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Closed-loop fault-tolerant control for uncertain nonlinear systems

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Summary. We are designing, perhaps for the first time, closed-loop fault-tolerant control for uncertain nonlinear systems. Our solution is based on a new algebraic estimation technique of the derivatives of a time signal, which

- yields good estimates of the unknown parameters and of the residuals, *i.e.*, of the fault indicators,
- is easily implementable in real time,
- is robust with respect to a large variety of noises, without any necessity of knowing their statistical properties.

Convincing numerical simulations are provided via a popular case-study in the diagnosis community, namely the three-tank system, which may be characterized as a flat *hybrid* system.

Key words: Fault diagnosis, fault-tolerant control, uncertain nonlinear systems, differential algebra, algebraic estimation techniques, derivatives of a noisy time signal, three-tank system.

1 Introduction

We are further developing recent works on closed-loop fault detection and isolation for linear [11] and nonlinear [10, 25] systems, which may contain uncertain parameters. This important subject which is attracting more and more attention (see, *e.g.*, [2, 3, 5, 19] and the references therein) is treated in the nonlinear case like in [10, 25], *i.e.*, via *differential algebra* and the estimation techniques of [16].

Introducing on-line *accommodation*, or *fault-tolerant control*, *i.e.*, the possibility of still controlling a nonlinear system if a fault does occur, is the main novelty of this article. We are therefore achieving in the context of diagnosis one of the fundamental aims of nonlinear control (see, *e.g.*, [27, 30, 37] and the references therein), *i.e.*, we are able to combine on-line parameter estimation, and closed-loop fault-tolerant control. The two main ingredients of our solution are:

- an algebraic estimation technique [15] which permits to obtain the derivatives of various orders of a noisy time signal¹, and thus excellent estimates of the unknown parameters and of the *residuals*, *i.e.*, of the fault indicators.
- *Differential flatness* (see [12, 13] and [31, 32, 33, 35]): we all know that this standpoint is already playing a crucial rôle in many concrete and industrial control applications.

Our solution moreover is robust with respect to a large variety of noises, without any necessity of knowing their statistical properties.

Our paper is organized as follows. Section 2 is introducing the basics of the differential algebraic setting. Its content with respect to fault variables completes and supersedes [10]. Section 3 recalls the techniques for estimating the derivatives of a noisy signal. Section 4 is devoted to the three-tank system, which is perhaps the most popular case-study in the fault-diagnosis community (see, *e.g.*, [29] and the references therein). Several simulations are illustrating our results which may be favorably compared to some recent studies on this subject (see, *e.g.*, [22]), where only off-line diagnosis was obtained. A short conclusion indicates some prolongations.

Acknowledgement. It is an honor and a pleasure for the authors to dedicate this work to Prof. M. Zeitz for his 65th birthday as a tribute to his wonderful scientific achievements. At least a few words in German are in order:

gewidmet Herrn Prof. Dr.-Ing. M. Zeitz zum 65. Geburtstag.

2 Differential algebra and nonlinear systems

We will not recall here the basics of the approach to nonlinear systems via differential fields², which is already well covered in the control literature (see, *e.g.*, [6, 12, 33, 35] and the references therein).

¹ This method was introduced in [16] where it gave a quite straightforward solution for obtaining nonlinear state *reconstructors*, *i.e.*, nonlinear state estimation, which are replacing asymptotic observers and (sub)optimal statistical filters, like the extended Kalman filters. See [9, 14] for applications in signal processing.

² All differential rings and fields (see, *e.g.*, [26] and [4]) are of characteristics zero and are *ordinary*, *i.e.*, they are equipped with a single derivation $\frac{d}{dt}$. A differential ring R is a commutative ring such that, $\forall a, b \in R$,

Notation. Write $k\langle X \rangle$ (resp. $k\{X\}$) the differential field (resp. ring) generated by the differential field k and the set X .

2.1 Perturbed uncertain nonlinear systems and fault variables

Let k_0 be a given differential ground field. Let $k = k_0(\Theta)$ be the differential field extension which is generated by a finite set $\Theta = (\theta_1, \dots, \theta_\alpha)$ of *uncertain parameters*, which are assumed to be constant³, i.e., $\dot{\theta}_\iota = 0$, $\iota = 1, \dots, \alpha$. A *nonlinear system* is a differential field extension K/k , which is generated by the sets S , π , \mathbf{W} , i.e., $K = k\langle S, \pi, \mathbf{W} \rangle$, where

1. S is a finite set of system variables,
2. $\pi = (\pi_1, \dots, \pi_r)$ denotes the *perturbation*, or *disturbance, variables*,
3. $\mathbf{W} = (\mathbf{w}_1, \dots, \mathbf{w}_q)$ denotes the *fault variables*.

They satisfy the following properties:

- The perturbation and fault variables do not “interact”, i.e., the differential extensions $k\langle \pi \rangle/k$ and $k\langle \mathbf{W} \rangle/k$ are *linearly disjoint* (see, e.g., [28]).
- The fault variables are assumed to be *independent*, i.e., \mathbf{W} is a differential transcendence basis of $k\langle \mathbf{W} \rangle/k$.
- Set $k\{S^{\text{nom}}, \mathbf{W}^{\text{nom}}\} = k\{S, \pi, \mathbf{W}\}/(\pi)$, where
 - $(\pi) \subset k\{S, \pi, \mathbf{W}\}$ is the differential ideal generated by π ,
 - the *nominal* fault variables $S^{\text{nom}}, \mathbf{W}^{\text{nom}}$ are the canonical images of S, \mathbf{W} .

Assume that the ideal (π) is *prime*⁴. The *nominal system* is K^{nom}/k , where $K^{\text{nom}} = k\langle S^{\text{nom}}, \mathbf{W}^{\text{nom}} \rangle$ is the quotient field of $k\{S^{\text{nom}}, \mathbf{W}^{\text{nom}}\}$.

- Set $k\{S^{\text{pure}}\} = k\{S^{\text{nom}}, \mathbf{W}^{\text{nom}}\}/(\mathbf{W}^{\text{nom}})$ where
 - the differential ideal $(\mathbf{W}^{\text{nom}}) \subset k\{S^{\text{nom}}, \mathbf{W}^{\text{nom}}\}$ is generated by \mathbf{W}^{nom} ,
 - the *pure* system variables S^{pure} are the canonical images of S^{nom} .

Assume that the ideal $(\mathbf{W}^{\text{nom}})$ is prime. The *pure system* is K^{pure}/k , where $K^{\text{pure}} = k\langle S^{\text{pure}} \rangle$ is the quotient field of $k\{S^{\text{pure}}\}$.

$$\begin{aligned} \frac{d}{dt}(a+b) &= \dot{a} + \dot{b} \\ \frac{d}{dt}(ab) &= \dot{a}b + a\dot{b} \end{aligned}$$

A differential field is a differential ring which is a field. A *constant* is an element $c \in R$ such that $\dot{c} = 0$.

³ This assumption may be easily removed in our general presentation.

⁴ An ideal \mathfrak{J} of a ring R is said to be *prime* [28] if, and only if, one of the two following equivalent conditions is verified:

- the quotient ring R/\mathfrak{J} is entire, i.e., without non-trivial zero divisors,
- $\forall x, y \in R$ such that $xy \in \mathfrak{J}$, then $x \in \mathfrak{J}$ or $y \in \mathfrak{J}$.

The assumptions for (π) and below for $(\mathbf{W}^{\text{nom}})$ being prime are thus natural.

A *dynamics* is a system where a finite subset $\mathbf{u} = (u_1, \dots, u_m) \subset S$ of *control* variables has been distinguished, such that the extension $K^{\text{pure}}/k\langle\mathbf{u}^{\text{pure}}\rangle$ is differentially algebraic. The control variables verify the next two properties:

- they do not interact with the fault variables, *i.e.*, the fields $k\langle\mathbf{u}\rangle$ and $k\langle\mathbf{W}\rangle$ are linearly disjoint over k .
- they are *independent*, *i.e.*, the components of \mathbf{u} are differentially algebraically independent over k .

An *input-output system* is a dynamics where a finite subset $\mathbf{y} = (y_1, \dots, y_p) \subset S$ of output variables has been distinguished. Only input-output systems will be considered in the sequel.

2.2 Differential flatness

A system K/k is said to be (*differentially*) *flat* if, and only if, there exists a finite set $\mathbf{z} = (z_1, \dots, z_m)$ of elements in the algebraic closure of K such that

- its components are differentially algebraically independent over k ,
- the algebraic closures of K^{pure} and $k\langle\mathbf{z}^{\text{pure}}\rangle$ are the same.

The set \mathbf{z} is called a *flat* output. It means that

- any pure system variable is a function of the components of the pure flat output and of their derivatives up to some finite order,
- any component of the pure flat output is a function of the pure system variables and of their derivatives up to some finite order,
- the components of the flat output are not related by any nontrivial differential relation.

The next property is well known [12, 13]:

Proposition 1. *Take a flat dynamics with independent control variables, then the cardinalities of \mathbf{z} and \mathbf{u} are equal.*

2.3 Detectability, isolability and parity equations for fault variables

The fault variable \mathbf{w}_ι , $\iota = 1, \dots, q$, is said to be *detectable* if, and only if, the field extension $K^{\text{nom}}/k\langle\mathbf{u}^{\text{nom}}, \mathbf{W}_\iota^{\text{nom}}\rangle$, where $\mathbf{W}_\iota^{\text{nom}} = \mathbf{W}^{\text{nom}} \setminus \{\mathbf{w}_\iota^{\text{nom}}\}$, is differentially transcendental. It means that \mathbf{w}_ι is indeed “influencing” the output.

A subset $\mathbf{W}' = (\mathbf{w}_{\iota_1}, \dots, \mathbf{w}_{\iota_{q'}})$ of the set \mathbf{W} of fault variables is said to be

- *Differentially algebraically isolable* if, and only if, the extension

$$k\langle\mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}}, \mathbf{W}'^{\text{nom}}\rangle/k\langle\mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}}\rangle \quad (1)$$

is differentially algebraic. It means that any component of \mathbf{W}'^{nom} satisfies a *parity differential equation*, *i.e.*, an algebraic differential equations where the coefficients belong to $k\langle\mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}}\rangle$.

- *Algebraically isolable* if, and only if, the extension (1) is algebraic. It means that the parity differential equation is of order 0, *i.e.*, it is an algebraic equation with coefficients $k\langle \mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}} \rangle$.
- *Rationally isolable* if, and only if, \mathbf{W}^{nom} belongs to $k\langle \mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}} \rangle$. It means that the parity equation is a linear algebraic equation, *i.e.*, any component of \mathbf{W}^{nom} may be expressed as a rational function over k in the variables $\mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}}$ and their derivatives up to some finite order.

The next property is obvious:

Proposition 2. *Rational isolability \Rightarrow algebraic isolability \Rightarrow differentially algebraic isolability.*

When we will say for short that fault variables are *isolable*, it will mean that they are differentially algebraically isolable.

Proposition 3. *Assume that all fault variables belonging to \mathbf{W}' are isolable, then*

$$\text{card}(\mathbf{W}') \leq \text{card}(\mathbf{y})$$

Proof. The differential transcendence degree⁵ of the extension

$$k\langle \mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}}, \mathbf{W}^{\text{nom}} \rangle / k$$

(resp. $k\langle \mathbf{u}^{\text{nom}}, \mathbf{y}^{\text{nom}} \rangle / k$) is equal to $\text{card}(\mathbf{u}) + \text{card}(\mathbf{W}')$ (resp. is less than or equal to $\text{card}(\mathbf{u}) + \text{card}(\mathbf{y})$). The equality of those two transcendence degrees implies our result.

2.4 Observability and identifiability

A system variable x , a component of the state for instance, is said to be *observable* [7, 8] if, and only if, x^{pure} is algebraic over $k\langle \mathbf{u}^{\text{pure}}, \mathbf{y}^{\text{pure}} \rangle$. It means in other words that x^{pure} satisfies an algebraic equation with coefficients in $k\langle \mathbf{u}^{\text{pure}}, \mathbf{y}^{\text{pure}} \rangle$. It is known [7, 8] that under some natural and mild conditions this definition is equivalent to the classic nonlinear extension of the Kalman rank condition for observability (see, *e.g.*, [24]).

A parameter θ is said to be *algebraically* (resp. *rationally*) *identifiable* [7, 8] if, and only if, it is algebraic over (resp. belongs to) $k\langle \mathbf{u}^{\text{pure}}, \mathbf{y}^{\text{pure}} \rangle$.

3 Estimation of the time derivatives⁶

Consider a real-valued time function $x(t)$ which is assumed to be analytic on some interval $t_1 \leq t \leq t_2$. Assume for simplicity's sake that $x(t)$ is analytic around $t = 0$ and introduce its truncated Taylor expansion

⁵ See, *e.g.*, [28] for the definition of the transcendence degree of a field extension.

See [26] for its obvious generalization to differential fields.

⁶ See [16] and [9] for more details and related references.

$$x(t) = \sum_{\nu=0}^N x^{(\nu)}(0) \frac{t^\nu}{\nu!} + o(t^N)$$

Approximate $x(t)$ in the interval $(0, \varepsilon)$, $\varepsilon > 0$, by a polynomial $x_N(t) = \sum_{\nu=0}^N x^{(\nu)}(0) \frac{t^\nu}{\nu!}$ of degree N . The usual rules of symbolic calculus in Schwartz's distributions theory [34] yield

$$x_N^{(N+1)}(t) = x(0)\delta^{(N)} + \dot{x}(0)\delta^{(N-1)} + \dots + x^{(N)}(0)\delta$$

where δ is the Dirac measure at 0. From $t\delta = 0$, $t\delta^{(\alpha)} = -\alpha\delta^{(\alpha-1)}$, $\alpha \geq 1$, we obtain the following triangular system of linear equations for determining estimated values $[x^{(\nu)}(0)]_e$ of the derivatives⁷ $x^{(\nu)}(0)$:

$$t^\alpha x^{(N+1)}(t) = t^\alpha ([x(0)]_e \delta^{(N)} + [\dot{x}(0)]_e \delta^{(N-1)} + \dots + [x^{(N)}(0)]_e \delta) \quad (2)$$

$\alpha = 0, \dots, N$

The time derivatives of $x(t)$ and the Dirac measures and its derivatives are removed by integrating with respect to time both sides of equation (2) at least N times:

$$\int^{(\nu)} \tau_1^\alpha x^{(N+1)}(\tau_1) = \int^{(\nu)} \tau_1^\alpha ([x(0)]_e \delta^{(N)} + [\dot{x}(0)]_e \delta^{(N-1)} + \dots + [x^{(N)}(0)]_e \delta) \quad (3)$$

$\nu \geq N, \alpha = 0, \dots, N$

where $\int^{(\nu)} = \int_0^t \int_0^{\tau_{\nu-1}} \dots \int_0^{\tau_1}$ is an iterated integral. A quite accurate value of the estimates may be obtained with a small time window $[0, t]$.

Remark 1. Those iterated integrals are moreover low pass filters⁸. They are attenuating highly fluctuating noises, which are usually dealt with in a statistical setting. We therefore do not need any knowledge of the statistical properties of the noises (see [14]).

4 Application to the three-tank system

4.1 Process description

The three-tank system can be conveniently represented as in [1] by:

⁷ Those quantities are *linearly identifiable* [15].

⁸ Those iterated integrals may be replaced by more general low pass filters, which are defined by strictly proper rational transfer functions.

$$\begin{cases} \dot{x}_1 = -D\mu_1 \text{sign}(x_1 - x_3) \sqrt{|x_1 - x_3|} \\ \quad + u_1/S + w_1/S \\ \dot{x}_2 = D\mu_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} \\ \quad - D\mu_2 \text{sign}(x_2) \sqrt{|x_2|} \\ \quad + u_2(t)/S + w_2/S \\ \dot{x}_3 = D\mu_1 \text{sign}(x_1 - x_3) \sqrt{|x_1 - x_3|} \\ \quad - D\mu_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} + w_3/S \\ y_1 = x_1 + w_4 \\ y_2 = x_2 + w_5 \\ y_3 = x_3 + w_6 \end{cases} \quad (4)$$

where x_i , $i = 1, 2, 3$, is the liquid level in tank i . The control variables u_1 , u_2 are the input flows. The actuator and/or system faults w_1 , w_2 , w_3 represent power losses and/or leaks; w_4 , w_5 , w_6 are sensor faults. The constant parameters D , S are well known physical quantities. The viscosity coefficients μ_i , $i = 1, 2, 3$, are constant but uncertain.

The next result is an immediate consequence of proposition 3:

Proposition 4. *The fault variables w_1, \dots, w_6 are not simultaneously isolable.*

The pure system corresponding to system (4) may be called a *flat hybrid* system: it is flat in each one of the four regions defined by $x_1 > x_3$ or $x_1 < x_3$, and $x_2 > x_3$ or $x_2 < x_3$. In all possible cases, x_1 , x_3 are the components of a flat output.

4.2 Control

From the single outflow rate in tank 2 we may assume that system (4) is staying in the region defined by $x_1 < x_3$ and/or $x_3 < x_2$. We obtain the following pure open loop control, where $x_1^* = F_1$ and $x_3^* = F_3$,

$$u_1^* = S \left(\dot{F}_1 + D\mu_1 \sqrt{F_1 - F_3} \right)$$

and

$$u_2^* = S \left(\dot{F}_3 - D\mu_3 \sqrt{F_3 - x_2^*} + D\mu_2 \sqrt{x_2^*} \right)$$

where

$$x_2^* = F_3 - \left(\frac{-\dot{F}_3 + D\mu_1 \sqrt{F_1 - F_3}}{D\mu_3} \right)^2$$

The loop is closed via a nonlinear extension (see, also, [20, 21]⁹) of the classic *proportional-integral* (PI) controller:

$$u_1 = u_1^* + SD\mu_1 \sqrt{y_1 - y_3} - SD\mu_1 \sqrt{x_1^* - x_3^*} - P_1 S e_1 - P_2 S \int e_1$$

⁹ Those references also contain most useful material on the control of uncertain nonlinear systems.

$$u_2 = u_2^* - SD\mu_3\sqrt{y_3 - y_2} + SC_2\sqrt{y_2} + SD\mu_3\sqrt{x_3^* - x_2^*} - SD\mu_2\sqrt{x_2^*} - P_3Se_3 - P_4S \int e_3 \quad (5)$$

where $e_i = y_i - F_i^*$ is the tracking error. Set for the gain coefficients

$$P_1 = P_3 = 2.10^{-2}, \quad P_2 = P_4 = 2.10^{-4}$$

4.3 Simulation results

General principles

The estimations of the uncertain parameters and of the *residuals*¹⁰ are achieved via the estimations of the first order derivatives of the output variables.

Remark 2. In order to test the robustness of our approach, we have added a zero-mean Gaussian noise of variance 0.005.

Estimations of the viscosity coefficients

The values of the known system parameters are $D = 0.0144$, $S = 0.0154$. The nominal flatness-based reference trajectories are computed via the following nominal numerical values of the viscosity coefficients

$$\mu_1 = \mu_3 = 0.5, \quad \mu_2 = 0.675$$

whereas their true values are

$$\mu_1^{\text{real}} = \mu_1(1 + 0.33), \quad \mu_2^{\text{real}} = \mu_2(1 - 0.33), \quad \mu_3^{\text{real}} = \mu_3$$

The system behavior in the fault free case is presented figure 1. Those viscosity coefficients are algebraically identifiable:

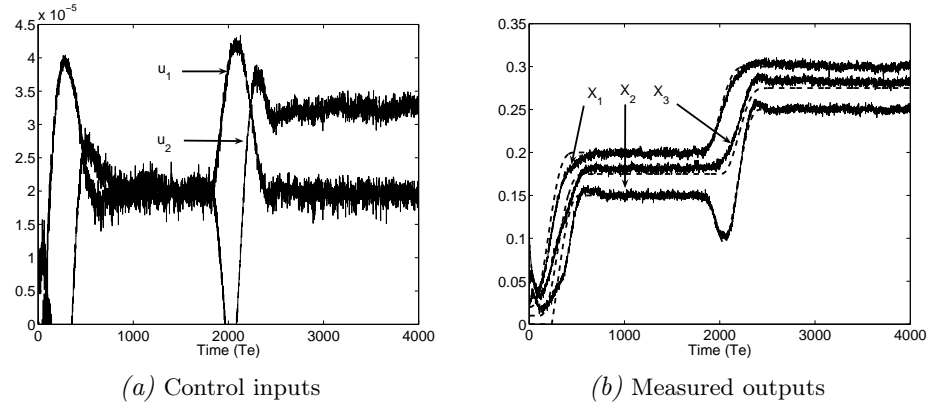
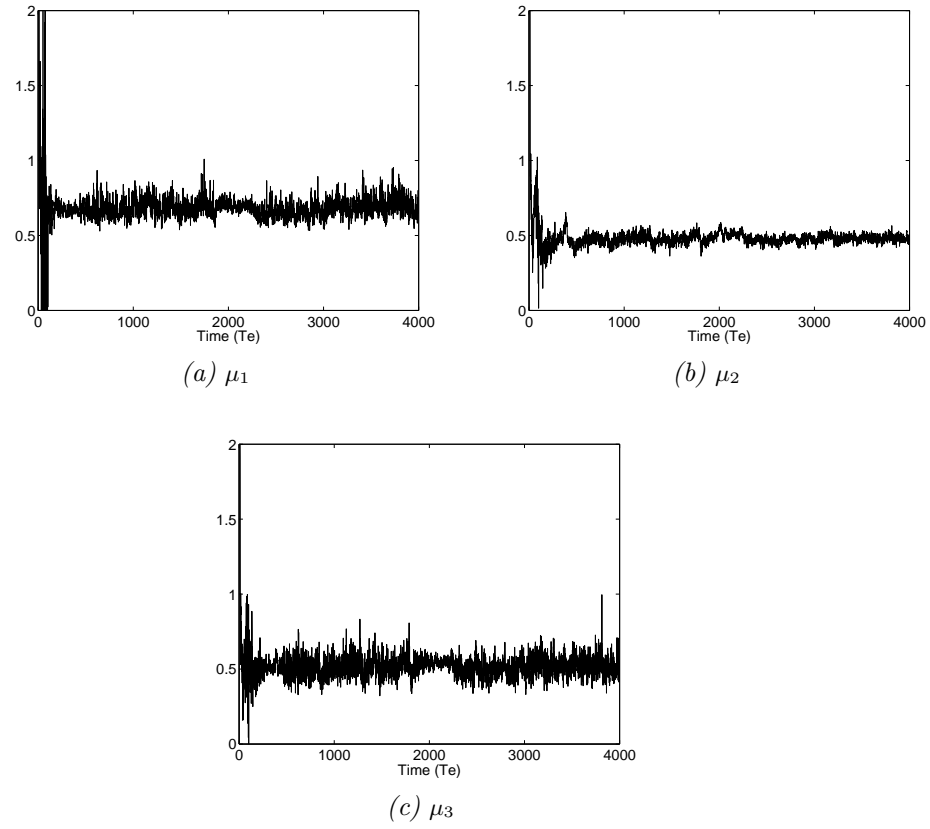
$$\begin{aligned} \mu_1 &= \frac{-(S\dot{y}_1 - u_1)}{SD\sqrt{y_1 - y_3}} \\ \mu_3 &= \frac{-(S\dot{y}_1 + S\dot{y}_3 - u_1)}{SD\sqrt{y_3 - y_2}} \\ \mu_2 &= \frac{-(S\dot{y}_1 + S\dot{y}_2 + S\dot{y}_3 - u_1 - u_2)}{SD\sqrt{y_2}} \end{aligned}$$

Their estimations, which yield

$$[\mu_1]_c = 0.6836, \quad [\mu_2]_c = 0.4339, \quad [\mu_3]_c = 0.4819$$

are presented in figure 2. After a short period of time has elapsed, those estimates become available for the implementation of our diagnosis and accommodation schemes.

¹⁰ A residual is a *fault indicator* which is usually deduced from some parity equation. Here it is obtained via the estimates of the unknown coefficients and of the derivatives of the control and output variables.

**Fig. 1.** Fault-free case**Fig. 2.** Estimations of the viscosity coefficients

Actuator and system faults

Fault diagnosis

Assuming only the existence of the fault variables w_1, w_2 yields their algebraic isolability:

$$\begin{aligned} w_1 &= S [\dot{y}_1 + \mu_1 D \sqrt{y_1 - y_3}] - u_1 \\ w_2 &= S [\dot{y}_2 - \mu_3 D \sqrt{y_3 - y_2} + \mu_2 D \sqrt{y_2}] - u_2 \end{aligned}$$

Convenient residuals r_1, r_2 are obtained by replacing in the above equations the viscosity coefficients μ_1, μ_2, μ_3 by their estimated values $[\mu_1]_c, [\mu_2]_c, [\mu_3]_c$:

$$\begin{aligned} r_1 &= S [\dot{y}_1 + [\mu_1]_c D \sqrt{y_1 - y_3}] - u_1 \\ r_2 &= S [\dot{y}_2 - [\mu_3]_c D \sqrt{y_3 - y_2} + [\mu_2]_c D \sqrt{y_2}] - u_2 \end{aligned}$$

Fault-tolerant control

Using the closed loop control u_1 and u_2 and the residual estimation, define a fault-tolerant control by

$$\begin{aligned} u_1^{[\text{FTC}]} &= u_1 + u_{a1} \\ u_2^{[\text{FTC}]} &= u_2 + u_{a2} \end{aligned}$$

where

- u_1, u_2 are given by formula (5),
- the additive control variables u_{a1}, u_{a2} are defined by

$$u_{a1} = -r_1, \quad u_{a2} = -r_2$$

The simulations are realized by assuming a detection delay T_{di} of the fault variable w_i .

Simulation comments

Figures 3-(c)-(d), 4-(c)-(d) and 5-(c)-(d) indicate an excellent fault diagnosis for the following three classic cases (see, *e.g.*, [17]):

1. $w_1 = -0.5u_1, w_2 = -0.5u_2$, for $t > 1000T_e$, where T_e is the sampling period (figures 3),
2. $w_1 = -0.5u_1, w_2 = -0.5u_2$ for $t > 2000T_e$ (figures 4),
3. $w_1 = (-0.5 - \frac{t}{16000T_e})u_1, w_2 = (-0.5 - \frac{t}{16000T_e})u_2$, for $t > 1000T_e$ (figures 5).

The behavior for the residuals changes at time $t = 500T_e$. This is due to the fact that the nominal value of μ_i is being used for $t < 500T_e$. The interest of the fault-tolerant control is demonstrated in figures 3, 4 and 5. Note that the simulations were realized with a delay of $T_{di} = 100T_e$ for the fault-tolerant control.

In figure 6 the system is corrupted by two major faults variables, where $w_1 = -0.9u_1, w_2 = -0.9u_2$ for $t > 1000T_e$. The fault-tolerant control is then saturating the actuator. The output references cannot be reached.

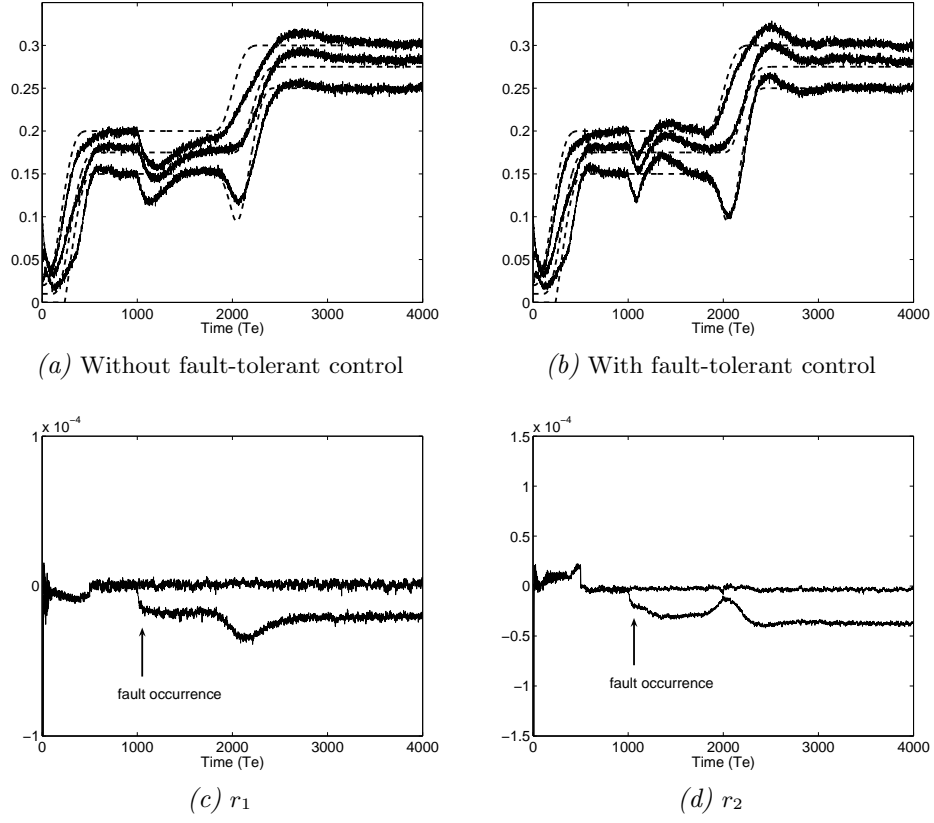


Fig. 3. Fault occurrence in steady mode

Combination of system and sensor faults

Fault diagnosis

We associate here the leak w_2 and the sensor fault w_4 , which is algebraically isolable:

$$w_4 = y_1 - y_3 - \left(\frac{\dot{y}_3 + \mu_3 D(y_3 - y_2) \sqrt{y_3 - y_2}}{\mu_1 D} \right)^2$$

It yields in the same way as before the residual

$$r_4 = y_1 - y_3 - \left(\frac{\dot{y}_3 + [\mu_3]_c D(y_3 - y_2) \sqrt{y_3 - y_2}}{[\mu_1]_c D} \right)^2$$

Fault-tolerant control

The leak w_2 is accommodated as in section 4.3. For the sensor fault w_4 , accommodation is most simply achieved by subtracting r_4 from the measurement y_1 when closing the loop (5).

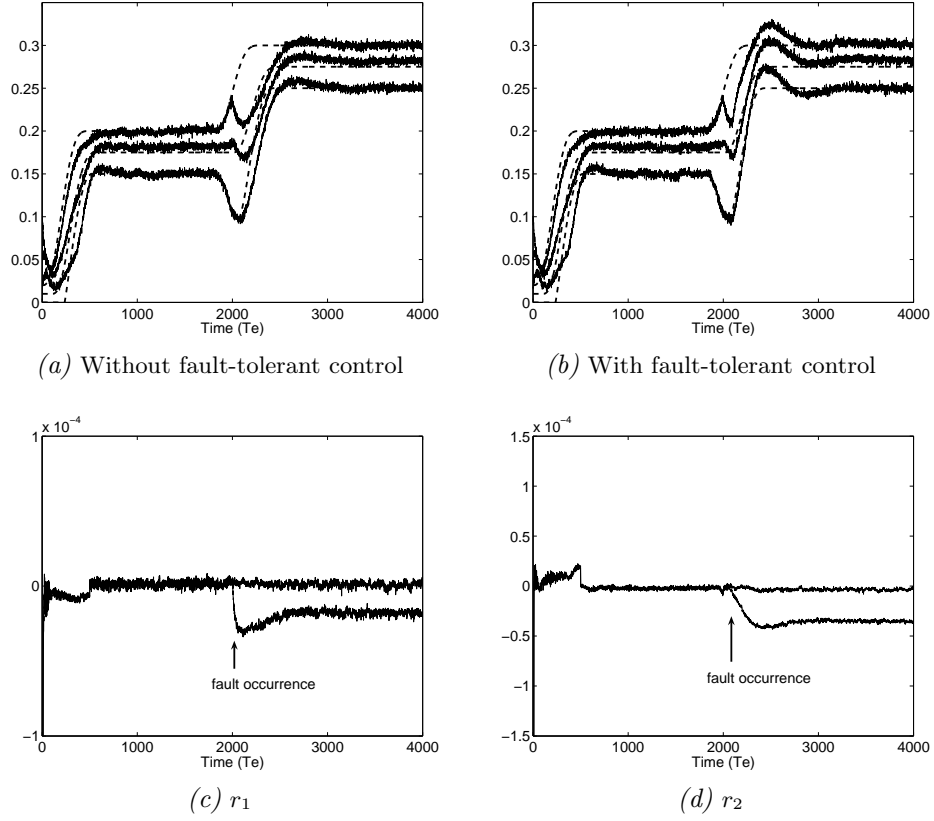


Fig. 4. Fault occurrence in dynamical mode

Simulation comments

Figures 7-(c)-(d) (resp. 7-(a)-(b)) show an excellent fault diagnosis (resp. accommodation) for $w_2(t) = -0.3[\mu_2]_c D\sqrt{x_2}$, for $t > 1000T_e$, and $w_4(t) = 0.02$, for $t > 2500T_e$.

Remark 3. Figures 8-(a) and 8-(b), when compared to figures 1-(b) and 7-(b), show quite better performances of the feedback loop (5) when the nominal values of the viscosity coefficients are replaced by the estimated ones.

5 Conclusion

This communication should be viewed as a first draft of a full paper which will comprise also state estimation [16, 36] and many more examples. Those simple

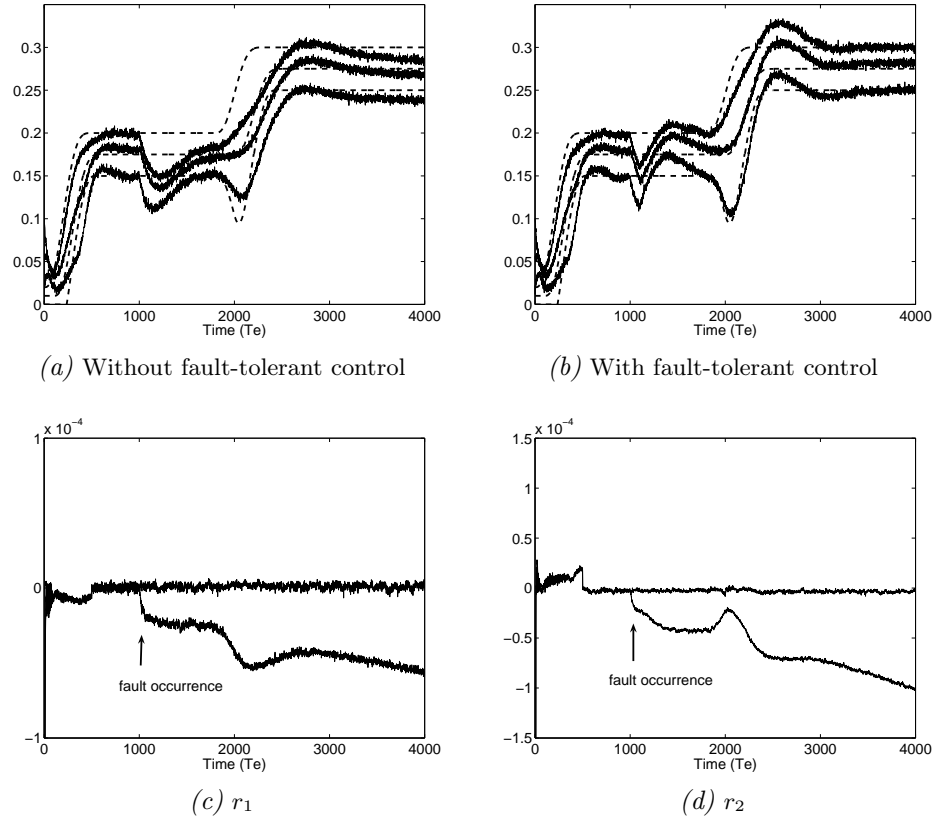


Fig. 5. Fault of type 3

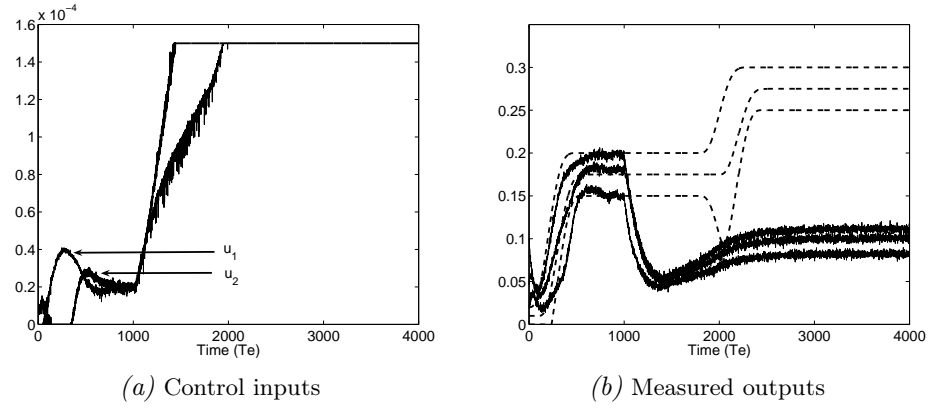
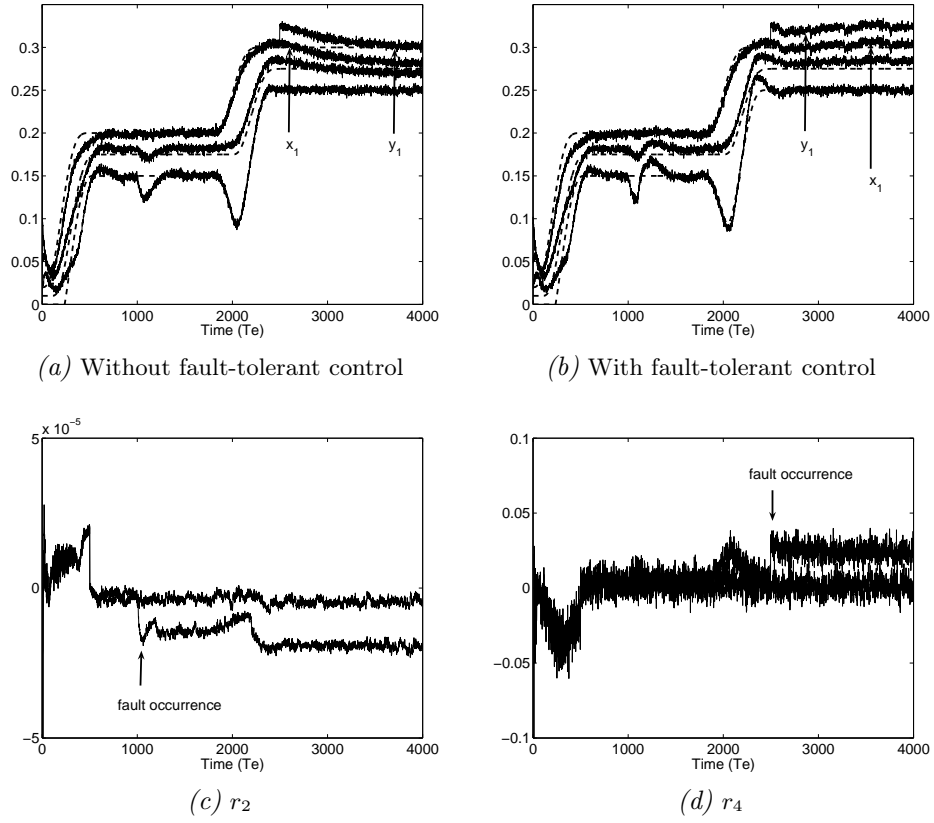


Fig. 6. Actuator saturations

**Fig. 7.** Leak and sensor faults

solutions of long-standing problems in nonlinear control are robust with respect to a large variety of perturbations and may be quite easily implemented in real time. They were made possible by a complete change of viewpoint on estimation techniques, where the classic asymptotic and/or probabilistic methods are abandoned¹¹.

Further studies will demonstrate the possibility of controlling nonlinear systems with poorly known models, *i.e.*, not only with uncertain parameters.

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¹¹ See [16] for more details.

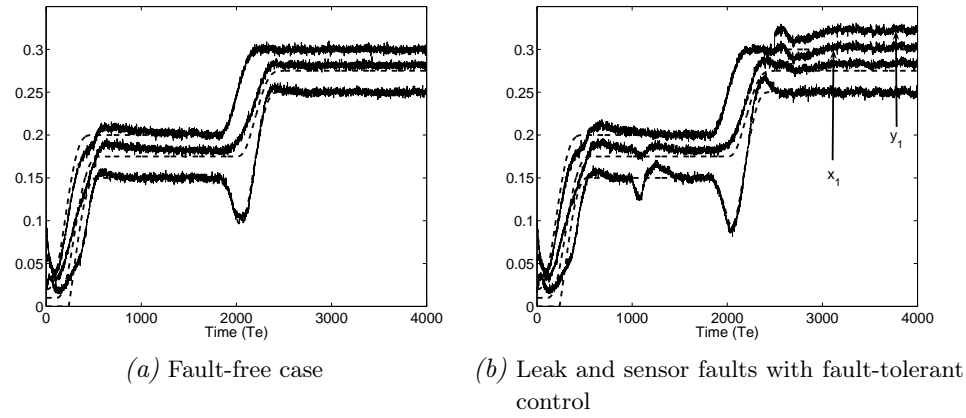


Fig. 8. Control feedback with the estimated viscosity coefficients

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Index

accomodation, 2	identifiability, 5
detectability, 4	input-output system, 4
differential algebra, 2	isolability, 4
dynamics, 4	nonlinear system, 2
estimation of the time derivatives, 5	observability, 5
fault indicator, 8	residual, 8
fault variable, 3	three-tank system, 6
fault-tolerant control, 2	uncertain parameters, 3
flat hybrid system, 7	
flat system, 4	

