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A REGULARIZATION PROCESS TO IMPLEMENT SELF-ORGANIZING NEURONAL NETWORKS

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Abstract. Kohonen Self-Organizing maps are interesting computational structures because of their original properties, including adaptive topology and competition, their biological plausibility and their successful applications to a variety of real-world applications. In this paper, this neuronal model is presented, together with its possible implementation with a variational approach. We then explain why, beyond the interest for understanding the visual cortex, this approach is also interesting for making easier and more efficient the choice of this neuronal technique for real-world applications.

1 INTRODUCTION. Kohonen Self-Organizing Maps (SOM) (1,2) are among the oldest, best known and most frequently used Artificial Neural Networks (ANN) (3). There are several reasons to this success. This model, whose algorithm will be described below, is rather simple and lays emphasis on fundamental neuronal principles such as competition and self-organization. Though simple, this non-linear model has been extensively studied from a theoretical point of view (4, 5), to better understand its convergence properties as well as its relationships to several statistical non-supervised classification tools (6). This model is also interesting because it can be presented as biologically inspired (1,2,7), particularly illustrating how cortical maps could process perceptual and motor information. Finally, the model can also be presented with an excellent ration efficiency/difficulty to implement, since it has been used in many real-world and even industrial applications related to intelligent data processing, robotics, control, optimisation, medical applications, etc (8,9,10,11).

Recently, the authors of the present paper have started a collaboration to specify SOM model using a variational approach (12,13). Some of us are interested in creating cerebral models with biologically inspired neuronal units, implementing them on autonomous robots, while others are specialists in computer vision, able to implement regularization processes with Partial Differential Equations (PDE) (12). Working together on SOM algorithm was an opportunity for all of us to work on the understanding of the functioning of the visual cortex, seen both as a neuronal system and an image processing device.

In this paper, we propose to explain that this approach, giving a solid mathematical framework to SOM computing, is not only interesting for better understanding the visual cortex; it also comes with very interesting properties that give a new view to the usefulness of SOM for real-world industrial applications. To assess this statement, we first present the SOM algorithm, together with its current interests and limitations, explain how we propose to use a variational approach to revisit this algorithm and more generally discuss the advantages of such an approach for concrete applications of SOM.

2 KOHONEN'S SOM. A Self-Organizing Map (1,2) is a layer of neurons N_V, where V is generally a 2D vector, corresponding to the dimensionality of a map. Other dimensionalities are also possible. The neural weights are subject to unsupervised learning. For a learning data set of size s_{max}, elements of an input sequence I_P^s ($s \in \{0 \ldots s_{max}\{$) are proposed as input to the neuronal map. In the framework of image processing, each element is a small piece of image (P is a 2D vector) but for other applications, other formats are possible (generally a 1D vector of parameters). From the neuron point of view, each unit in the neuronal map consequently receives an input of size P and its weight vector W has the same size. When an input is proposed, several steps are computed:

2.1 Neuronal activation. Each neuron in the map can compute its activity A_V from a distance operator d from the input to the weight vector, such that the smallest is the distance the greatest is the activity.

$A_V = f(|I-W|)$, where f() is a toric sigmoid-like profile.

The distance operator has to be chosen as a function of the objects to be compared. Then, the neuron with the maximal activity will inhibit the others and remain the only one active. This can be simply computed as a winner-take-all algorithm or can be implemented in a neuron-like way with inhibitory lateral connectivity between neurons in the map.

2.2 Learning. Only the winner neuron and its neighbouring neurons learn, i.e. update their weights.

$\Delta W \equiv g(\left| I-W \right|^2)(I-W)$

The neighbourhood is defined with the function g, generally corresponding either to a Gaussian function or a so-called mexican hat. To ensure convergence, the size of such profile, together with the learning rate decrease with time.

2.3 Analysis. From random initial weights, this very simple mechanism displays several interesting properties. Without lateral connectivity in the map, the algorithm can be compared to k-means algorithm; with lateral connectivity, we can think of a.k.o. clustering or hierarchical classification (3,6). The most activated neuron changes its weights as a function of its distance to the input to be even more similar to this input. Lateral connectivity makes the neighbourhood change in the same way and makes a topology emerge in the map. Former neurons tend to converge to complementary inputs leading to a sparse but usually representative view of the input set. This fundamental property (close units perform close operations) is also observed in the cortex and is the basis of biological plausibility of this kind of algorithm.

As it can be remarked, the neuronal model described here is very general and several aspects must be specified to have a precise algorithm. For example, the distance function must be adapted to processed data, which is not so a simple issue in the case of pieces of image. Similarly, the neighbourhood shape, size and evolution in time can be critical to obtain stable results. Up to now, only restricted configurations of SOM have allowed theoretical studies leading to convergence proofs (5).

Another critical point with SOM is that of dimensionality; 2D maps are often selected because of the 2D analogy with the cortical sheet or because such a representation yields a more simple and intuitive visualisation (see e.g. (8) for a concrete example). Now, it is clear that complex real-world problems are often specified in a high dimensionality and their representations in 2D are often too poor. There is a priori no limitations to design Kohonen's "maps" in higher dimensions, except that computations can become prohibitive and results very difficult to interpret, while sparse representations in higher dimension are subject to the curse of dimensionality. Several studies have explored multi-map configurations (14) but obtaining stable and easy-to-master algorithms remain a difficult challenge. We now present the principle of using a variational approach to implement SOM and explain how it could contribute to overcome these limitations.

3 REGULARIZATION MECHANISM. The domain of computer vision is close from that of neural networks in the sense that its main task is to compute maps of numerical values. It has a long experience of criteria optimisation in such maps through a very efficient theory. A criterion to be minimized is expressed, deriving an Euler-Lagrange equation from which Partial Differential Equations are derived to iteratively estimate variables of interest. Such a regularization mechanism is interesting because it is known to converge toward a local optimum, even if input data are corrupted with noise or incomplete.

For example, in computer vision, a smoothing mechanism can be written as the minimization of the local variation of the image s (12):

$$L = \int ||\nabla s||^2$$

Which yields a PDE of the form:

$$s' = \Delta s \ \text{, since } \nabla L = -2\Delta s$$

Following this track, Cottet and El Ayyadi (15) have proposed an adaptive diffusion mechanism with a directional Laplacian (the direction being defined by the gradient of the image itself), such that the diffusion is isotropic in regions with low contrast and anisotropic in contrasted areas. From a computer vision point of view, this gives an edge-preserving smoothing, obtained from an iterative process which is shown to be convergent under mild hypotheses. The authors have also underlined (and discussed from a biologically inspired point of view) the powerfulness of such a mechanism, where the results from the previous step of computation can be fed-back (in a very biologically plausible way) to influence the feed-forward diffusion process.

On the same basis, other works has been proposed for segmentation or binarization, as reported for example in (12). This latter reference is also interesting because it underlines that this kind of regularization process, set of iterative local numerical calculus can be implemented – compiled- as a neuronal network and an implementation for several well known networks, like the analog Hopfield network and also (to a certain extend) event-driven spiking networks.

4 A VARIATIONAL APPROACH FOR SOM. Here we propose a similar approach to implement Self-Organizing maps (13), from some criteria to be minimised as proposed in (16):

$$L = \int_s\int_V\int_P \psi(|I^s - w_V|^2 + \int_V\int_P \phi(|\nabla_P W_V|) + \int_V\int_P \xi(|\nabla_V W_V|)$$

The first term (called input term) explains that weights have to remain close to inputs; the second term (intra-neuron term with diffusion) explains that the weight vector of one neuron has to remain stable through examples; the third term (inter-neuron term with diffusion) explains that close neurons have close weights. A PDE derived from this global criterion is proposed to yield dynamics of the neuronal network, together with its learning rule. As proposed by (13), such a criterion is also a convergence function, useful to control convergence of the neuronal implementation.

5 DISCUSSION. From a general point of view, such an approach is interesting because it proposes an original view of neural networks, particularly in the framework of biological inspiration. It explains that such a distributed system can be viewed as globally solving an optimisation problem. When this problem has adequate characteristics, it can be solved up to a good approximation, using iterative equations implemented locally on each unit of the distributed system. A neuronal analogy exists, in some contexts..

In the original case that we have considered here, namely Kohonen's Self-Organizing Maps, this approach is interesting because it can help giving convergence properties but also because it can propose original ways to overcome some limitations evoked above. Indeed, in the same way as exploited in (15), distance function and neighbourhood function could act as re-entrant parameters, adapting local computation of the neuronal map to the design of these functions that could be made by hand or approximated elsewhere, i.e. in other maps. This latter possibility refers to the possible implementation of a multi-map system where parameters estimated in a map control the behavior of another map, which is also an hypothesis discussed in (14), from biological inspiration.

This kind of mechanism and the corresponding properties are particularly interesting to use SOM in the framework of industrial applications. We have explained above that such neuronal models are extensively used for such real-world applications, even if convergence results are poor and if some internal mechanisms have to be chosen, or rather tuned, by hand. The variational approach that we propose here could be a way to propose a more robust and a more secure self-organising process, which is a fundamental point for industrial applications. At the moment, we are evaluating such properties for computer vision applications.

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