 as $\tau \to \infty$. The shift parameter $\tau_o(x)$ is chosen at each x so that $h_{0o}(x) + \omega_o(x) = f_o(x)$. Since

(8.12) $$o < (h_{0o} + \omega_o) + h_{0+} + \omega_+) + (h_{0-} + \omega_-) = -\,3h_{0o} + (\omega_o + \omega_+ + \omega_-)$$

then any suitable initial value of h_{0o} can be attained from (8.10) by a proper choice of τ_o.

We only briefly describe the rest of the expansion since it is exactly as in [2] (only the tanh form of h_{0o} was left out of that account). From (8.6) and (8.4) we see that

$$(8.13) \qquad < \psi_i,\ h_1(\tau) > \ = - \int_{\tau}^{\infty} < \psi_i,\ V \frac{\partial}{\partial x} h_o(s) > ds$$

This gives the initial values for ρ_1 and m_1 in (5.9) and (5.12) since (8.9) implies that

$$(8.14) \qquad \begin{cases} \rho_1(t = o) = - <\psi_1,\ h_1\ (\tau = o) > \\ m_1(t = o) = - <\psi_2,\ h_2\ (\tau = o) > \end{cases}$$

The two relations in (8.13) allow (8.6) to be reduced to a single linear first order equation for h_{1o} with a negative linear coefficient. Since the inhomogeneities in the equation decrease to o exponentially as $\tau \to \infty$, so will h_{1o}. The initial value of h_{1o} is found from the initial values of ω, ρ_1 and m_1 through (8.3.) and (5.7).

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