Fuzzy Relative Positioning for On-Line Handwritten Stroke Analysis

François Bouteruche, Sébastien Macé, Eric Anquetil

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François Bouteruche, Sébastien Macé, Éric Anquetil
IRISA - INSA
Campus Universitaire de Beaulieu
35042, Rennes Cedex, France
{Francois.Bouteruche, Sebastien.Mace, Eric.Anquetil}@irisa.fr

Abstract

This paper deals with the qualitative and robust modelling of the relative positioning of on-line handwritten strokes. We exploit the fuzzy approach to take the imprecision of such relations into account. We first transpose a well-formalized method which proved itself in the domain of image analysis to the on-line case; it aims at evaluating the relation "to be in a given direction*" relatively to a reference. Our first contribution is a solution to deal with the particular nature of on-line strokes, which are constituted of non-connected points. Our second and main contribution is a method to learn automatically fuzzy relative position relationships. It aims at evaluating the relation "to be* in a given position*" relatively to a reference using jointly the direction and the distance. We test the impact of this new fuzzy positioning approach on one possible application: the recognition of handwritten graphic gestures, which requires spatial context information to be discriminated. Whereas the recognition rate is 52.95% without any spatial information, we obtain a maximum of 95.75% when we use learnt relative position relationships.*

Keywords: On-line interpretation, Handwritten stroke analysis, Fuzzy positioning, Learning.

1. Introduction

One of the major issues of on-line handwritten stroke interpretation is the context evaluation, which enables to associate the semantics to a stroke. For instance, in a handwriting note-taking application with sketch recognition capabilities, it is impossible to discriminate strokes representing circles, letters "o" and numbers "0" without any information on their context. Moreover, the context identification allows driving the shape recognition process [5, 10]. Here, we focus on this problem and particularly on the spatial context modelling. The challenge is to describe qualitatively and robustly the relative position of objects, *i.e.* how they are located one relatively to another.

Numerous domains of on-line handwritten stroke recognition face the problem of relative spatial position relationships between strokes: chinese character recognition [9], mathematical expression recognition [6], diagram recognition [12]... Several approaches use relative positional information such as [7] or [12] which are based on the probalistic frameworks. However, these methods do not address the specific problem of describing qualitatively the position relationship between two strokes. So far, this problem is not very present in the literature. Existing method are mostly empirical and *ad hoc*, which is not satisfactory, since they can not be adapted to other domains dealing with handwritten stroke interpretation. Moreover, they often give an "all-or-nothing" answer, which we believe is not suitable. Indeed, let us consider the reference object R and the analysed object A on figure 1: although A is clearly not on the left nor below the object R and is clearly on the right, it is more difficult to make a decision about the relation "above".

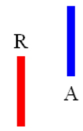

Figure 1. An ambiguous relation between two objects

Since it is not possible to make a hard decision about relative positioning, fuzzy set theory [13] is an interesting approach to model such imprecise relations.

This paper proposes two main contributions: first, it introduces the adaptation to the on-line case of a robust and well formalized approach of fuzzy relative positioning which proved itself in the domain of image analysis [2]. It is based on the definition of relative directional relationships. Then, we present an original method to automatically learn fuzzy relative position relationships, which is as far as we know completely new, even in image analysis. In this paper, we use the term *relative directional relationship* to represent a relation which evaluates if an analysed object A is in a given direction relatively to a reference object R. When the distance to the reference is taken into account, we use the term *relative position relationship*. Both relationships are *relative positioning relationships*.

We test the pertinence of these methods on the recognition of on-line handwritten graphic gestures; their particularity is that some of them have the same shape, but have different semantics depending on their context. It is then impossible to discriminate them without any context.

The following section presents a short state of the art on the fuzzy methods to model the relative positioning between objects. Section 3 focuses on a particular approach, based on mathematical morphology, and then section 4 explains its application and adaptation to the on-line domain. Section 5 presents how relative position relationships can be learnt, and section 6 highlights the corresponding pertinent results. Finally, we summarize our future work.

2. State of the art in fuzzy positioning

There is a lack of generic and well formalized methods based on fuzzy set theory to model relative positioning in the on-line domain. Approaches based on *ad hoc* fuzzy positioning exist. We first present a typical one and then investigate the approaches in the image analysis domain.

2.1. *Ad hoc* fuzzy relative positioning in the on-line domain

In the on-line domain, objects of interest are handwritten strokes. To our knowledge, very few authors propose stroke relative positioning methods based on fuzzy set theory. The approach of Zhang *et al.* [14] to deal with on-line mathematical expression recognition is typical of the existing methods. They define fuzzy regions for each relation they consider (*e.g.* superscript, inline, *etc.*) relatively to an object; some of these regions overlap, which represents the fact that there is not a strict limit between, for instance, the superscript and the inline relations. The method consists then of determining in which of these fuzzy regions the centroid (calculated based on the bounding box and on the symbol class) of the analysed object is located. Even though this approach is a first step toward the fuzzy positioning in on-line interpretation, it remains quite *ad hoc* and can not be generalized to any domain.

Since the fuzzy methods evaluating the relative positioning in the on-line case are quite limited, we focused on the approaches proposed in the image analysis domain.

2.2. Fuzzy relative directional relationship in the domain of image analysis

In the domain of image analysis, objects of interest are sets of pixels representing them. The goal is then to qualify how an analysed object belongs to a space region defined relatively to a reference object.

The *aggregation method* [8] consists of measuring the angle θ between the line connecting two points P and Q and the horizontal line. The membership function $\mu_{R\_relation\_A}$ of a relation must be defined depending on this angle (*e.g.* the relation *right_of* will have high membership values for low values of θ). If R and A are object constituted of several points, the authors evaluate each $\mu_{R\_relation\_A}(\theta(Q,P))$, with $Q \in R$ and $P \in A$; they use an aggregation operator (*e.g.* the mean operator) to compute the membership value of the relation on R and A.

Instead of evaluating $\mu_{R\_relation\_A}(\theta(Q,P))$ for each $Q \in R$ and $P \in A$, the *centroid method* only evaluates it for one particular point of each object, which is their centroid [8] or another characteristic point.

The *compatibility method* [11] also requires defining the membership function $\mu_{R\_relation\_A}$ of each available relative directional relationship. Then, in order to evaluate the degree to which one of them holds between two objects R and A, the normalized histogram of angle between each $Q \in R$ and $P \in A$ is computed; it is then compared with each of the considered relations using a compatibility operator of a distribution to a fuzzy set.

Bloch *et al.* [2, 3] propose a different way to deal with relative positioning based on a *morphological approach*. They propose a fuzzy adaptation of the classical mathematical morphological operators, and exploit them to define relative directional relationships. This offers the advantage of taking the shape of the objects into account.

The work we present in this paper is the adaptation of this fuzzy morphological approach to the on-line domain. We have chosen this approach for three main reasons. Firstly, it takes the shape of the objects into account. Secondly, it is flexible and enables to model a large panel of relations. These properties interest us especially for the objective of learning relative position relationships. Finally, as shown in [2], it offers results that fit the intuition.

3. A morphological approach for fuzzy relative directional relationships

In this section, we present a subset of the works of Bloch *et al.* which is based on a fuzzy adaptation of mathematical morphology; the interested reader can refer himself for instance to [2, 3, 4] for more information.

The principle of Bloch *et al.* approach for evaluating the relative directional relationships of an analysed object relatively to a reference object can be divided into two steps [2]. First, they define a fuzzy landscape around the reference object: it is a fuzzy set which defines the membership value of each point of the space to the relative directional relationship under examination. Then, they compare the analysed object to the fuzzy landscape in order to evaluate how well the object matches with the relation.

We present each of these steps, and apply them on the example presented in figure 1.

3.1. Defining a fuzzy landscape

Defining a fuzzy landscape consists of defining the fuzzy set $\mu_\alpha(R)$, which represents the adequation of any point P of the space S with the directional relation defined by the angle α relatively to the reference object R: points which satisfy to a high degree the relation "to be in the direction $\overrightarrow{u}_\alpha$ with respect to R" will have a high membership value. Although there is a large spectrum of possibilities, we present the definitions proposed in [2], because they correspond to the ones we exploit.

Let us denote by P any point of the space S, and by Q any point in R. Let $\beta(P,Q)$ be the angle between the vector $\overrightarrow{QP}$ and the direction $\overrightarrow{u}_\alpha$, computed in $[0,\pi]$. $\beta(P,Q)$

is given by the equation:

$$\beta(P,Q) = \arccos(\frac{\overrightarrow{QP}.\overrightarrow{u}_\alpha}{||\overrightarrow{QP}||}), \text{ and } \beta(P,P)=0. \quad (1)$$

For each point P, Bloch *et al.* determine the point Q of the reference object R which leads to the smallest angle β, denoted by β_{min}. This is illustrated by figure 2.

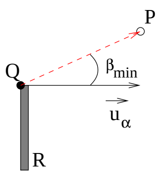

Figure 2. Definition of β_{min}.

The fuzzy landscape $\mu_\alpha(R)$ can then be defined as:

$$\mu_\alpha(R)(P) = f(\beta_{min}(P)) \quad (2)$$

where f is a decreasing function of $[0,\pi]$ into $[0,1]$. The authors chose a simple linear function, leading to the following definition of the fuzzy landscape, in the crisp case:

$$\mu_\alpha(R)(P) = \max(0, 1 - \frac{2\beta_{min}(P)}{\pi}) \quad (3)$$

Figure 3 presents examples of fuzzy landscapes defined by a crisp reference object; they correspond to the relative directional relationships *right_of*, *above*, *left_of* and *below* defined relatively the object R presented in figure 1.

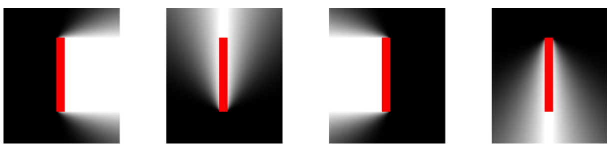

Figure 3. Fuzzy landscapes for the directional relationships *right_of*, *above*, *on_the_left* and *below* relatively to the object R presented in figure 1 (high grey values correspond to high membership values).

In the domain of image analysis, references and analysed objects can be represented as spatial fuzzy sets in order to take the imprecision of the image into account. Thus, the reference object is defined by a membership function μ_R, which represents for each point Q of the space to which degree it belongs to R. Then, to determine the fuzzy landscape, the membership value of the point P to the relative directional relationship is combined with the membership of the point Q to the reference object R. So, the fuzzy landscape is defined using a t-norm t as:

$$\mu_\alpha(R)(P) = \max_{Q\in S} t[\mu_R(Q), f(\beta(P,Q))] \quad (4)$$

The authors highlight the possible interpretation of this approach in terms of morphological operations [2]; indeed, $\mu_\alpha(R)$ is exactly the fuzzy dilatation of μ_R by ν, where ν is a fuzzy structuring element defined on S as:

$$\forall P \in S, \nu(P) = \max[0, 1 - \frac{2}{\pi}\arccos(\frac{\overrightarrow{OP}.\overrightarrow{u}_\alpha}{||\overrightarrow{OP}||})] \quad (5)$$

where O is the center of the structuring element. The dilatation equation is then [4]:

$$\forall P \in S, D_\nu(\mu)(P) = \max_{Q\in S} t[\mu_R(Q), \nu(P-Q)] \quad (6)$$

In fact, it is the fuzzy structuring element that defines the relationship. A modification of this structuring element can modify the evaluated relationship. This will play an important role on the learning strategy (see section 5).

3.2. Comparing an object with a fuzzy landscape

Once the fuzzy landscape has been defined, we want to determine the degree to which an analysed object A satisfies the corresponding relation. Bloch *et al.* propose to exploit the fuzzy pattern-matching approach; in particular, the evaluation of the matching between two possibility distributions consists of two numbers, a necessity degree N (pessimistic evaluation) and a possibility degree Π (optimistic evaluation). An average value can also be defined:

$$M_\alpha^R(A) = \frac{1}{|A|}\sum_{x\in S}\mu_A(x)\mu_\alpha(R)(x) \quad (7)$$

where $|A|$ denotes the fuzzy cardinality of A:

$$|A| = \sum_{x\in S}\mu_A(x)$$

This value is useful from a practical point of view, because it provides a global evaluation of the matching of the fuzzy analysed object A with the fuzzy landscape $\mu_\alpha(R)$. This is the value we exploit in particular in the continuation of this paper because it is easily usable by classifiers.

Let us go back to the example presented figure 1. Table 1 presents the results obtained for directional relationships of the object A relatively to the object R. Note that these results fit the intuition, since according to them, A is not on the left of R nor below; on the contrary, the *right_of* relation has high membership values, and the *above* one has middle values.

Table 1. Average values for relative directional relationships obtained for the objects of figure 1.

relationship	left	right	above	below
average (M)	0.0	0.81	0.66	0.04

4. Fuzzy relative directional relationships: transposition to the on-line domain

In this section, we transpose the morphological approach to model relative directional relationships to the on-line domain. We first present the effect of the direct application of the theory and point out a limit. Then we propose a way to overcome it.

4.1. Direct application

In the on-line domain, the considered objects are handwritten strokes. A stroke is constituted of a sequence of points. It is then quite natural to consider both the reference object and the analysed object as crisp: their membership functions μ_R and μ_A, defined in section 3.1, are binary functions. Their values are 1 for the points of the corresponding object and 0 for the others. The membership degree of the point P to the fuzzy landscape is then, as seen previously:

$$\begin{aligned} \mu_\alpha(R)(P) &= \max_{Q\in S} t[\nu(P-Q), \mu_R(Q)] \\ &= \max_{Q\in R} \nu(P-Q) \\ &= f(\beta_{min}(P)). \end{aligned} \tag{8}$$

Figure 5 presents on the left the result of the direct application of the equation to compute the fuzzy landscape corresponding to a particular relative directional relationships, here *right_of* of a handwritten stroke. It highlights a limit of this approach: as the objects are seen as points which are non-connected, the relationships are computed only relatively to these points. This produces an unsatisfactory "comb effect".

4.2. Dealing with objects represented by non-connected points

Several techniques are possible to overcome this problem; we could for instance re-sample the points of the object. We propose an approach which does not need a modification of the stroke points and which considers the substrokes between consecutive points as straight lines. It comes from the fact that the ideal angle can be reached for a point of the substroke which is located between two consecutive points of the stroke. Thus, we consider each pair of angle value β, this time computed in $[0, 2\pi]$, for each pair of consecutive points: if one is in $[0, \frac{\pi}{2}]$ and the other in $[\frac{3\pi}{2}, 2\pi]$, it implies that there is a point between them so that $\beta = 0$, and as a consequence $\beta_{min} = 0$. This process is illustrated on the relative directional relationship *right_of* on figure 4: on the left, P is on the right of a point of the substroke between two points Q_1 and Q_2 of the reference stroke, which is not the case on the right.

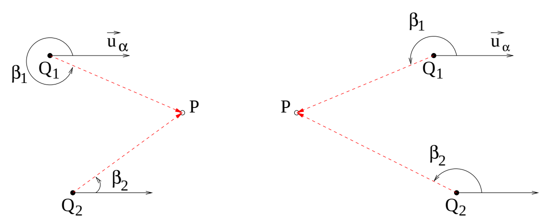

Figure 4. Calculus of β for the relative directional relationship *right_of* of two points P of the space.

The result of this algorithm is shown on the right of figure 5: the "comb effect" has disappeared of the fuzzy landscape, which enables to exploit the morphological approach in the on-line context.

Note that this technique does not require adding new points and is not more complex than the classical one; it could even allow limiting the calculation to a subset of points of a stroke if several points are aligned.

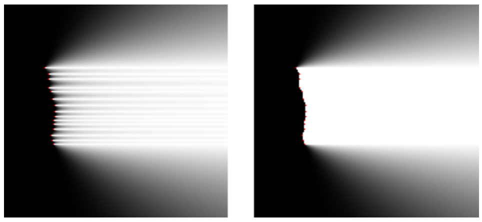

Figure 5. Fuzzy landscape computation of the *right_of* relative directional relationship: effect of the direct application of the equations producing a "comb effect" (on the left), and effect of the proposed solution to overcome it (on the right).

5. Learning fuzzy relative position relationships

The works of Bloch *et al.* and their adaptation to the particularity of on-line handwritten strokes presented in the previous section are a powerful framework to define relative directional relationships to evaluate the direction of a handwritten stroke relatively to a reference stroke. With the proposed fuzzy structuring element, the value of $\mu_\alpha(R)(P)$ is 0 as soon as the angle between the vector $\overrightarrow{QP}$ and the direction $\overrightarrow{u}_\alpha$ is greater than $\pi/2$. Thus, to have a good representation of the direction of the analysed stroke relatively to its reference, we must define *a priori* four relative directional relationships: *right_of*, *left_of*, *above* and *below* (see figure 3). In addition, to have a full description of its spatial context, we must associate a distance relationship to these directional relationships, as mentioned in the introduction.

However in the case of contextual on-line handwritten stroke recognition, we dispose of supervised examples which are composed of three elements: a reference handwritten stroke, a handwritten stroke to analyse and the class of this stroke. As the spatial context of the stroke plays a major role in the interpretation process, it is of high interest to take advantage of these supervised examples to refine the positioning. We propose to define a dedicated fuzzy relative position relationship for each class of analysed strokes, and to learn them from data. They will take into account both the direction and the distance of the analysed stroke relatively to its reference to evaluate to which degree it satisfies the relationship "to be *in the position* of the class i relatively to the reference".

As mentioned in section 3.1, the fuzzy structuring element $\nu(P)$ defines the evaluated spatial relationship. In the next section, we propose a method to learn from the data a fuzzy structuring element which jointly evaluates the direction and the distance of an analysed stroke relatively to its reference. Then, we propose interpretations of the membership functions μ_R and μ_A that are more adapted to the on-line case than those used in the image analysis domain (see section 3.1).

5.1. Learning the fuzzy structuring element $\nu(P)$

To learn the fuzzy structuring element of a fuzzy relative position relationship dedicated to a class i, we extract for each example of this class, which is composed of an analysed stroke and a reference stroke, all the vector $\overrightarrow{QP}$ between each couple of reference point Q and analysed point P. From all these samples, we compute the average vector $\overrightarrow{c}^{i}_{pos}$ and the covariance matrix Cov^{i}_{pos}. The average vector $\overrightarrow{c}^{i}_{pos}$ models both the average direction and the average distance between the reference points and the analysed points. We then define the fuzzy structuring element $\nu(P)$ as a hyper-ellipsoidal radial basis functions with $\overrightarrow{c}^{i}_{pos}$ as centre and Cov^{i}_{pos} defining its shape using the Mahalanobis distance:

$$\nu(P) = \frac{1}{1 + d_{Cov^{i}_{pos}}(\overrightarrow{OP}, \overrightarrow{c}^{i}_{pos})} \qquad (9)$$

where O is the centre of the fuzzy structuring element.

5.2. Interpretation and learning of the membership functions μ_R and μ_A

A slight but important difference in the definition and interpretation of μ_R and μ_A between the domains of image analysis and on-line handwritten stroke analysis must be pointed out. As see in section 3.1, in the domain of image analysis, the functions μ_R and μ_A define the reference object R and the analysed object A in the image space. For each point of the image space, they evaluate to which degree the considered point belongs to the object.

In section 4, we use a direct adaptation of the image analysis interpretation of μ_R and μ_A which conducts to the simplification of the equation 6. In this case, it only depends on the structuring element $\nu(P)$.

However, another interpretation of this membership functions is possible in the on-line case. We can define and interpret μ_R as a membership function evaluating to which degree $\mu_R(Q)$ a point Q of the reference must be taken into account in computing the satisfaction degree of the considered relationship. Indeed, in on-line handwritten stroke analysis, we must face out the variety of handwriting styles. Thus, some points of the reference could be not typical and then should not have the same weight than more typical ones in the evaluation. We propose to define and to interpret μ_A in the same way.

We learn these membership functions on the supervised examples of the class i associated to the relationship we want to define. To learn the membership function μ^{i}_{R}, we extract from the database the reference strokes of all the examples of the class i. Then we translate them in a space with the barycentre of the segments representing them as origin and compute the average point c^{i}_{R} and the covariance matrix Cov^{i}_{R}. The membership function μ^{i}_{R} is then defined as a hyper-ellipsoidal radial basis functions with c^{i}_{R} as centre and Cov^{i}_{R} defining its shape using the Mahalanobis distance like in equation 9. We do the same operations to learn the membership functions μ^{i}_{A}.

6. Exploitation for on-line graphic gesture recognition

6.1. The benchmark

To test the pertinence of the different proposed methods to describe the spatial context, we apply them on an extended version of the benchmark used in [5]. It consists in the recognition of 18 on-line handwritten graphic gestures. 2 of them correspond to the addition of a stroke to a character. The 16 others (see table 2) correspond to an accentuation of their reference character (acute, grave, cedilla, *etc.*), to a punctuation symbol (coma, dot, apostrophe, *etc.*) or to an editing gesture (space, caret return, *etc.*). As several subsets of gestures have the same shape, the only way to discriminate them is to use spatial context. So, this benchmark fits our need to test the different methods of relative positioning. The examples of the benchmark have been written on PDA by 14 writers. The training database contains 4243 examples of 8 writers and the test database contains 2393 examples of 6 writers. None of the writers is common to both data sets.

Table 2. The 16 handwritten gestures

acute	grave	circumflex	dieresis
accent deletion	cedilla	cedilla deletion	case switch
character deletion	last character deletion	caret return	insertion
space	apostrophe	comma	dot

6.2. The experimental protocol

To recognize these 18 on-line handwritten graphic gestures, we used a Radial Basis Function Network (RBFN) [1]. To describe the shape of the gestures, we use 9 classical shape features. These 9 features are common to all tests. To describe the spatial context, we use the different proposed methods to generate spatial context features. First, to evaluate the need of spatial context, we test the performance without any spatial context features (*test 1*). Then, we test it using 4 features that describe the position in x-coordinate and y-coordinate of the bounding box of the analysed stroke relatively to the bounding box of the reference (*test 2*).

We then used the fuzzy relative directional relationships presented in section 3 and 4 to describe the direction of the analysed sample relatively to its reference. We defined the 4 four directional relationships given in section 5: *right_of*, *left_of*, *above* and *below*. We associate to these 4 features a distance features that evaluate the average distance between the points of the analysed stroke and the points of the reference. We made two performance tests using these 5 features, the first one using the direct application of the fuzzy directional relationships (*test 3*) and the second one using the proposed optimisation (see section 4.2, *test 4*).

Finally, we used the fuzzy learnt relative position relationships proposed in section 5. We learnt a relationship for each gesture to recognize. Thus, 18 spatial context features were associated to the 9 shape features (*test 5*). Figure 6 presents four of these learnt fuzzy relative position relationships (corresponding to the *grave*, *dieresis*, *dot* and *carret return* classes).

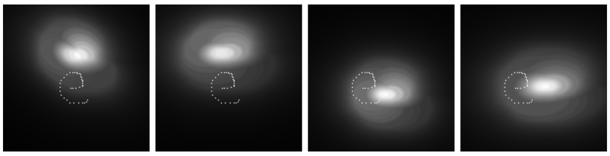

Figure 6. Fuzzy relative position relationships for the *grave*, *dieresis*, *dot* and *carret return* classes relatively to a stroke.

6.3. Results analysis

Table 3 gives the recognition rates obtained for the 5 tests. The 52.95% recognition rate on test 1 shows the need of spatial context information to discriminate the graphic gestures. The error rate decrease between test 2 and test 3 shows the benefits of using the fuzzy relative directional relationships associated to a distance to model more precisely the spatial context. Even if the proposed optimisation in section 4.2 allows a better evaluation of the fuzzy landscape (see figure 5), the results obtained on test 4 are the same than those on test 3. The ability of the RBFN to deal with the noise generated by the "comb effect" explains these results. Finally, the 95.75% recognition rate obtained on test 5 proves that, in a contextual handwritten stroke recognition problem, it is more valuable to describe the spatial context of a stroke with position relationships than with a distance feature and associated to a directional relationships. The benefit is a 18.63% decrease of the error rate on this benchmark.

Table 3. Performance charts.

Test	Recognition Rate	Test	Recognition Rate
test 1	52.95%	test 2	92.27%
test 3	94.90%	test 4	94.90%
test 5	95.75%		

7. Conclusion and future work

We propose an adaptation to the on-line domain of fuzzy relative directional relationships used in the domain of image analysis. These relationships associated to a distance allow describing the spatial context of a stroke by its position relatively to a reference. Then, we propose a new method to learn automatically fuzzy relative position relationships that evaluate if a handwritten stoke is "*in the position* of a class relatively to a reference". These relationships allow describing the spatial context of a stroke by the adequation of its position relatively to a reference with the position of a class relatively to this reference.

The experiments show the benefits, compared to basic spatial context features, of using fuzzy relative directional relationships to describe more precisely the spatial context of a stroke. They also show the benefits of using the proposed learnt fuzzy relative position relationships for a contextual handwritten stroke recognition problem.

Our future works will focus on a general evaluation of the different fuzzy relative positioning methods in the on-line case. We will also integrate the learnt fuzzy relative position relationships in handwritten stroke analysis methods using spatial context information [5, 10].

8. Acknowledgements

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