Color Mixing Correction for Post-printed Patterns on Colored Background Using Modified Particle Density Model

Misako Suwa
Language and Media Processing Lab,
Fujitsu Labs. Ltd.,
4-1-1 Kamikodanaka, Nakahara-ku,
Kawasaki-shi, 211-8588, JAPAN
suwa@jp.fujitsu.com

Katsuhito Fujimoto
Language and Media Processing Lab,
Fujitsu Labs. Ltd.,
4-1-1 Kamikodanaka, Nakahara-ku,
Kawasaki-shi, 211-8588, JAPAN
fujimoto.kat@jp.fujitsu.com

Abstract

Color mixing occurs between background and foreground colors when a pattern is post-printed on a colored area because ink is not completely opaque. This paper proposes a new method for the correction of color mixing in line pattern such as characters and stamps, by using a modified particle density model. Parameters of the color correction can be calculated from two sets of foreground and background colors. By employing this method, the colors of foreground patterns on any background can be transformed into the color on the base background color.

Keywords: Color mixing correction, post-printed pattern, modified particle density model, line-shaped distribution

1. Introduction

Color mixing occurs between the foreground and background colors when a pattern is *post-printed* on a colored region because the ink of the pattern is not completely opaque. (In this paper, the term "post-printed" implies hand-printed, typed, stamped, carbon-copied, etc.) Moreover, Gaussian-type transitions from the foreground to the background color occur near the boundaries of a pattern in a scanned image [4]. Therefore, the color of a pattern post-printed on a colored background generally differs from that of a pattern post-printed on a white background. Fig. 1 shows the images of character strings written on white and red backgrounds by using the same blue ballpoint pen. The square located on the right side of each character string indicates the color of the character stroke in each image. The average RGB values of the character strokes in Fig. 1 (a) and (b) are (29, 35, 150) and (47, 10, 27), respectively. Evidently, the color of the square in Fig. 1 (b) cannot be observed as blue. During the extraction of a pattern of a specific color, erroneous extraction or extraction failure may occur due to color mixing. In order to prevent these errors, it is essential to correct the color mixing between the foreground and the background.

The Kubelka-Munk model (KMM) [2] is widely used for modeling the color mixing effects of light scattering materials. Ref. [3] describes the application of this model to watercolor synthesis. However, the KMM is a nonlinear complex model; therefore, it is very difficult to apply it to our problem. On the other hand, Terai *et al.* have proposed a particle density model (PDM) [1] for estimating the base colors for watercolor prints. This model assumes virtual opaque particles with peculiar colors and describes a color layer as the distribution of these particles with a uniform density. The PDM can be treated easily and its concept does not limit the color material to watercolors. This model is explained in detail in the next chapter.

In this study, we attempt to apply the PDM to the correction of color mixing in linear patterns post-printed on colored backgrounds. However, it cannot be assumed that the particle density is constant because the line patterns in a scanned image exhibit severe color blurring due to scanning and uneven ink density. Therefore, we introduce a parameter that indicates the density spread and propose a modified particle density model (MPDM) for line patterns.

Fig. 1. Color mixing between foreground and background colors: (a) RGB values: background (255, 255, 251) and foreground (29, 35, 150) (b) RGB values: background (252, 35, 28) and foreground (47, 10, 27)

2. PDM

The PDM assumes virtual opaque particles with peculiar colors and describes a uniform color layer as the distribution of these particles with a constant density. It also describes color mixing as the state in which the particles of the lower layer are visible in the space between the particles of the upper layer. This is shown in Fig. 2. We assume that uniform color layers are printed on paper N times such that they overlap and the i-th layer with the pigment color and particle density $(\rho_i, \mathbf{T}_i)$ is printed on the (i–1)-th layer. Then, the color $\mathbf{C}_i$ observed through the i-th layer satisfies the following equation:

$$\mathbf{C}_i = \rho_i \mathbf{T}_i + (1-\rho_i)\mathbf{C}_{i-1},$$
$$0.0 \le \rho_i \le 1.0, \quad (1)$$

where $\mathbf{C}_0$ is the paper color and $\mathbf{C}_{i-1}$ is the color observed through the $(i-1)$-th layer. $\mathbf{C}_i$ and $\mathbf{T}_i$ are RGB color vectors. Therefore, the color mixing between two color layers occurs as follows. Let $\mathbf{C}_A$ be the observed color of ink A when it is printed on $\mathbf{C}_0$ and $\mathbf{C}_B$ be the observed color of ink B when it is printed on $\mathbf{C}_0$. Their particle densities and pigment colors are $(\rho_A, \mathbf{T}_A)$ and $(\rho_B, \mathbf{T}_B)$, respectively. When ink B is printed on ink A, the observed color $\mathbf{C}_{BA}$ is a mixture of A and B. $\mathbf{C}_A$, $\mathbf{C}_B$, and $\mathbf{C}_{BA}$ satisfy the following equations:

$$\mathbf{C}_A = \rho_A \mathbf{T}_A + (1-\rho_A)\mathbf{C}_0,$$
$$\mathbf{C}_B = \rho_B \mathbf{T}_B + (1-\rho_B)\mathbf{C}_0, \quad (2)$$
$$\mathbf{C}_{BA} = \rho_B \mathbf{T}_B + (1-\rho_B)\mathbf{C}_A.$$

These relations are shown in Fig. 2.

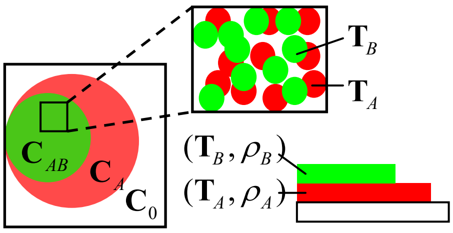


Fig. 2. Color mixing using the PDM

3. Color Mixing Correction

3.1. Color Mixing Correction Using PDM

We consider the correction of color mixing due to the overlapping of the color layers by using the PDM. The correction implies the color transformation of a pattern on a certain background color into that on a predefined base background color. The base color is generally white.

Let $\mathbf{C}_{A0}$ and $\mathbf{C}_{A1}$ be the observed colors of the patterns produced with ink A and post-printed on the colored areas $\mathbf{C}_{B0}$ and $\mathbf{C}_{B1}$, respectively. The following relations are derived from Eq. 1 and 2:

$$\mathbf{C}_{A0} = \rho_A \mathbf{T}_A + (1-\rho_A)\mathbf{C}_{B0},$$
$$\mathbf{C}_{A1} = \rho_A \mathbf{T}_A + (1-\rho_A)\mathbf{C}_{B1}, \quad (3)$$

where $(\rho_A, \mathbf{T}_A)$ and $(\rho_B, \mathbf{T}_B)$ are the particle densities and pigment colors. By subtracting the lower equation from the upper one, we obtain

$$\mathbf{C}_{A0} = \mathbf{C}_{A1} + (1-\rho_A)(\mathbf{C}_{B0} - \mathbf{C}_{B1}). \quad (4)$$

When ρ_A is known, $\mathbf{C}_{A0}$ can be calculated by observing $\mathbf{C}_{A1}$, $\mathbf{C}_{B0}$, and $\mathbf{C}_{B1}$. When the base background color is input into $\mathbf{C}_{B0}$, $\mathbf{C}_{A0}$ can be considered as the foreground color on the base background color. The relations among these color vectors are shown in Fig. 3. In this model, $\{\mathbf{C}_{A0}, \mathbf{C}_{B0}, \mathbf{C}_{A0}, \mathbf{C}_{B1}, \mathbf{T}_A\}$ exist on the same plane.

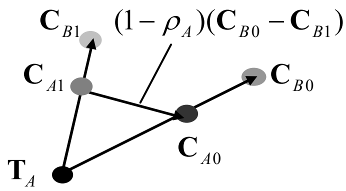
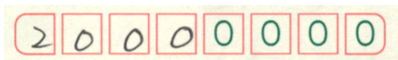


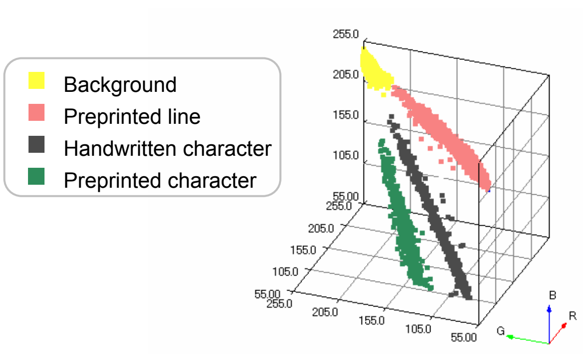
Fig. 3. Relation among color vectors

3.2. Color Distribution of Line Pattern

Before describing our technique, we will briefly explain the characteristics of the color distribution of a scanned image. When a pattern has a uniform color, the RGB values of each pixel in the pattern are distributed around an ideal point in the color space. In fact, the shape of the color cluster of the uniform-colored background appears like this distribution. However, the shape of the color cluster of a line pattern is significantly different. A figure that consists of narrow strokes, such as printed/hand-printed characters, symbols, stamps, *etc.* are termed “line pattern”. In this case, the cluster shape resembles a straight line segment because of Gaussian-type blurring near the boundary of a pattern by scanning or uneven ink density. This effect cannot be ignored particularly in the case of line pattern. Therefore, we assume that the particle density of the PDM is not constant and varies from 0 to 1.



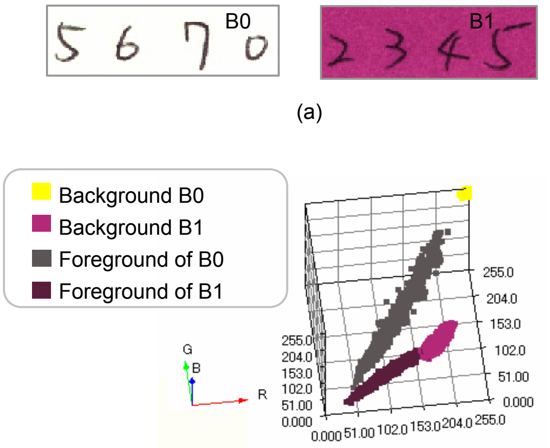
(a)



(b)

Fig. 4. Line-shaped distribution 1 (a) Pre- and post-printed patterns on single background color (b) Color distributions of each pixel

Fig. 4 and 5 show the examples of line-shaped clusters in color space. Fig. 4 (a) shows an image of a single-color background on which characters and frame lines are printed. Fig. 4 (b) shows the distribution of foreground and background pixels in a color space. As we can see from the figure, the elements of each foreground cluster spread toward the center of the background cluster. On the other hand, Fig. 5 (a) shows two images of character strings written on different colored backgrounds with the same pen. The color values of each pixel in both images are plotted in the same color space, as shown in Fig. 5 (b). In this case, the elements of each foreground cluster spread toward the center of their corresponding background color.



(a)

(b)

Fig. 5. Line-shaped distribution 2 (a) Hand-printed patterns on two different backgrounds by using the same pen (b) Color distributions of each pixel

3.3. Color Mixing Correction Using MPDM

By considering the shape of the color cluster mentioned above, we attempt to modify the PDM for line patterns. We introduce a parameter t that corresponds to the spread of the particle density along an axis, on which the variance of the distribution is the largest, i.e.,

$$\rho_A = \rho_A(t) = t. \tag{5}$$

Therefore, Eq. 3 can be rewritten as follows:

$$\begin{aligned} \mathbf{C}_{A0}(t_0) &= \rho_A(t_0)\mathbf{T}_A + (1-\rho_A(t_0))\mathbf{C}_{B0}, \\ \mathbf{C}_{A1}(t_1) &= \rho_A(t_1)\mathbf{T}_A + (1-\rho_A(t_1))\mathbf{C}_{B1}. \end{aligned} \tag{6}$$

When $\mathbf{C}_{A0}(t_P)$ linearly corresponds to $\mathbf{C}_{A1}(t_P)$ at any point of t_0 = t_1 = t_P, the following equations can be obtained from Eq. 4:

$$\mathbf{C}_{A0}(t) = \mathbf{C}_{A1}(t) + (1-\rho_A(t))(\mathbf{C}_{B0} - \mathbf{C}_{B1}), \tag{7}$$

where

$$0.0 \le t \le 1.0, \tag{8}$$

$$\begin{aligned} \mathbf{C}_{A0}(0) &= \mathbf{C}_{B0}, \mathbf{C}_{A1}(0) = \mathbf{C}_{B1}, \\ \mathbf{C}_{A0}(1) &= \mathbf{C}_{A1}(1) = \mathbf{T}_A. \end{aligned} \tag{9}$$

Eq. 6 indicates two lines that intersect at the location vector $\mathbf{T}_A$. Thus, after the line approximations of the color distributions and calculation of their intersection point, the transformation equation between the two distributions can be obtained as Eq. 7. The equation can be used to transform the background color into the base color, and mixing between the colors of a post-printed pattern and its background can be corrected (see Fig. 6). We term this model the modified particle density model (MPDM).

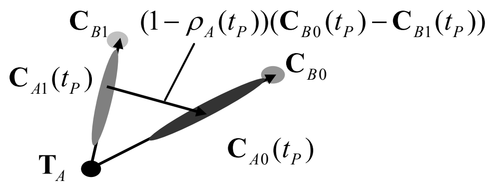


Fig. 6. Color mixing using the PDM

Using Eq. 7, $\mathbf{C}_{A0}{}^P$ on the base background color can be calculated from $\mathbf{C}_{A1}{}^P$ on any background color when $\mathbf{T}_\mathrm{A}$ is known. The next paragraph describes the calculation of $\mathbf{T}_\mathrm{A}$.

3.4. Calculation of $\mathbf{T}_A$

We further describe the application of the model to real images. The fore- and background pixels of the image are determined using the binarization beforehand. The pigment color $\mathbf{T}_\mathrm{A}$ is a crossing point of $\mathbf{C}_{A0}(t_\mathrm{P})$ and $\mathbf{C}_{A1}(t_\mathrm{P})$ as you can see from Eq. 6. At first, the line approximations are executed for the distributions of $\mathbf{C}_{A0}$ and $\mathbf{C}_{A1}$. For simplicity, we draw lines between $\overline{\mathbf{C}_{B0}}$ and $\overline{\mathbf{C}_{A0}}$, $\overline{\mathbf{C}_{B1}}$ and $\overline{\mathbf{C}_{A1}}$, respectively. Where, $\overline{\mathbf{C}_*}$ is a center of distribution $\mathbf{C}_*$. These lines are written in the following form,

$$\begin{aligned} l_0 &: \mathbf{x}_0 = \mathbf{a}_0 + s_0\mathbf{b}_0, \\ l_1 &: \mathbf{x}_1 = \mathbf{a}_1 + s_1\mathbf{b}_1. \end{aligned} \tag{10}$$

Where, $\mathbf{x}_0$ and $\mathbf{x}_1$ are the position vectors of arbitrary points on l_0 and l_1, and $\mathbf{a}_0$ and $\mathbf{a}_1$ are fixed points of position vectors, respectively. $\mathbf{b}_0$ and $\mathbf{b}_1$ are the direction vectors of l_0 and l_1, and s_0 and s_1 are the parameters. When $\mathbf{b}_0$ and $\mathbf{b}_1$ satisfy the following condition, then we calculate $\mathbf{T}_A$.

$$\cos^{-1}\left(\frac{\mathbf{b}_0 \cdot \mathbf{b}_1}{|\mathbf{b}_0||\mathbf{b}_1|}\right) > \theta_{th}. \tag{11}$$

Above θ_{th} is a threshold of the angle between the two vectors. Two lines l_0 and l_1 should intersect at $\mathbf{T}_A$, however, they are generally twisted because of calculation errors or fitting errors of the model. Therefore, $\mathbf{T}_A$ is calculated as follows. When the distance between point $\mathbf{x}_{0n}$ on l_0 and point $\mathbf{x}_{1n}$ on l_1 is the shortest, $\mathbf{T}_A$ is approximately estimate as

$$\mathbf{T}_A \sim \frac{\mathbf{x}_{0n} + \mathbf{x}_{1n}}{2} \tag{12}$$

The points $\mathbf{x}_{0n}$ and $\mathbf{x}_{1n}$ can be obtained by executing partially differentiate distance function

$$g^2 \equiv (\mathbf{x}_0 - \mathbf{x}_1)^2 = (\mathbf{a}_0 - \mathbf{a}_1 + s_0\mathbf{b}_0 - s_1\mathbf{b}_1)^2. \quad (13)$$

with respect to s_0 and s_1. Therefore,

$$\frac{\partial g^2}{\partial s_0} = 2\mathbf{b}_0 \cdot (\mathbf{a}_0 - \mathbf{a}_1 + s_0\mathbf{b}_0 - s_1\mathbf{b}_1) = 0,$$
$$\frac{\partial g^2}{\partial s_1} = -2\mathbf{b}_1 \cdot (\mathbf{a}_0 - \mathbf{a}_1 + s_0\mathbf{b}_0 - s_1\mathbf{b}_1) = 0. \quad (14)$$

The solutions s_{0n} and s_{1n} of the above equations are

$$s_{0n} = \frac{(\mathbf{b}_0 - (\mathbf{b}_0 \cdot \mathbf{b}_1)\mathbf{b}_1) \cdot (\mathbf{a}_0 - \mathbf{a}_1)}{1 - (\mathbf{b}_0 \cdot \mathbf{b}_1)^2},$$
$$s_{1n} = \frac{(\mathbf{b}_1 - (\mathbf{b}_0 \cdot \mathbf{b}_1)\mathbf{b}_0) \cdot (\mathbf{a}_0 - \mathbf{a}_1)}{1 - (\mathbf{b}_0 \cdot \mathbf{b}_1)^2}. \quad (15)$$

Consequently, we can calculate $\mathbf{x}_{0n}$ and $\mathbf{x}_{1n}$ as follows:

$$\mathbf{x}_{0n} = \mathbf{a}_0 + s_{0n}\mathbf{b}_0,$$
$$\mathbf{x}_{1n} = \mathbf{a}_1 + s_{1n}\mathbf{b}_1. \quad (16)$$

Then lines l_0' and l_1' are drawn between $\mathbf{T}_A$ and $\overline{\mathbf{C}_{B0}}$, $\mathbf{T}_A$ and $\overline{\mathbf{C}_{B1}}$. The lines are terminated at these points. It implies that no foreground pixels are assumed to be distributed beyond these points. We can write l_0' and l_1' as

$$l_0' : \mathbf{x}_0 = t\mathbf{T}_A + (1-t)\overline{\mathbf{C}_{B0}},$$
$$l_1' : \mathbf{x}_1 = t\mathbf{T}_A + (1-t)\overline{\mathbf{C}_{B1}}. \quad (17)$$

These equations are the final line-approximation of the distributions and they correspond to Eq. 6. These relations of vectors and lines are depicted in Fig. 8. An example of the line approximation is shown in Fig. 9. The original image is also attached to each distribution.

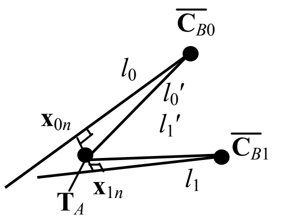


Fig. 8. Calculation of $\mathbf{T}_A$

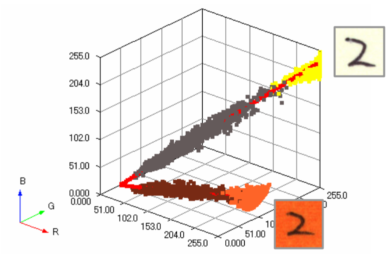


Fig. 9. Calculation of $\mathbf{T}_A$

After the calculation of $\mathbf{T}_A$, we estimate the value of parameter t at each point. Suppose the color $\mathbf{C}_{A1}{}^P$ is observed at the point P on a pattern post-printed on the background color $\mathbf{C}_{B1}$. We draw perpendicular line from $\mathbf{C}_{A1}{}^P$ to l_1' which is a fitting line of the distribution of $\mathbf{C}_{A1}$, and obtain the crossing point $\mathbf{C}_{A1}{}^{P\prime}$. These relations are shown in Fig. 10. The parameter t_P of the particle density can be calculated approximately as follows:

$$t_P \sim \frac{|\mathbf{C}_{A1}{}^{P\prime} - \overline{\mathbf{C}_{B1}}|}{|\mathbf{T}_A - \overline{\mathbf{C}_{B1}}|}. \quad (18)$$

Then Eq. 7 can be approximated as,

$$\mathbf{C}_{A0}{}^P \sim \mathbf{C}_{A1}{}^P - (1-t_P)\,(\overline{\mathbf{C}_{B1}} - \overline{\mathbf{C}_{B0}}). \quad (19)$$

After one estimate $\mathbf{T}_A$ for target pattern, the color of each pixels of the pattern on any background colors can be transformed to that of the base background color according to Eq. 19.

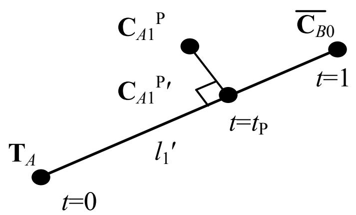


Fig. 10. Relation of the parameters and the vectors

3.5. Application to Images of Any Background Colors

The pigment color $\mathbf{T}_A$ in this model is intrinsic value for ink and is not depend on a background color. Once it is determined, it can be applied to color correction of images that have any background colors if those foreground patterns are printed with the same ink. The algorithm flow of the color mixing correction is as follows:

- Preprocessing: Each pixel of images is classified into the foreground or background by binarization. We used the method of Cheriet *et al.*[5]
- Learning transformation parameters: The color distributions of training images are calculated and approximated as lines. These images must have different background colors satisfying the condition Eq. 11 and contain only the target patterns. Then, $\mathbf{T}_A$ for the targeted ink is calculated.
- Color mixing correction: At first, the average background color of a test image is calculated and color values of each pixel are observed. Then they are substituted into Eq. 21 for targeted ink and obtain the corrected colors.

4. Experiments and Results

In this section, the results of performance evaluation experiments are reported. In each experiment, images of 12+1 different colored forms including a base color were used. The characters were written on them by blue and black ballpoint pens with oil and gel inks. These images are 24 bit bitmap images of 300 dpi. The parameters of scanner {highlight, shadow, gamma} ware set to {255, 0, 2.2}, respectively. The threshold θ_{th} was set to 5 degree.

The base colored form and one of 12 colored form were used for determined the transformation parameters. Remainder 11 forms were used as test data. Cross-validation is executed for 12 colored forms. The accuracy of the correction was evaluated by using angle between the fitting line of the color distribution of the post-printed patterns on each colored-area and the fitting line of the color distribution of the patterns written on the base background color. Average RGB values of the background colors and the ink colors are shown in the table 1 and 2, respectively. The best and worst results of the color mixing correction are shown in the table 3, 4, and 5. In the table 3 and 5, the term "before" implies the angle before the color correction, "after" implies the angle after the color correction by proposed method. In this experiment an angle is measured in degrees. Color mixing correction by PDM with constant particle density does not change the angle. Therefore, the result takes the same value as "before". The values of degrees were calculated by taking the average of 11 test data. "No." implies the number of the training data. Table 4 shows average RGB values of before and after correction of foreground colors with black and blue oil pens. Ideal RGB values for after correction are added at the bottom of each table.

Table 1. Average RGB values of background color.

No.	R	G	B	No.	R	G	B
1	250	46	36	8	129	91	144
2	245	80	60	9	180	78	103
3	160	197	213	10	161	47	108
4	253	107	34	11	197	131	78
5	249	99	130	12	224	112	89
6	194	204	126	base	254	254	243
7	133	185	133				

Table 2. Average RGB values of pens.

Ink	Gel			Oil		
Pen	R	G	B	R	G	B
Black	64	66	59	100	86	88
Blue	67	74	133	93	97	168

Table 3. The best correction results. No. implies the number of background color of training data.

Ink	O/G	Before	After	No.
Black	Oil	17.28	0.73	2
	Gel	16.29	0.47	2
Blue	Oil	15.07	6.05	12
	Gel	15.10	3.69	12

Table 4. The best correction results of foreground colors with black and blue oil pens

No.	Before			After		
	R	G	B	R	G	B
1	162	89	91	164	149	149
3	141	140	152	164	154	156
4	165	113	87	165	153	149
5	162	114	131	163	154	157
6	149	142	129	164	154	156
7	138	140	134	165	155	158
8	138	114	135	169	156	156
9	149	108	124	167	156	158
10	151	99	130	173	164	162
11	155	129	118	166	156	158
12	159	123	122	165	156	157
Ideal	161	149	151			

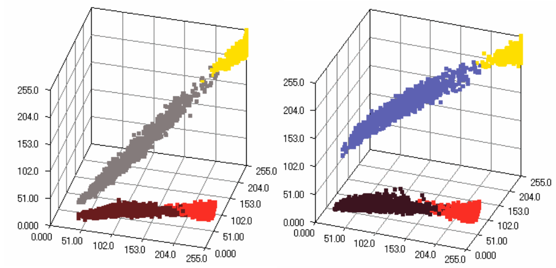
No.	Before			After		
	R	G	B	R	G	B
1	149	90	105	151	169	184
2	149	108	121	153	168	189
3	136	148	208	166	165	214
4	152	114	95	153	168	181
5	152	117	172	154	170	207
6	137	144	159	158	160	197
7	124	138	166	158	157	197
8	123	114	179	168	174	211
9	139	111	159	162	176	205
10	132	93	162	162	178	206
11	143	131	143	159	167	196
Ideal	153	156	209			

Table 5. The worst correction results.

Ink	O/G	Before	After	BG No.
Black	Oil	17.28	1.28	6
	Gel	16.29	0.91	7
Blue	Oil	15.07	7.46	7
	Gel	15.10	5.57	6

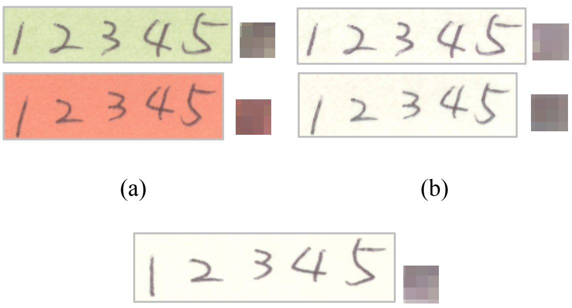
As we can see from table 3, 4, and 5, our method could correct color mixing both for the best data and worst data, especially for the case of black pens. The accuracy of the color correction of blue pens is worse compared with that of black pens. This is because that the color distribution of the foreground pixels for blue pens tends to away from straight line. In addition to that, 2 fitting lines tend to be on the twisted position depending on the background color. This means that the precondition of "opaque particle" applies not so well for the pen-inks of general colors, unlike the water paints. On the other hand, the color distribution of the foreground pixels of the black pens can be approximated as a line well and two fitting lines are intersecting at one point in good accuracy. This fact is shown in Fig. 11.

The examples of the color correction results are shown in Fig. 12 and 13. The left images are the pattern on the colored background and the right images are the corrected results. We also show their ideal images in (c). The small square located on the right side of each image indicates the magnification of the character stroke. As we can see from this figure, the character colors of middle images become close to the character colors of right images.



(a) (b)

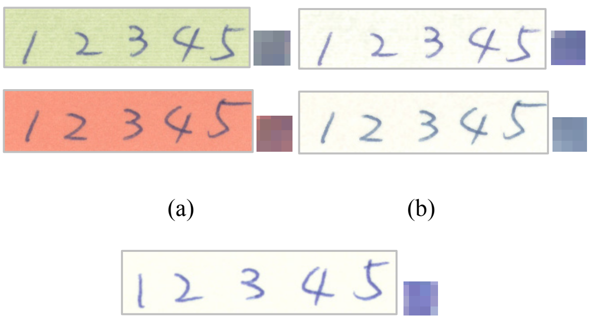
Fig. 11. Color distribution of foreground pixels on the No. 1 (a) Black oil pen (b) Blue oil pen



(a) (b)

(c)

Fig. 12. Results of color mixing correction for black oil pens (a) Characters written on colored background (b) Corrected result (c) Ideal image



(a) (b)

(c)

Fig. 13. Results of color mixing correction for blue oil pens (a) Characters written on colored background (b) Corrected result (c) Ideal image

5. Conclusions and Remarks

In this paper, we have proposed the modified particle density model for the line patterns post-printed on any colored areas. We introduced the parameter t to the particle density of the PDM, because the shape of the color distribution of such patterns is line-shaped and not point-centered. We have presented the possibility that our model correct the color mixing of the patterns post-printed on the colored area.

In this time, we only had stated the method of color mixing correction. Next time, we would like to present a total system that extracts the target-colored pattern from multiple-colored patterns.

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