Strategies for Replica Placement in Tree Networks

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Abstract: In this paper, we discuss and compare several policies to place replicas in tree networks, subject to server capacity and QoS constraints. The client requests are known beforehand, while the number and location of the servers are to be determined. The standard approach in the literature is to enforce that all requests of a client be served by the closest server in the tree. We introduce and study two new policies. In the first policy, all requests from a given client are still processed by the same server, but this server can be located anywhere in the path from the client to the root. In the second policy, the requests of a given client can be processed by multiple servers.

One major contribution of this paper is to assess the impact of these new policies on the total replication cost. Another important goal is to assess the impact of server heterogeneity, both from a theoretical and a practical perspective. In this paper, we establish several new complexity results, and provide several efficient polynomial heuristics for NP-complete instances of the problem. These heuristics are compared to an absolute lower bound provided by the formulation of the problem in terms of the solution of an integer linear program.

Key-words: Replica placement, tree networks, access policy, scheduling, complexity results, heuristics, heterogeneous clusters.

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Stratégies de placement de répliques sur des arbres

Résumé : Dans ce rapport nous présentons et comparons plusieurs politiques de placement de répliques sur des arbres, prenant en compte à la fois des contraintes liées à la capacité de traitement de chaque serveur et des contraintes de type QoS (qualité de service). Les requêtes des clients sont connues avant exécution, alors que le nombre et l'emplacement des répliques (serveurs) sont à déterminer par l'algorithme de placement. L'approche classique impose que toutes les requêtes d'un client donné soient traitées par un seul serveur, à savoir le plus proche du client dans l'arbre. Nous introduisons deux nouvelles politiques de placement. Dans la première, chaque client a toujours un serveur unique, mais ce dernier peut être situé n'importe où sur le chemin qui mène du client à la racine dans l'arbre. Avec la deuxième politique, les requêtes d'un même client peuvent être traitées par plusieurs serveurs sur ce même chemin.

Nous montrons que ces deux nouvelles politiques de placement sont à même de réduire fortement le coût total de la réplication. Un autre objectif de ce travail est l'analyse de l'impact de l'hétérogénéité de la plate-forme, à la fois d'un point de vue théorique et pratique. Sur le plan théorique, nous établissons plusieurs résultats de complexité, dans les cadres homogène et hétérogène, pour l'approche classique et les nouvelles politiques. Sur le plan pratique, nous concevons des heuristiques polynomiales pour les instances combinatoires du problème. Nous comparons les performances de ces heuristiques en les rapportant à une borne inférieure absolue sur le coût total de la réplication; cette borne est obtenue par relaxation d'un programme linéaire en nombre entiers qui caractérise la solution optimale du problème.

Mots-clés : Placement de répliques, réseaux en arbre, ordonnancement, complexité, heuristiques, grappes de calcul hétérogènes.

Contents

1 Introduction

In this paper, we consider the general problem of replica placement in tree networks. Informally, there are clients issuing requests to be satisfied by servers. The clients are known (both their position in the tree and their number of requests), while the number and location of the servers are to be determined. A client is a leaf node of the tree, and its requests can be served by one or several internal nodes. Initially, there are no replica; when a node is equipped with a replica, it can process a number of requests, up to its capacity limit. Nodes equipped with a replica, also called servers, can only serve clients located in their subtree (so that the root, if equipped with a replica, can serve any client); this restriction is usually adopted to enforce the hierarchical nature of the target application platforms, where a node has knowledge only of its parent and children in the tree.

The rule of the game is to assign replicas to nodes so that some optimization function is minimized. Typically, this optimization function is the total utilization cost of the servers. If all the nodes are identical, this reduces to minimizing the number of replicas. If the nodes are heterogeneous, it is natural to assign a cost proportional to their capacity (so that one replica on a node capable of handling 200 requests is equivalent to two replicas on nodes of capacity 100 each).

The core of the paper is devoted to the study of the previous optimization problem, called Replica Placement in the following. Additional constraints are introduced, such as guaranteeing some Quality of Service (QoS): the requests must be served in limited time, thereby prohibiting too remote or hard-to-reach replica locations. Also, the flow of requests through a link in the tree cannot exceed some bandwidth-related capacity. We focus on optimizing the total utilization cost (or replica number in the homogeneous case). There is a bunch of possible extensions: dealing with several object types rather than one, including communication time into the objective function, taking into account an update cost of the replicas, and so on. For the sake of clarity we devote a special section (Section 8) to formulate these extensions, and to describe which situations our results and algorithms can still apply to.

We point out that the distribution tree (clients and nodes) is fixed in our approach. This key assumption is quite natural for a broad spectrum of applications, such as electronic, ISP, or VOD service delivery. The root server has the original copy of the database but cannot serve all clients directly, so a distribution tree is deployed to provide a hierarchical and distributed access to replicas of the original data. On the contrary, in other, more decentralized, applications (e.g. allocating Web mirrors in distributed networks), a two-step approach is used: first determine a "good" distribution tree in an arbitrary interconnection graph, and then determine a "good" placement of replicas among the tree nodes. Both steps are interdependent, and the problem is much more complex, due to the combinatorial solution space (the number of candidate distribution trees may well be exponential).

Many authors deal with the Replica Placement optimization problem, and we survey related work in Section 9. The objective of this paper is twofold: (i) introducing two new access policies and comparing them with the standard approach; (ii) assessing the impact of server heterogeneity on the problem.

In most, if not all, papers from the literature, all requests of a client are served by the closest replica, i.e. the first replica found in the unique path from the client to the root in the distribution tree. This *Closest* policy is simple and natural, but may be unduly restrictive, leading to a waste of resources. We introduce and study two different approaches: in the first one, we keep the restriction that all requests from a given client are processed by the same replica, but we allow client requests to "traverse" servers so as to be processed by other replicas located higher in the path (closer to the root). We call this approach the *Upwards* policy. The trade-of to explore is the following: the *Closest* policy assigns replicas at proximity of the clients, but may need to allocate too many of them if some local subtree issues a great number of requests. The *Upwards* policy will ensure a better resource usage, load-balancing the process of requests on a larger scale; the possible drawback is that requests will be served by remote servers, likely to take longer time to process them. Taking QoS constraints into account would typically be more important for the *Upwards* policy.

In the second approach, we further relax access constraints and grant the possibility for a client to be assigned several replicas. With this *Multiple* policy, the processing of a given client's requests will be split among several servers located in the tree path from the client to the root. Obviously, this policy is the most flexible, and likely to achieve the best resource usage. The only drawback is the (modest) additional complexity induced by the fact that requests must now be tagged with the replica server ID in addition to the client ID. As already stated, one major objective of this paper is to compare these three access policies, *Closest*, *Upwards* and *Multiple*.

The second major contribution of the paper is to assess the impact of server heterogeneity, both from a theoretical and a practical perspective. Recently, several variants of the Replica Placement optimization problem with the *Closest* policy have been shown to have polynomial complexity. In this paper, we establish several new complexity results. Those for the homogeneous case are surprising: for the simplest instance without QoS nor bandwidth constraints, the *Multiple* policy is polynomial (as *Closest*) while *Upwards* is NP-hard. The three policies turn out to be NP-complete for heterogeneous nodes, which provides yet another example of the additional difficulties induced by resource heterogeneity. On the more practical side, we provide an optimal algorithm for the *Multiple* problem with homogeneous nodes, and several heuristics for all three policies in the heterogeneous case. We compare these heuristics through simulations conducted for problem instances without QoS nor bandwidth constraints. Another contribution is that we are able to assess the absolute performance of the heuristics, not just comparing one to the other, owing to a lower bound provided by a new formulation of the Replica Placement problem in terms of an integer linear program: the relaxation of this program to the rational numbers provides a lower bound to the solution cost (which is not always feasible).

The rest of the paper is organized as follows. Section 2 is devoted to a detailed presentation of the target optimization problems. In Section 3 we introduce the three access policies, and we give a few motivating examples. Next in Section 4 we proceed to the complexity results for the simplest version of the Replica Placement problem, both in the homogeneous and heterogeneous cases. Section 5 deals with the formulation for the Replica Placement problem in terms of an integer linear program. In Section 6 we introduce several polynomial heuristics to solve the Replica Placement problem with the different access policies. These heuristics are compared through simulations, whose results are analyzed in Section 7. Section 8 discusses various extensions to the Replica Placement problem while Section 9 is devoted to an overview of related work. Finally, we state some concluding remarks in Section 10.

2 Framework

This section is devoted to a precise statement of the Replica Placement optimization problem. We start with some definitions and notations. Next we outline the simplest instance of the problem. Then we describe several types of constraints that can be added to the formulation.

2.1 Definitions and notations

We consider a distribution tree $\mathcal{T}$ whose nodes are partitioned into a set of clients $\mathcal{C}$ and a set of nodes $\mathcal{N}$. The set of tree edges is denoted as $\mathcal{L}$. The clients are leaf nodes of the tree, while $\mathcal{N}$ is the set of internal nodes. It would be easy to allow *client-server* nodes which play both the rule of a client and of an internal node (possibly a server), by dividing such a node into two distinct nodes in the tree, connected by an edge with zero communication cost.

A *client* $i \in \mathcal{C}$ is making requests to database objects. For the sake of clarity, we restrict the presentation to a single object type, hence a single database. We deal with several object types in Section 8.

A *node* $j \in \mathcal{N}$ may or may not have been provided with a replica of the database. Nodes equipped with a replica (*i.e.* servers) can process requests from clients in their subtree. In other words, there is a unique path from a client i to the root of the tree, and each node in this path is eligible to process some or all the requests issued by i when provided with a replica.

Let r be the root of the tree. If $j \in \mathcal{N}$, then $\mathsf{children}(j)$ is the set of children of node j. If $k \neq r$ is any node in the tree (leaf or internal), $\mathsf{parent}(k)$ is its parent in the tree. If $l : k \rightarrow k' = \mathsf{parent}(k)$ is any link in the tree, then $\mathsf{succ}(l)$ is the link $k' \rightarrow \mathsf{parent}(k')$ (when it exists). Let $\mathsf{Ancestors}(k)$ denote the set of ancestors of node k, i.e. the nodes in the unique path that leads from k up to the root r (k excluded). If $k' \in \mathsf{Ancestors}(k)$, then $\mathsf{path}[k \rightarrow k']$ denotes the set of links in the path from k to k'; also, $\mathsf{subtree}(k)$ is the subtree rooted in k, including k.

We introduce more notations to describe our system in the following.

- **Clients** $i \in \mathcal{C}$ – Each client i (leaf of the tree) is sending r_i requests per time unit. For such requests, the required QoS (typically, a response time) is denoted q_i, and we need to ensure that this QoS will be satisfied for each client.
- **Nodes** $j \in \mathcal{N}$ – Each node j (internal node of the tree) has a processing capacity W_j, which is the total number of requests that it can process per time-unit when it has a replica. A cost is also associated to each node, sc_j, which represents the price to pay to place a replica at this node. With a single object type it is quite natural to assume that sc_j is proportional to W_j: the more powerful a server, the more costly. But with several objects we may use non-related values of capacity and cost.
- **Communication links** $l \in \mathcal{L}$ – The edges of the tree represent the communication links between nodes (leaf and internal). We assign a communication time comm_l on link l which is the time required to send a request through the link. Moreover, BW_l is the maximum number of requests that link l can transmit per time unit.

2.2 Problem instances

For each client $i \in \mathcal{C}$, let $\mathsf{Servers}(i) \subseteq \mathcal{N}$ be the set of servers responsible for processing at least one of its requests. We do not specify here which access policy is enforced (e.g. one or multiple servers), we defer this to Section 3. Instead, we let $r_{i,s}$ be the number of requests from client i processed by server s (of course, $\sum_{s \in \mathsf{Servers}(i)} r_{i,s} = r_i$). In the following, R is the set of replicas:

$$R = \{s \in \mathcal{N} | \ \exists i \in C \ , \ s \in \mathsf{Servers}(i)\} \, .$$

2.2.1 Constraints

Three main types of constraints are considered.

Server capacity – The constraint that no server capacity can be exceeded is present in all variants of the problem:

$$\forall s \in R, \sum_{i \in \mathcal{C} | s \in \mathsf{Servers}(i)} r_{i,s} \leq \mathsf{W}_s$$

QoS – Some problem instances enforce a quality of service: the time to transfer a request from a client to a replica server is bounded by a quantity q_i. This translates into:

$$\forall i \in \mathcal{C}, \forall s \in \mathsf{Servers}(i), \sum_{l \in \mathsf{path}[i \rightarrow s]} \mathsf{comm}_l \leq \mathsf{q}_i .$$

Note that it would be easy to extend the QoS constraint so as to take the computation cost of a request in addition to its communication cost. This former cost is directly related to the computational speed of the server and the amount of computation (in flops) required for each request.

Link capacity – Some problem instances enforce a global constraint on each communication link $l \in \mathcal{L}$:

$$\sum_{i \in \mathcal{C}, s \in \mathsf{Servers}(i) | l \in \mathsf{path}[i \rightarrow s]} r_{i,s} \leq \mathsf{BW}_l$$

2.2.2 Objective function

The objective function for the Replica Placement problem is defined as:

$$\text{Min} \sum_{s \in R} \mathsf{sc}_s$$

As already pointed out, it is frequently assumed that the cost of a server is proportional to its capacity, so in some problem instances we let $\mathsf{sc}_s = \mathsf{W}_s$.

2.2.3 Simplified problems

We define a few simplified problem instances in the following:

QoS=distance – We can simplify the expression of the communication time in the QoS constraint and only consider the distance (in number of hops) between a client and its server(s). The QoS constraint is then

$$\forall i \in \mathcal{C}, \forall s \in \mathsf{Servers}(i),\ d(i,s) \leq \mathsf{q}_i$$

where the distance $d(i,s) = |\mathsf{path}[i \to s]|$ is the number of communication links between i and s.

No QoS – We may further simplify the problem, by completely suppressing the QoS constraints. In this case, the servers can be anywhere in the tree, their location is indifferent to the client.

No link capacity – We may consider the problem assuming infinite link capacity, i.e. not bounding the total traffic on any link in an admissible solution.

Only server capacities – The problem without QoS and link capacities reduces to finding a valid solution of minimal cost, where "valid" means that no server capacity is exceeded. We name Replica Cost this fundamental problem.

Replica counting – We can further simplify the previous Replica Cost problem in the homogeneous case: with identical servers, the Replica Cost problem amounts to minimize the number of replicas needed to solve the problem. In this case, the storage cost sc_j is set to 1 for each node. We call this problem Replica Counting.

3 Access policies

In this section we review the usual policies enforcing which replica is accessed by a given client. Consider that each client i is making r_i requests per time-unit. There are two scenarios for the number of servers assigned to each client:

Single server – Each client i is assigned a single server $\mathsf{server}(i)$, that is responsible for processing all its requests.

Multiple servers – A client i may be assigned several servers in a set $\mathsf{Servers}(i)$. Each server $s \in \mathsf{Servers}(i)$ will handle a fraction $r_{i,s}$ of the requests. Of course $\sum_{s \in \mathsf{Servers}(i)} r_{i,s} = r_i$.

To the best of our knowledge, the single server policy has been enforced in all previous approaches. One objective of this paper is to assess the impact of this restriction on the performance of data replication algorithms. The single server policy may prove a useful simplification, but may come at the price of a non-optimal resource usage.

In the literature, the single server strategy is further constrained to the *Closest* policy. Here, the server of client i is constrained to be the first server found on the path that goes from i upwards to the root of the tree. In particular, consider a client i and its server $\mathsf{server}(i)$. Then any other client node i' residing in the subtree rooted in $\mathsf{server}(i)$ will be assigned a server in that subtree. This forbids requests from i' to "traverse" $\mathsf{server}(i)$ and be served higher (closer to the root in the tree).

We relax this constraint in the *Upwards* policy which is the general single server policy. Notice that a solution to *Closest* always is a solution to *Upwards*, thus *Upwards* is always better than *Closest* in terms of the objective function. Similarly, the *Multiple* policy is always better than *Upwards*, because it is not constrained by the single server restriction.

The following sections illustrate the three policies. Section 3.1 provides simple examples where there is a valid solution for a given policy, but none for a more constrained one. Section 3.2 shows that *Upwards* can be arbitrarily better than *Closest*, while Section 3.3 shows that *Multiple* can be arbitrarily better than *Upwards*. We conclude with an example showing that the cost of an optimal solution of the Replica Counting problem (for any policy) can be arbitrarily higher than the obvious lower bound

$$\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil ,$$

where W is the server capacity.

3.1 Impact of the access policy on the existence of a solution

We consider here a very simple instance of the Replica Counting problem. In this example there are two nodes, s_1 being the unique child of s_2, the tree root (see Figure 1). Each node can process $\mathsf{W} = 1$ request.

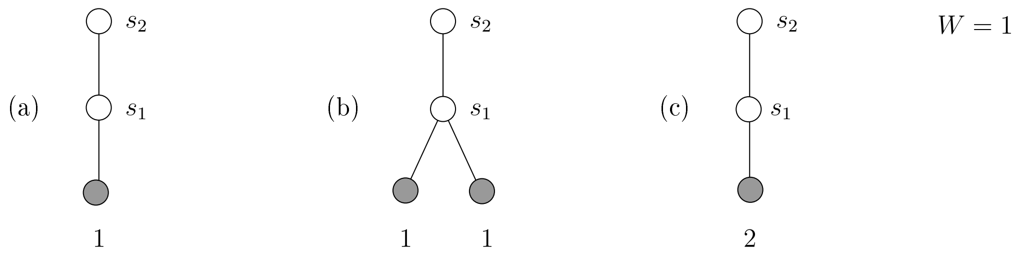


Figure 1: Access policies.

- If s_1 has one client child making 1 request, the problem has a solution with all three policies, placing a replica on s_1 or on s_2 indifferently (Figure 1(a)).
- If s_1 has two client children, each making 1 request, the problem has no more solution with *Closest*. However, we have a solution with both *Upwards* and *Multiple* if we place replicas on both nodes. Each server will process the request of one of the clients (Figure 1(b)).
- Finally, if s_1 has only one client child making 2 requests, only *Multiple* has a solution since we need to process one request on s_1 and the other on s_2, thus requesting multiple servers (Figure 1(c)).

This example demonstrates the usefulness of the new policies. The *Upwards* policy allows to find solutions when the classical *Closest* policy does not. The same holds true for *Multiple* versus *Upwards*. In the following, we compare the cost of solutions obtained with different strategies.

3.2 *Upwards* versus *Closest*

In the following example, we construct an instance of Replica Counting where the cost of the *Upwards* policy is arbitrarily lower than the cost of the *Closest* policy. We consider the tree network of Figure 2, where there are $2n + 2$ internal nodes, each with $\mathsf{W}_j = W = n$, and $2n + 1$ clients, each with $r_i = r = 1$.

With the *Upwards* policy, we place three replicas in s_{2n}, s_{2n+1} and s_{2n+2}. All requests can be satisfied with these three replicas.

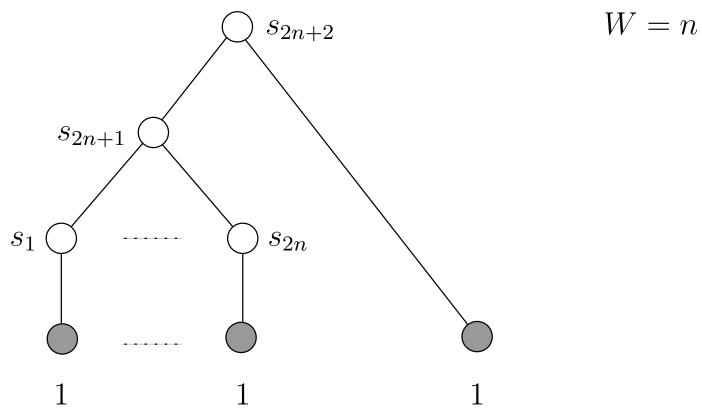


Figure 2: *Upwards* versus *Closest*

When considering the *Closest* policy, first we need to place a replica in s_{2n+2} to cover its client. Then,

- Either we place a replica on s_{2n+1}. In this case, this replica is handling n requests, but there remain n other requests from the $2n$ clients in its subtree that cannot be processed by s_{2n+2}. Thus, we need to add n replicas between $s_1..s_{2n}$.
- Otherwise, $n-1$ requests of the $2n$ clients in the subtree of s_{2n+1} can be processed by s_{2n+2} in addition to its own client. We need to add $n+1$ extra replicas among $s_1, s_2, \ldots, s_{2n}$.

In both cases, we are placing $n+2$ replicas, instead of the 3 replicas needed with the *Upwards* policy. This proves that *Upwards* can be arbitrary better than *Closest* on some Replica Counting instances.

3.3 *Multiple* versus *Upwards*

In this section we build an instance of the Replica Counting problem where *Multiple* is twice better than *Upwards*. We do not know whether there exist instances of Replica Counting where the performance ratio of *Multiple* versus *Upwards* is higher than 2 (and we conjecture that this is not the case). However, we also build an instance of the Replica Cost problem (with heterogeneous nodes) where *Multiple* is arbitrarily better than *Upwards*.

We start with the homogeneous case. Consider the instance of Replica Counting represented in Figure 3, with $3n+1$ nodes of capacity $\mathsf{W}_j = W = 2n$. The root r has $n+1$ children, n nodes labeled s_1 to s_n and a client with $r_i = n$. Each node s_j has two children nodes, labeled v_j and w_j for $1 \le j \le n$. Each node v_j has a unique child, a client with $r_i = n$ requests; each node w_j has a unique child, a client with $r_i = n+1$ requests.

The *Multiple* policy assigns $n+1$ replicas, one to the root r and one to each node s_j. The replica in s_j can process all the $2n+1$ requests in its subtree except one, which is processed by the root.

For the *Upwards* policy, we need to assign one replica to r, to cover its client. This replica can process n other requests, for instance those from the client child of v_1. We need to place at least a replica in s_1 or in w_1, and $2(n-1)$ replicas in v_j and w_j for $2 \le j \le n$. This leads to a total of $2n$ replicas, hence a performance factor $\frac{2n}{n+1}$ whose limit is to 2 when n tends to infinity.

We now proceed to the heterogeneous case. Consider the instance of Replica Cost represented in Figure 4, with 3 nodes s_1, s_2 and s_3, and 2 clients. The capacity of s_1 and s_2 is $\mathsf{W}_1 = \mathsf{W}_2 = n$ while that of s_3 is $\mathsf{W}_3 = Kn$, where K is arbitrarily large. Recall that in the Replica Cost problem, we let $\mathsf{sc}_j = \mathsf{W}_j$ for each node. *Multiple* assigns 2 replicas, in s_1 and s_2, hence has cost $2n$. The *Upwards* policy assigns a replica to s_1 to cover its child, and then cannot

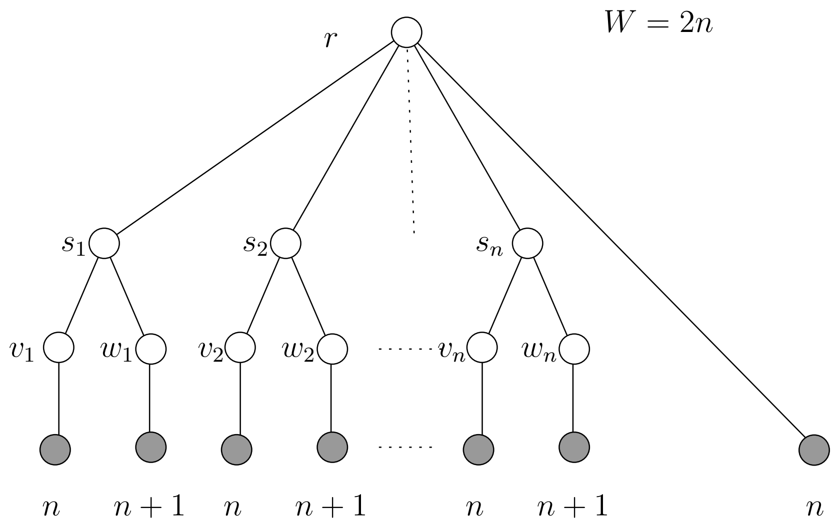


Figure 3: *Multiple* versus *Upwards*, homogeneous platforms.

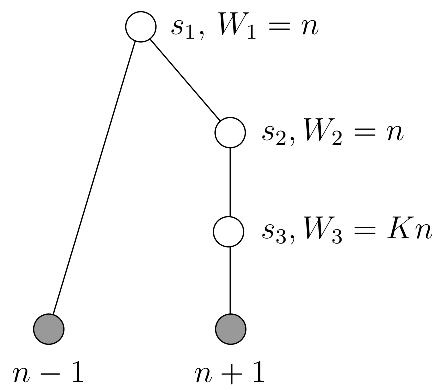


Figure 4: *Multiple* versus *Upwards*, heterogeneous platforms.

use s_2 to process the requests of the child in its subtree. It must place a replica in s_3, hence a final cost $n + Kn = (K+1)n$ arbitrarily higher than *Multiple*.

3.4 Lower bound for the Replica Counting problem

Obviously, the cost of an optimal solution of the Replica Counting problem (for any policy) cannot be lower than the obvious lower bound $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil$, where W is the server capacity. Indeed, this corresponds to a solution where the total request load is shared as evenly as possible among the replicas.

The following instance of Replica Counting shows that the optimal cost can be arbitrarily higher than this lower bound. Consider Figure 5, with $n+1$ nodes of capacity $\mathsf{W}_j = W$, The root r has $n+1$ children, n nodes labeled s_1 to s_n, and a client with $r_i = W$. Each node s_j has a unique child, a client with $r_i = W/n$ (assume without loss of generality that W is divisible by n). The lower bound is $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil = \frac{2W}{W} = 2$. However, each of the three policies *Closest*, *Upwards* and *Multiple* will assign a replica to the root to cover its client, and will then need n extra replicas, one per client of s_j, $1 \leq j \leq n$. The total cost is thus $n+1$ replicas, arbitrarily higher than the lower bound.

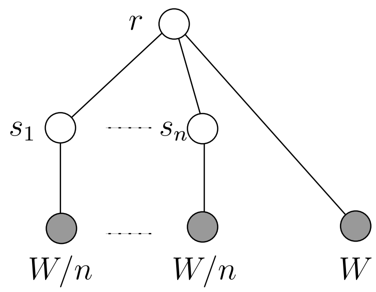


Figure 5: The lower bound cannot be approximated for Replica Counting.

All the examples in Sections 3.1 to 3.4 give an insight of the combinatorial nature of the Replica Placement optimization problem, even in its simplest variants Replica Cost and Replica Counting. The following section corroborates this insight: most problems are shown NP-hard, even though some variants have polynomial complexity.

4 Complexity results

One major goal of this paper is to assess the impact of the access policy on the problem with homogeneous vs heterogeneous servers. We restrict to the simplest problem, namely the Replica Cost problem introduced in Section 2.2.3. We consider a tree $\mathcal{T} = \mathcal{C} \cup \mathcal{N}$, no QoS constraint, and infinite link capacities. Each client $i \in \mathcal{C}$ has r_i requests; each node $j \in \mathcal{N}$ has processing capacity W_j and storage cost $\mathsf{sc}_j = W_j$. This simple problem comes in two flavors, either with homogeneous nodes ($\mathsf{W}_j = \mathsf{W}$ for all $j \in \mathcal{N}$), or with heterogeneous nodes (servers with different capacities/costs).

In the single server version of the problem, we need to find a server $\mathsf{server}(i)$ for each client $i \in \mathcal{C}$. Let $\mathsf{Servers}$ be the set of servers chosen among the nodes in $\mathcal{N}$. The only constraint is that server capacities cannot be exceeded: this translates into

$$\sum_{i \in \mathcal{C}, \mathsf{server}(i) = j} r_i \leq W_j \quad \text{for all } j \in \mathsf{Servers}.$$

The objective is to find a valid solution of minimal storage cost $\sum_{j \in \mathsf{Servers}} W_j$. Note that with homogeneous nodes, the problem reduces to find the minimum number of servers, i.e. to the Replica Counting problem. As outlined in Section 3, there are two variants of the single server version of the problem, namely the *Closest* and the *Upwards* strategies.

In the *Multiple* policy with multiple servers per client, let $\mathsf{Servers}$ be the set of servers chosen among the nodes in $\mathcal{N}$; for any client $i \in \mathcal{C}$ and any node $j \in \mathcal{N}$, let $r_{i,j}$ be the number of requests from i that are processed by j ($r_{i,j} = 0$ if $j \notin \mathsf{Servers}$). We need to ensure that

$$\sum_{j \in \mathcal{N}} r_{i,j} = r_i \quad \text{for all } i \in \mathcal{C}.$$

The capacity constraint now writes

$$\sum_{i \in \mathcal{C}} r_{i,j} \leq W_j \quad \text{for all } j \in \mathsf{Servers},$$

while the objective function is the same as for the single server version.

The decision problems associated with the previous optimization problems are easy to formulate: given a bound on the number of servers (homogeneous version) or on the total storage cost (heterogeneous version), is there a valid solution that meets the bound?

	Homogeneous	**Heterogeneous**
Closest	polynomial [2, 9]	NP-complete
Upwards	NP-complete	NP-complete
Multiple	polynomial	NP-complete

Table 1: Complexity results for the different instances of the Replica Cost problem.

Table 1 captures the complexity results. These complexity results are all new, except for the *Closest*/Homogeneous combination. The NP-completeness of the *Upwards*/Homogeneous case comes as a surprise, since all previously known instances were shown to be polynomial, using dynamic programming algorithms. In particular, the *Closest*/Homogeneous variant remains polynomial when adding communication costs [2] or QoS constraints [9]. Previous NP-completeness results involved general graphs rather than trees, and the combinatorial nature of the problem came from the difficulty to extract a good replica tree out of an arbitrary communication graph. Here the tree is fixed, but the problem remains combinatorial due to resource heterogeneity.

4.1 With homogeneous nodes and the *Multiple* strategy

Theorem 1. *The instance of the* Replica Counting *problem with the Multiple strategy can be solved in polynomial time.*

Proof. We outline below an optimal algorithm to solve the problem. The proof of optimality is quite technical, so the reader may want to skip it at first reading. □

4.1.1 Algorithm for multiple servers

We propose a greedy algorithm to solve the Replica Counting problem. Let W be the total number of requests that a server can handle.

This algorithm works in three passes: first we select the nodes which will have a replica handling exactly W requests. Then a second pass allows us to select some extra servers which are fulfilling the remaining requests. Finally, we need to decide for each server how many requests of each client it is processing.

We assume that each node i knows its parent $\mathsf{parent}(i)$ and its children $\mathsf{children}(i)$ in the tree. We introduce a new variable which is the flow coming up in the tree (requests which are not already fulfilled by a server). It is denoted by flow_i for the flow between i and $\mathsf{parent}(i)$. Initially, $\forall i \in \mathcal{C}$ $\mathsf{flow}_i = r_i$ and $\forall i \in \mathcal{N}$ $\mathsf{flow}_i = -1$. Moreover, the set of replicas is empty in the beginning: $repl = \emptyset$.

Pass 1– We greedily select in this step some nodes which will process W requests and which are as close to the leaves as possible. We place a replica on such nodes (see Algorithm 1). Procedure **pass1** is called with r (root of the tree) as a parameter, and it goes down the tree recursively in order to compute the flows. When a flow exceeds W, we place a replica since the corresponding server will be fully used, and we remove the processed requests from the flow going upwards.

At the end, if $flow_r = 0$ or ($flow_r \leq \mathsf{W}$ and $r \notin repl$), we have an optimal solution since all replicas which have been placed are fully used and all requests are satisfied by adding a replica in r if $flow_r \neq 0$. In this case we skip pass 2 and go directly to pass 3.

Otherwise, we need some extra replicas since some requests are not satisfied yet, and the root cannot satisfy all the remaining requests. To place these extra replicas, we go through pass 2.

Pass 2– In this pass, we need to select the nodes where to add replicas. To do so, while there are too many requests going up to the root, we select the node which can process the highest number of requests, and we place a replica there. The number of requests that a node

procedure **pass1** (node $s \in \mathcal{N}$)
begin
  $flow_s = 0$;
  for $i \in \textsf{children}(s)$ **do**
    if $flow_i == -1$ **then** **pass1**(i); // *Recursive call.*
    $flow_s = flow_s + flow_i$;
  end
  if $flow_s \geq \textsf{W}$ **then** $flow_s = flow_s - \textsf{W}; repl = \{s\} \cup repl$;
end

Algorithm 1: Procedure pass1

$j \in \mathcal{N}$ can eventually process is the minimum of the flows between j and the root r, denoted $uflow_j$ (for *u*seful *flow*). Indeed, some requests may have no server yet, but they might be processed by a server on the path between j and r, where a replica has been placed in pass 1. Algorithm 2 details this pass.

If we exit this pass with $finish = -1$, this means that we have tried to place replicas on all nodes, but this solution is not feasible since there are still some requests which are not processed going up to the root. In this case, the original problem instance had no solution.

However, if we succeed to place replicas such that $flow_r = 0$, we have a set of replicas which succeed to process all requests. We then go through pass 3 to assign requests to servers, i.e. to compute how many requests of each client should be processed by each server.

while $flow_r \neq 0$ **do**
  $freenode = \mathcal{N} \setminus repl$;
  if $freenode == \emptyset$ **then** $finish = -1$; exit the loop;
  // *At each step, assign 1 replica and re-compute flows.*
  $child = \textsf{children}(r); uflow_r = flow_r$;
  while $child! = \emptyset$ **do**
    remove j from $child$;
    $uflow_j = \min(flow_j, uflow_{\textsf{parent}(j)})$;
    $child = child \cup \textsf{children}(j)$;
  end
  // *The useful flows have been computed, select the max.*
  maxuflow=0;
  for $j \in freenode$ **do**
    if $uflow_j > maxuflow$ **then** $maxuflow = uflow_j$; $maxnode = j$;
  end
  if $maxuflow \neq 0$ **then**
    $repl = repl \cup \{maxnode\}$;
    // *Update the flows upwards.*
    for $j \in \textsf{Ancestors}(maxnode) \cup \{maxnode\}$ **do** $flow_j = flow_j - maxuflow$;
  end
  else $finish = -1$; exit the loop;
end

Algorithm 2: Pass 2

Pass 3– This pass is in fact straightforward, starting from the leaves and distributing the requests to the servers from the bottom until the top of the tree. We decide for instance to affect requests from clients starting to the left. Procedure **pass3** is called with r (root of the tree) as a parameter, and it goes down the tree recursively (c.f. Algorithm 3). For $i \in \mathcal{C}$, r'_i is the number of requests of i not yet affected to a server (initially $r'_i = r_i$). $w_{s,i}$ is the number of requests of client i affected to server $s \in \mathcal{N}$, and $w_s \leq \textsf{W}$ is the total number of requests affected to s. $C(s)$ is the set of clients in $\mathsf{subtree}(s)$ which still have some requests not affected. Initially, $C(i) = \{i\}$ for $i \in \mathcal{C}$, and $C(s) = \emptyset$ otherwise.

Note that a server which was computing W requests in pass 1 may end up computing fewer requests if one of its descendants in the tree has earned a replica in pass 2. But this does not affect the optimality of the result, since we keep the same number of replicas.

```
procedure pass3 (node s ∈ N)
begin
    w_s = 0;
    for i ∈ children(s) do
        if C(i) = ∅ then pass3(i); // Recursive call.
        C(s) = C(s) ∪ C(i);
    end
    if s ∈ repl then
        for i ∈ C(s) do
            if r'(i) ≤ W − w_s then C(s) = C(s) \ {i}; w_{s,i} = r'_i; w_s = w_s + r'_i; r'_i = 0;
        end
        if C(s) ≠ ∅ then Let i ∈ C(s); x = W − w_s; r'_i = r'_i − x; w_{s,i} = x; w_s = W;
    end
end
```

Algorithm 3: Procedure pass3

The proof in Section 4.1.3 shows the equivalence between the solution built by this algorithm and any optimal solution, thus proving the optimality of the algorithm. The following example illustrates the step by step execution of the algorithm.

4.1.2 Example

Figure 6(a) provides an example of network on which we are placing replicas with the *Multiple* strategy. The network is thus homogeneous and we fix $\mathsf{W} = 10$.

Pass 1 of the algorithm is quite straightforward to unroll, and Figure 6(b) indicates the flow on each link and the saturated replicas are the black nodes.

During pass 2, we select the nodes of maximum useful flow. Figure 6(c) represents these useful flows; we see that node n_4 is the one with the maximum useful flow (7), so we assign it a replica and update the useful flows. All the useful flows are then reduced down to 1 since there is only 1 request going through the root n_1. The first node of maximum useful flow 1 to be selected is n_2, which is set to be a replica of pass 2. The flow at the root is then 0 and it is the end of pass 2.

Finally, pass 3 affects the servers to the clients and decides which requests are served by which replica (Figure 6(d)). For instance, the client with 12 requests shares its requests between n_{10} (10 requests) and n_2 (2 requests). Requests are affected from the bottom of the tree up to the top. Note that the root n_1, even though it was a saturated replica of pass 1, has only 5 requests to proceed in the end.

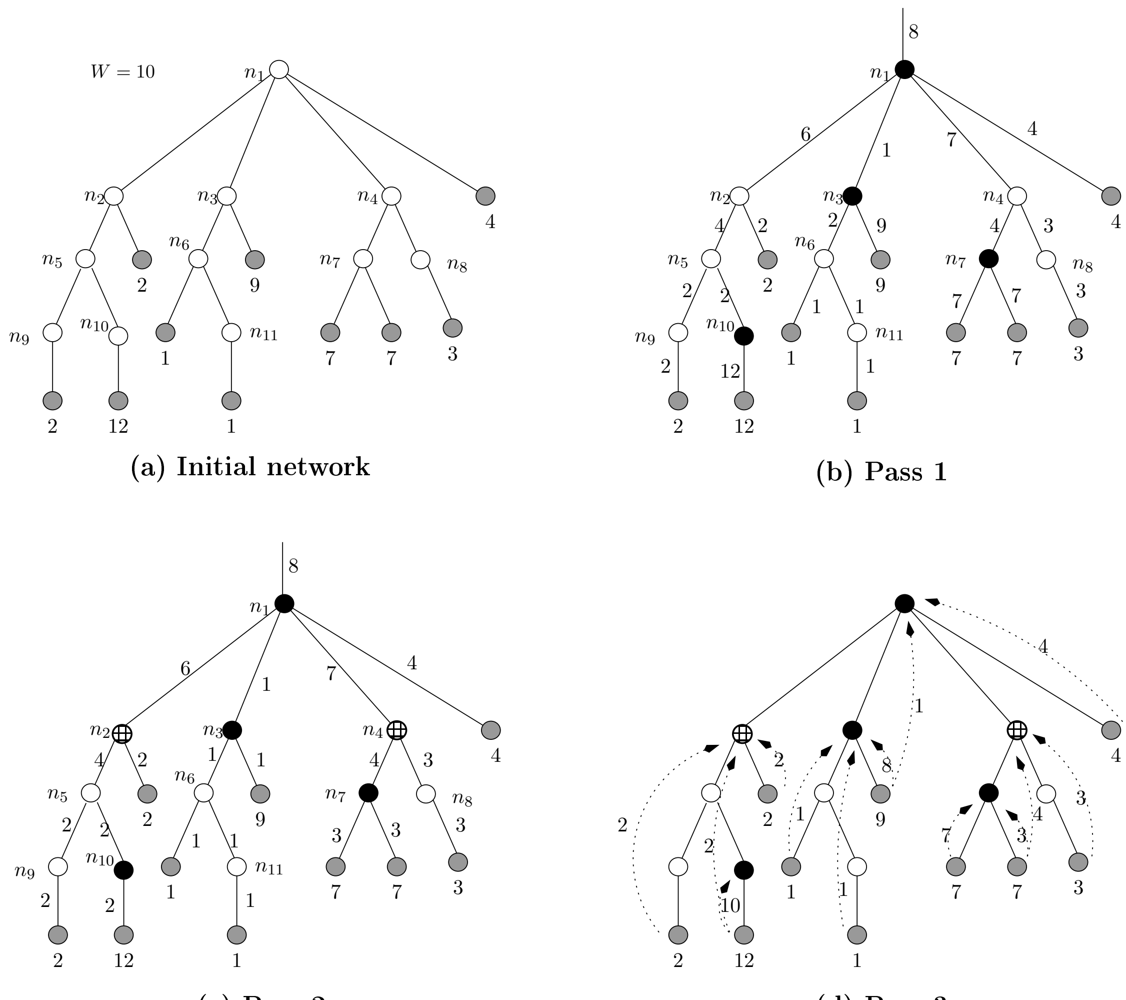
4.1.3 Proof of optimality

Let R_{opt} be an optimal solution to an instance of the problem. The core of the proof consists in transforming this solution into an equivalent canonical optimal solution R_{can}. We will then show that our algorithm is building this canonical solution, and thus it is producing an optimal solution.

Each server $s \in R_{opt}$ is serving $w_{s,i}$ requests of client $i \in \mathsf{subtree}(s) \cap \mathcal{C}$, and

$$w_s = \sum_{i \in \mathsf{subtree}(s) \cap \mathcal{C}} w_{s,i} \leq W.$$

For each $i \in \mathcal{C}$, $w_{s,i} = 0$ if $s \in \mathcal{N}$ is not a replica, and, $\sum_{s \in Ancests(i)} w_{s,i} = r_i$.



(a) Initial network **(b) Pass 1**

(c) Pass 2 **(d) Pass 3**

Figure 6: Algorithm for the Replica Counting problem with the *Multiple* strategy.

We define the *flow* of node k, flow_k, by the number of requests going through this node up to its parents. Thus, for $i \in \mathcal{C}$, $flow_i = r_i$, while for a node $s \in \mathcal{N}$,

$$flow_s = \sum_{i \in \mathsf{children}(s)} flow_i - w_s.$$

The *total flow* going through the tree, $tflow$, is defined in a similar way, except that we do not remove from the flow the requests processed by a replica, *i.e.* $tflow_s = \sum_{i \in \mathsf{children}(s)} tflow_i$. We thus have

$$tflow_s = \sum_{i \in \mathsf{subtree}(s) \cap \mathcal{C}} r_i.$$

These variables are completely defined by the network and the optimal solution R_{opt}.

A first lemma shows that it is possible to change request assignments while keeping an optimal solution. The flows need to be recomputed after any such modification.

Lemma 1. *Let $s \in \mathcal{N} \cap R_{opt}$ be a server such that $w_s < \mathsf{W}$.*

- *If $tflow_s \geq \mathsf{W}$, we can change the request assignment between replicas of the optimal solution, in such a way that $w_s = \mathsf{W}$.*

- *Otherwise, we can change the request assignment so that* $w_s = tflow_s$.

Proof. First we point out that the clients in $\mathsf{subtree}(s)$ can all be served by s, and since R_{opt} is a solution, these requests are served by a replica somewhere in the tree. We do not modify the optimality of the solution by changing the $w_{s,i}$, it just affects the flows of the solution. Thus, for a given client $i \in \mathsf{subtree}(s) \cap \mathcal{C}$, if there is a replica $s' \neq s$ on the path between i and the root, we can change the assignment of the requests of client i. Let $x = max(w_{s',i}, \mathsf{W} - w_s)$. Then we move x requests, *i.e.* $w_{s',i} = w_{s',i} - x$ and $w_{s,i} = w_{s,i} + x$. From the definition of $tflow_s$, we obtain the result, if we move all possible requests to s until there are no more requests in the subtree or until s is processing W requests. □

We now introduce a new definition, completely independent from the optimal solution but related to the tree network. The *canonical flow* is obtained by distinguishing nodes which receive a flow greater than W from the other nodes. We compute the canonical flow $cflow$ of the tree, independently of the replica placement, and define a subset of nodes which are *saturated*, SN. We also compute the number of saturated nodes in $\mathsf{subtree}(k)$, denoted nsn_k, for any node $k \in \mathcal{C} \cup \mathcal{N}$ of the tree.

For $i \in \mathcal{C}$, $cflow_i = r_i$ and $nsn_i = 0$, and we then compute recursively the canonical flows for nodes $s \in \mathcal{N}$. Let $f_s = \sum_{i \in \mathsf{children}(s)} cflow_i$ and $x_s = \sum_{i \in \mathsf{children}(s)} nsn_i$. If $f_s \geq \mathsf{W}$ then $s \in SN$, $cflow_s = f_s - \mathsf{W}$ and $nsn_s = x_s + 1$. Otherwise, s is not saturated, $cflow_s = f_s$ and $nsn_s = x_s$.

We can deduce from these definitions the following results:

Proposition 1. *A non saturated node always has a canonical flow being less than* W*:*
$\forall s \in \mathcal{N} \setminus SN \;\; cflow_s < \mathsf{W}$

Lemma 2. *For all nodes* $s \in \mathcal{C} \cup \mathcal{N}$, $cflow_s = tflow_s - nsn_s \times \mathsf{W}$.

Corollary 1. *For all nodes* $s \in \mathcal{C} \cup \mathcal{N}$, $tflow_s \geq nsn_s \times \mathsf{W}$.

Proof. Proposition 1 is trivial due to the definition of the canonical flow.

Lemma 2 can be proved recursively on the tree.

- This property is true for the clients: for $i \in \mathcal{C}$, $nsn_i = 0$ and $tflow_i = cflow_i = r_i$.
- Let $s \in \mathcal{N}$, and let us assume that the proposition is true for all children of s. Then,

$$\forall j \in \mathsf{children}(s) \; cflow_j = tflow_j - nsn_j \times \mathsf{W}.$$

  – If $s \notin SN$, $nsn_s = \sum_{j \in \mathsf{children}(s)} nsn_j$ and

$$cflow_s = \sum_{j \in \mathsf{children}(s)} cflow_j = \sum_{j \in \mathsf{children}(s)} (tflow_j - nsn_j \times \mathsf{W}) = tflow_s - nsn_s \times \mathsf{W}$$

  – If $s \in SN$, $nsn_s = \left(\sum_{j \in \mathsf{children}(s)} nsn_j\right) + 1$ and

$$cflow_s = \sum_{j \in \mathsf{children}(s)} cflow_j - \mathsf{W} = \sum_{j \in \mathsf{children}(s)} (tflow_j - nsn_j \times \mathsf{W}) - \mathsf{W}$$

$$= tflow_s - (nsn_s - 1) \times \mathsf{W} - \mathsf{W} = tflow_s - nsn_s \times \mathsf{W}$$

which proves the result. Corollary 1 is trivially deduced from Lemma 2 since $cflow$ is a positive function. □

We also show that it is always possible to move a replica into a free server which is one of its ancestors in the tree, while keeping an optimal solution:

Proposition 2. *Let* R_{opt} *be an optimal solution, and let* $s \in R_{opt}$. *If* $\exists s' \in \mathsf{Ancestors}(s) \setminus R_{opt}$ *then* $R'_{opt} = \{s'\} \cup R_{opt} \setminus \{s\}$ *is also an optimal solution.*

Proof. s' can handle all requests which were processed by s since $s \in \mathsf{subtree}(s')$. We just need to redefine $w_{s',i} = w_{s,i}$ for all $i \in \mathcal{C}$ and then $w_{s,i} = 0$. $\square$

We are now ready to transform R_{opt} into a new optimal solution, R_{sat}, by redistributing the requests among the replicas and moving some replicas, in order to place a replica at each saturated node, and affecting W requests to this replica. This transformation is done starting at the leaves of the tree, and considering all nodes of SN. Nothing needs to be done for the leaves (the clients) since they are not in SN.

Let us consider $s \in SN$, and assume that the optimal solution has already been modified to place a replica, and assign it W requests, on all nodes in $subSN = SN \cap \mathsf{subtree}(s) \setminus \{s\}$.

We need to differentiate two cases:

1. If $s \in R_{opt}$, we do not need to move any replica. However, if $w_s \neq \mathsf{W}$, we change the assignment of some requests while keeping the same replicas in order to obtain a workload of W on server s. We do not remove requests from the saturated servers of $subSN$ which have already been filled. Corollary 1 ensures that $tflow_s \geq nsn_s \times \mathsf{W}$, and $(nsn_s - 1) \times \mathsf{W}$ requests should not move since they are affected to the $nsn_s - 1$ servers of $subSN$. There are thus still more than W requests of clients of $\mathsf{subtree}(s)$ which can possibly be moved on s using Lemma 1.
2. If $s \notin R_{opt}$, we need to move a replica of R_{opt} and place it in s without changing the optimality of the solution. We differentiate two subcases.
   (a) If $\exists s_1 \in \mathsf{subtree}(s) \cap R_{opt} \setminus SN$, then the replica placed on s_1 can be moved in s by applying Proposition 2. Then, if $w_s \neq \mathsf{W}$, we apply case 1 above to saturate the server.
   (b) Otherwise, all the replicas placed in $\mathsf{subtree}(s)$ are also in SN, and the flow consumed by the already modified optimal algorithm is exactly $(nsn_s - 1) \times \mathsf{W}$. It is easy to see that the flow (of the optimal solution) at s is exactly equal to the total flow minus the consumed flow. Therefore, $flow_s = tflow_s - (nsn_s - 1) \times \mathsf{W}$, and with the application of Corollary 1, $flow_s \geq \mathsf{W}$.
   The idea now consists in affecting the requests of this flow to node s by removing work from the replicas upwards to the root, and rearrange the remaining requests to remove one replica. The flow $flow_s$ is going upwards to be processed by some of the nr_s replicas in $\mathsf{Ancestors}(s) \cap R_{opt}$, denoted $s_1, ..., s_{nr_s}$, s_1 being the closest node from s. We can remove W of these requests from the flow and affect them to a new replica placed in s. Let $w_{s_k,s} = \sum_{j \in \mathsf{subtree}(s) \cap \mathcal{C}} w_{s_k,j}$. We have $\sum_{k=1..nr_s} w_{s_k,s} = flow_s$. We move these requests from s_k to s, starting with $k = 1$. Thus, after the modification, $w_{s_1,s} = 0$. It is however possible that $w_{s_1} \neq 0$ since s_1 may process requests which are not coming from $\mathsf{subtree}(s)$. In this case, we are sure that we have removed enough requests from s_k, $k = 2..nr_s$ which can instead process requests still in charge of s_1. We can then remove the replica initially placed in s_1.
   This way, we have not changed the assignment on replicas in $subSN$, but we have placed a replica in s which is processing W requests. Since we have at the same time removed the first replica on the path from s to the root (s_1), we have not changed the number of replicas and the solution is still optimal.

Once we have applied this procedure up to the root, we have an optimal solution R_{sat} in which all nodes of SN have been placed a replica and are processing W requests. We will not change the assignment of these replicas anymore in the following. Free nodes in the new solution are called *F-nodes*, while replicas which are not in SN are called *PS-nodes*, for *partially saturated*.

In a next step, we further modify the R_{sat} optimal solution in order to obtain what we call the *canonical solution* R_{can}. To do so, we change the request assignment of the PS-nodes: we "saturate" some of them as much as we can and we integrate them into the subset of nodes SN, redefining the $cflow$ accordingly. At the end of the process, $SN = R_{can}$.

The $cflow$ is still the flow which has not been processed by a saturated node in the subtree, and thus we can express it in a more general way:

$$cflow_s = tflow_s - \sum_{s' \in SN \cap \mathsf{subtree}(s)} w_{s'}$$

Note that this is totally equivalent to the previous definition while we have not modified SN.

We also introduce a new flow definition, the *non-saturated flow* of s, $nsflow_s$, which counts the requests going through node s and not served by a saturated server anywhere in the tree. Thus,

$$nsflow_s = cflow_s - \sum_{i \in \mathsf{children}(s) \cap \mathcal{C}} \sum_{s' \in \mathsf{Ancestors}(s) \cap SN} w_{s',i}.$$

This flow represents the requests that can potentially be served by s while keeping all nodes of SN saturated.

Lemma 3. *In a saturated optimal solution, there cannot exist a PS-node in the subtree of another PS-node.*

Proof. The non-saturated flow is $nsflow_s \leq cflow_s$ since we further remove from the canonical flow some requests which are affected upwards in the tree to some saturated servers.

Let $s \in R_{sat} \setminus SN$ be a PS-node. Its canonical flow is $cflow_s < W$. It can potentially process all the requests of the subtree which are not affected to a saturated server upwards or downwards in the tree, thus $nsflow_s$ requests. Since $nsflow_s \leq cflow_s < W$, we can change the request assignment to assign all these $nsflow_s$ requests to s, removing eventually some work from other non-saturated replicas upwards or downwards which were processing these requests. Thus, the replica on node s is processing all the requests of $\mathsf{subtree}(s)$ which are not processed by saturated nodes.

If there was a non saturated replica in $\mathsf{subtree}(s)$, it could thus be removed since all the requests are processed by s. This means that a solution with a PS-node in the subtree of another PS-node is not optimal, thus proving the lemma. □

At this point, we can move the PS-nodes as high as possible in R_{sat}. Let s be a PS-node. If there is a free node s' in $\mathsf{Ancestors}(s)$ then we can move the replica from s to s' using Proposition 2. Lemma 3 ensures that there are no other PS-nodes in $\mathsf{subtree}(s')$.

All further modifications will only alter nodes which have no PS-nodes in their ancestors. We define $\mathcal{N}' = \{s | \mathsf{Ancestors}(s) \setminus SN = \emptyset\}$.

Let $s \in \mathcal{N}'$. $nsflow_s = cflow_s - \sum_{i \in \mathsf{children}(s) \cap \mathcal{C}} \sum_{s' \in \mathsf{Ancestors}(s)} w_{s',i}$ since all ancestors of s are in SN. Thus,

$$nsflow_s = \sum_{s' \in \mathsf{subtree}(s) \setminus SN} w_{s'}.$$

By definition, $\forall s \in \mathcal{N}$ $nsflow_s \leq cflow_s$. Moreover, if $s \notin SN$, then $nsflow_s = w_s$ since $\mathsf{subtree}(s) \setminus SN$ is reduced to s (no other PS-node under the PS-node s, from Lemma 3).

We introduce a new flow definition, the useful flow, which intuitively represents the number of requests that can possibly be processed on s without removing requests from a saturated server.

$$uflow_s = \min_{s' \in \mathsf{Ancestors}(s) \cup \{s\}} \{cflow_{s'}\}$$

Lemma 4. *Let $s \in \mathcal{N}'$. Then $nsflow_s \leq uflow_s$.*

Proof. Let $s' \in \mathsf{Ancestors}(s)$. Since $s \in \mathcal{N}'$, $s' \in SN$.

$$cflow_{s'} \geq nsflow_{s'} = \sum_{s'' \in \mathsf{subtree}(s') \setminus SN} w_{s''}$$

But since $s \in \mathsf{subtree}(s')$, $\mathsf{subtree}(s) \setminus SN \subseteq \mathsf{subtree}(s') \setminus SN$, hence $nsflow_s \leq nsflow_{s'}$. Note that $nsflow$ is a non decreasing function (when going up the tree).

Thus, $\forall s' \in \mathsf{Ancestors}(s) \cup \{s\}$, $nsflow_s \leq cflow_{s'}$, and by definition of the useful flow, $nsflow_s \leq uflow_s$. □

Now we start the modification of the optimal solution in order to obtain the canonical solution. At each step, we select a node $s \in \mathcal{N} \setminus SN$ maximizing the useful flow. If there are several nodes of identical $uflow$, we select the first one in a depth-first traversal of the tree. We will prove that we can affect $uflow_s$ requests to this node without unsaturating any server of SN. s is then considered as a saturated node, we recompute the canonical flows (and thus the useful flows) and reiterate the process until $cflow_r = 0$, which means that all the requests have been affected to saturated servers.

Let us explain how to reassign the requests in order to saturate s with $uflow_s$ requests. The idea is to remove some requests from $\mathsf{Ancestors}(s)$ in order to saturate s, and then to saturate the ancestors of s again, by affecting them some requests coming from other non saturated servers.

First, we note that $uflow_s \leq cflow_r = nsflow_r$. Thus,

$$uflow_s \leq \sum_{s' \in \mathcal{N} \setminus SN} w_{s'} = w_s + \sum_{s' \in PS} w_{s'}$$

where PS is the set of non saturated nodes without s. Let $x = uflow_s - w_s$. If $x = 0$, s is already saturated. Otherwise, we need to reassign x requests to s. From the previous equation, we can see that $\sum_{s' \in PS} w_{s'} \geq uflow_s - w_s = x$. There are thus enough requests handled by non saturated nodes which can be passed to s.

The number of requests of $\mathsf{subtree}(s) \cap \mathcal{C}$ handled by $\mathsf{Ancestors}(s)$ is

$$\sum_{s' \in \mathsf{Ancestors}(s)} \sum_{i \in \mathsf{subtree}(s) \cap \mathcal{C}} w_{s',i} = cflow_s - nsflow_s$$

by definition of the flow. Or $cflow_s - nsflow_s \geq uflow_s - w_s = x$ so there are at least x requests that s can take from its ancestors.

Let $a_1 = \mathsf{parent}(s), ..., a_k = r$ be the ancestors of s. $x_j = \sum_{i \in \mathsf{subtree}(s) \cap \mathcal{C}} w_{a_j,i}$ is the amount of requests that s can take from a_j. We choose arbitrary where to take the requests if $\sum_j x_j > x$, and do not modify the assignment of the other requests. We thus assume in the following that $\sum_j x_j = x$. Since these x_j requests are coming from a client in $\mathsf{subtree}(s)$, we can assign them to s, and there are now only $\mathsf{W} - x_j$ requests handled by a_j, which means that a_j is temporarily unsaturated. However, we have given x extra requests to s, hence s is processing $w_s + x = uflow_s$ requests.

We finally need to reassign requests to $a_j, j = 1..k$ in order to saturate these nodes again, taking requests out of nodes in PS (non saturated nodes other than s). This is done iteratively starting with $j = 1$ and going up to the root a_k. At each step j, we assume that $a_{j'}, j' < j$ have already been saturated again and we should not move requests away from them. However, we can still eventually take requests away from $a_{j''}, j'' > j$.

In order to saturate a_j, we need to take:

- either requests from $\mathsf{subtree}(a_j) \cap \mathcal{C}$ which are currently handled by $a_{j''}, j'' > j$, but without moving requests which are already affected to s (*i.e.* $\sum_{j''>j} x_{j''}$);
- or requests from non saturated servers in $\mathsf{subtree}(a_j)$, except requests from s and requests already given to s that should not be moved any more (*i.e.* $\sum_{j'<j} x_{j'}$).

The number of requests that we can potentially affect to a_j is therefore:

$$X = \sum_{s' \in \mathsf{subtree}(a_j) \setminus SN \setminus \{s\}} w_{s'} + \sum_{i \in \mathsf{subtree}(a_j) \cap \mathcal{C}} \sum_{s' \in \mathsf{Ancestors}(a_j)} w_{s',i} - \sum_{j'<j} x_{j'} - \sum_{j''>j} x_{j''}$$

Let us show that $X \geq x_j$. Then we can use these requests to saturate a_j again.

$$cflow_{a_j} = nsflow_{a_j} + \sum_{i \in \mathsf{subtree}(a_j) \cap \mathcal{C}} \sum_{s' \in \mathsf{Ancestors}(a_j)} w_{s',i} = w_s + X + \sum_{j'<j} x_{j'} + \sum_{j''>j} x_{j''} = X + w_s + x - x_j$$

But $cflow_{a_j} \geq uflow_s$ and $uflow_s - w_s = x$ so

$$X = cflow_{a_j} - w_s - x + x_j \geq uflow_s - w_s - x + x_j = x_j$$

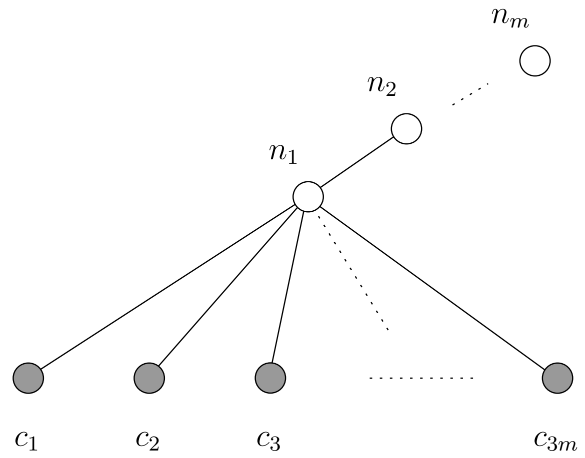


Figure 7: The platform used in the reduction for Theorem 2.

It is thus possible to saturate s and then keep its ancestors saturated. At this point, s becomes a node of SN and we can recompute the canonical and non saturated flows. We have removed $uflow_s$ requests which were processed by non saturated servers, so the $cflow$ and $nsflow$ of all ancestors of s, including s, should be decreased by $uflow_s$.

In particular, at the root, $cflow_r = cflow_r - uflow_s$, which proves that the contribution of s on $cflow_r$ is $uflow_s$.

In the last step of the proof, we show that the number of replicas in the modified canonical solution at the end of the iteration $R_{can} = SN$ has exactly the same number of replicas than R_{sat}. In the saturated solution, each PS-node s is processing $nsflow_s$ requests, while in the canonical solution, it is $uflow_s$. However, at every step when adding a saturated node s, we have $uflow_s$ greater than any of the $nsflow$s. It is thus easy to see that the number of nodes in the canonical solution is less or equal to the number of nodes in the saturated solution. Since the saturated solution is optimal, $|R_{can}| = |R_{sat}|$, which completes the proof.

Our algorithm builds R_{can} in polynomial time, which assesses the complexity of the problem.

4.2 With homogeneous nodes and the *Upwards* strategy

Theorem 2. *The instance of the* Replica Counting *problem with the Upwards strategy is NP-complete in the strong sense.*

Proof. The problem clearly belongs to the class NP: given a solution, it is easy to verify in polynomial time that all requests are served and that no server capacity is exceeded. To establish the completeness in the strong sense, we use a reduction from 3-PARTITION [3]. We consider an instance $\mathcal{I}_1$ of 3-PARTITION: given $3m$ positive integers $a_1, a_2, \ldots, a_{3m}$ such that $B/4 < a_i < B/2$ for $1 \leq i \leq 3m$, and $\sum_{i=1}^{3m} a_i = mB$, can we partition these integers into m triples, each of sum B? We build the following instance $\mathcal{I}_2$ of Replica Counting (see Figure 7):

- $3m$ clients c_i with $r_i = a_i$ for $1 \leq i \leq 3m$.
- m internal nodes n_j with $\mathsf{W}_j = \mathsf{sc}_j = B$ for $1 \leq j \leq m$.
  - The children of n_1 are all the $3m$ clients c_i, and its parent is n_2.
  - For $2 \leq j \leq m$, the only child of n_j is n_{j-1}. For $1 \leq j \leq m-1$, the parent of n_j is n_{j+1} (hence n_m is the root).

Finally, we ask whether there exists a solution with total storage cost mB, i.e. with a replica located at each internal node. Clearly, the size of $\mathcal{I}_2$ is polynomial (and even linear) in the size of $\mathcal{I}_1$.

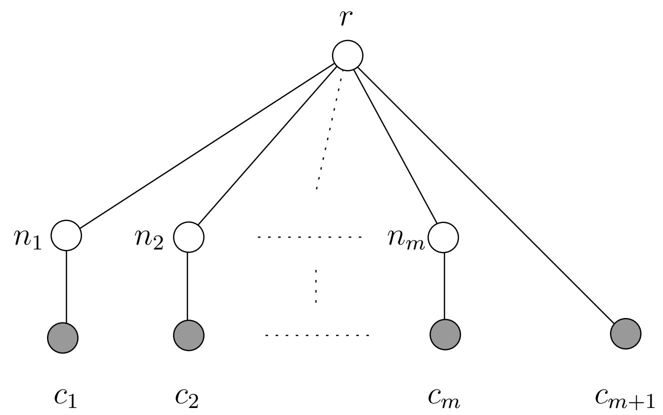


Figure 8: The platform used in the reduction for Theorem 3.

We now show that instance $\mathcal{I}_1$ has a solution if and only if instance $\mathcal{I}_2$ does. Suppose first that $\mathcal{I}_1$ has a solution. Let $(a_{k_1}, a_{k_2}, a_{k_3})$ be the k-triplet in $\mathcal{I}_1$. We assign the three clients c_{k_1}, c_{k_2} and c_{k_3} to server n_k. Because $a_{k_1} + a_{k_2} + a_{k_3} = B$, no server capacity is exceeded. Because the m triples partition the a_i, all requests are satisfied. We do have a solution to $\mathcal{I}_2$.

Suppose now that $\mathcal{I}_2$ has a solution. Let I_k be the set of clients served by node n_k if there is a replica located at n_k: then $\sum_{i \in I_k} a_i \leq B$. The total number of requests to be satisfied is $\sum_{i=1}^{3m} a_i = mB$, and there are at most m replicas of capacity B. Hence no set I_k can be empty, and $\sum_{i \in I_k} a_i \leq B$ for $1 \leq k \leq m$. Because $B/4 < a_i < B/2$, each I_k must be a triple. This leads to the desired solution of $\mathcal{I}_1$. $\square$

4.3 With heterogeneous nodes

Theorem 3. *All three instances of the* Replica Cost *problem with heterogeneous nodes are NP-complete.*

Proof. Obviously, the NP-completeness of the *Upwards* strategy is a consequence of Theorem 2. For the other two strategies, the problem clearly belongs to the class NP: given a solution, it is easy to verify in polynomial time that all requests are served and that no server capacity is exceeded. To establish the completeness, we use a reduction from 2-PARTITION [3]. We consider an instance $\mathcal{I}_1$ of 2-PARTITION: given m positive integers $a_1, a_2, \ldots, a_m$, does there exist a subset $I \subset \{1, \ldots, m\}$ such that $\sum_{i \in I} a_i = \sum_{i \notin I} a_i$. Let $S = \sum_{i=1}^{m} a_i$. We build the following instance $\mathcal{I}_2$ of Replica Cost (see Figure 8):

- $m+1$ clients c_i with $r_i = a_i$ for $1 \leq i \leq m$ and $r_{m+1} = 1$.
- $m+1$ internal nodes:
  - m nodes n_j, $1 \leq j \leq m$, with $\mathsf{W}_j = \mathsf{sc}_j = a_j$.
  - A root node r with $\mathsf{W}_r = \mathsf{sc}_r = S/2 + 1$. - The only child of n_j is c_j. The parent of n_j is r. The parent of c_{n+1} is r.

Finally, we ask whether there exists a solution with total storage cost $S+1$. Clearly, the size of $\mathcal{I}_2$ is polynomial (and even linear) in the size of $\mathcal{I}_1$. We now show that instance $\mathcal{I}_1$ has a solution if and only if instance $\mathcal{I}_2$ does. The same reduction works for both strategies, *Closest* and *Multiple*.

Suppose first that $\mathcal{I}_1$ has a solution. We assign a replica to each node n_i, $i \in I$, and one in the root r. Client c_i is served by n_i if $i \in \mathcal{I}$, and by the root r otherwise, i.e. if $i \notin \mathcal{I}$ or if $i = m+1$. The total storage cost is $\sum_{j \in I} W_j + W_r = S + 1$. Because $\mathsf{W}_r = S/2 + 1 = \sum_{i \notin I} r_i + r_{n+1}$, the capacity of the root is not exceeded. Note that the server allocation is compatible both with the *Closest* and *Multiple* policies. In both cases, we have a solution to $\mathcal{I}_2$.

Suppose now that $\mathcal{I}_2$ has a solution. Necessarily, there is a replica located in the root, otherwise client c_{n+1} would not be served. Let I be the index set of nodes n_j, $1 \leq j \leq n$, which have been allocated a replica in the solution of $\mathcal{I}_2$. For $j \notin I$, there is no replica in node n_j, hence all requests of client c_j are processed by the root, whose storage capacity is $S/2+1$. We derive that $\sum_{j \notin I} r_j \leq S/2$. Because the total storage capacity is $S+1$, the total storage capacity of nodes in I is $S/2$. The proof is slightly different for the two server strategies:

- For the *Closest* strategy, all requests from a client $c_j \in I$ are served by n_j, hence $\sum_{j \in I} r_j \leq S/2$. Since $\sum_{j \in I} r_j + \sum_{j \notin I} r_j = S$, we derive $\sum_{j \in I} r_j = \sum_{j \notin I} r_j = S/2$, hence a solution to $\mathcal{I}_2$.
- For the *Multiple* strategy, consider a server $j \in I$. Let r'_j be the number of requests from client c_j served by n_j, and r''_j be the number of requests from c_j served by the root r (of course $r_j = r'_j + r''_j$). All requests from a client c_j, $j \notin I$, are served by the root. Let $A = \sum_{j \in I} r'_j$, $B = \sum_{j \in I} r''_j$ and $C = \sum_{j \notin I} r_j$. The total storage cost is $A + B + S/2 + 1$, hence $A + B \leq S/2$. We have seen that $C \leq S/2$. But $A + B + C = S$, hence $B = 0$, and $A = C = S/2$, hence a solution to $\mathcal{I}_2$.

$\square$

5 Linear programming formulation

In this section, we express the Replica Placement optimization problem in terms of an integer linear program. We deal with the most general instance of the problem on a heterogeneous tree, including QoS constraints, and bounds on resource usage (both server and link capacities). We derive a formulation for each of the three server access policies, namely *Closest*, *Upwards* and *Multiple*. This is an important extension to a previous formulation due to [8].

While there is no efficient algorithm to solve integer linear programs (unless P=NP), this formulation is extremely useful as it leads to an absolute lower bound: we solve the integer linear program over the rationals, using standard software packages [1, 4]. Of course the rational solution will not be feasible, as it assigns fractions of replicas to server nodes, but it will provide a lower bound on the storage cost of any solution. This bound will be very helpful to assess the performance of the polynomial heuristics that are introduced in Section 6.

5.1 Single server

We start with single server strategies, namely the *Upwards* and *Closest* access policies. We need to define a few variables:

Server assignment

- x_j is a boolean variable equal to 1 if j is a server (for one or several clients)
- $y_{i,j}$ is a boolean variable equal to 1 if $j = \mathsf{server}(i)$
- If $j \notin Ancests(i)$, we directly set $y_{i,j} = 0$.

Link assignment

- $z_{i,l}$ is a boolean variable equal to 1 if link $l \in \mathsf{path}[i \to r]$ is used when client i accesses its server $\mathsf{server}(i)$
- If $l \notin \mathsf{path}[i \to r]$ we directly set $z_{i,l} = 0$.

The objective function is the total storage cost, namely $\sum_{j \in \mathcal{N}} \mathsf{sc}_j x_j$. We list below the constraints common to the *Closest* and *Upwards* policies: First there are constraints for server and link usage:

- Every client is assigned a server: $\forall i \in \mathcal{C}, \sum_{j \in \mathsf{Ancestors}(i)} y_{i,j} = 1$.
- All requests from $i \in \mathcal{C}$ use the link to its parent: $z_{i,i \to \mathsf{parent}(i)} = 1$
- Let $i \in \mathcal{C}$, and consider any link $l : j \to j' = \mathsf{parent}(j) \in \mathsf{path}[i \to r]$. If $j' = \mathsf{server}(i)$ then link $\mathsf{succ}(l)$ is not used by i (if it exists). Otherwise $z_{i,\mathsf{succ}(l)} = z_{i,l}$. Thus:

$$\forall i \in \mathcal{C}, \forall l : j \to j' = \mathsf{parent}(j) \in \mathsf{path}[i \to r], z_{i,\mathsf{succ}(l)} = z_{i,l} - y_{i,j'}$$

Next there are constraints expressing that server capacities and link bandwidths cannot be exceeded:

- The processing capacity of any server cannot be exceeded: $\forall j \in \mathcal{N}, \sum_{i \in \mathcal{C}} r_i y_{i,j} \leq \mathsf{W}_j x_j$. Note that this ensures that if j is the server of i, there is indeed a replica located in node j.
- The bandwidth of any link cannot be exceeded: $\forall l \in \mathcal{L}, \sum_{i \in \mathcal{C}} r_i z_{i,l} \leq \mathsf{BW}_l$.

Finally there remains to express the QoS constraints:

$$\forall i \in \mathcal{C}, \forall j \in \mathsf{Ancestors}(i), \mathsf{dist}(i,j) y_{i,j} \leq \mathsf{q}_i,$$

where $\mathsf{dist}(i,j) = \sum_{l \in \mathsf{path}[i \to j]} \mathsf{comm}_l$. As stated previously, we could take the computational time of a request into account by writing $(\mathsf{dist}(i,j) + \mathsf{comp}_j) y_{i,j} \leq \mathsf{q}_i$, where comp_j would be the time to process a request on server j.

Altogether, we have fully characterized the linear program for the *Upwards* policy. We need additional constraints for the *Closest* policy, which is a particular case of the *Upwards* policy (hence all constraints and equations remain valid).

We need to express that if node j is the server of client i, then no ancestor of j can be the server of a client in the subtree rooted at j. Indeed, a client in this subtree would need to be served by j and not by one of its ancestors, according to the *Closest* policy. A direct way to write this constraint is

$$\forall i \in \mathcal{C}, \forall j \in \mathsf{Ancestors}(i), \forall i' \in \mathcal{C} \cap \mathsf{subtree}(j), \forall j' \in \mathsf{Ancestors}(j), y_{i,j} \leq 1 - y_{i',j'}.$$

Indeed, if $y_{i,j} = 1$, meaning that $j = \mathsf{server}(i)$, then any client i' in the subtree rooted in j must have its server in that subtree, not closer to the root than j. Hence $y_{i',j'} = 0$ for any ancestor j' of j.

There are $O(s^4)$ such constraints to write, where $s = |\mathcal{C}| + |\mathcal{N}|$ is the problem size. We can reduce this number down to $O(s^3)$ by writing

$$\forall i \in \mathcal{C}, \forall j \in \mathsf{Ancestors}(i) \setminus \{r\}, \forall i' \in \mathcal{C} \cap \mathsf{subtree}(j), y_{i,j} \leq 1 - z_{i',j \to \mathsf{parent}(j)}.$$

5.2 Multiple servers

We now proceed to the *Multiple* policy. We define the following variables:

Server assignment

- x_j is a boolean variable equal to 1 if j is a server (for one or several clients)
- $y_{i,j}$ is an integer variable equal to the number of requests from client i processed by node j
- If $j \notin \mathit{Ancests}(i)$, we directly set $y_{i,j} = 0$.

Link assignment

- $z_{i,l}$ is an integer variable equal to the number of requests flowing through link $l \in \mathsf{path}[i \to r]$ when client i accesses any of its servers in $\mathsf{Servers}(i)$

  - If $l \notin \mathsf{path}[i \to r]$ we directly set $z_{i,l} = 0$.

The objective function is unchanged, as the total storage cost still writes $\sum_{j \in \mathcal{N}} \mathsf{sc}_j x_j$. But the constraints must be modified. First those for server and link usage:

- Every request is assigned a server: $\forall i \in \mathcal{C}, \sum_{j \in \mathsf{Ancestors}(i)} y_{i,j} = r_i$.
- All requests from $i \in \mathcal{C}$ use the link to its parent: $z_{i, i \to \mathsf{parent}(i)} = r_i$
- Let $i \in \mathcal{C}$, and consider any link $l : j \to j' = \mathsf{parent}(j) \in \mathsf{path}[i \to r]$. Some of the requests from i which flow through l will be processed by node j', and the remaining ones will flow upwards through link $\mathsf{succ}(l)$:

$$\forall i \in \mathcal{C}, \forall l : j \to j' = \mathsf{parent}(j) \in \mathsf{path}[i \to r], z_{i,\mathsf{succ}(l)} = z_{i,l} - y_{i,j'}$$

The other constraints on server capacities, link bandwidths and QoS are slightly modified:

- Servers: $\forall j \in \mathcal{N}, \sum_{i \in \mathcal{C}} y_{i,j} \leq \mathsf{W}_j x_j$. Note that this ensure that if j is the server for one or more requests from i, there is indeed a replica located in node j.
- Bandwidths: $\forall l \in \mathcal{L}, \sum_{i \in \mathcal{C}} z_{i,l} \leq \mathsf{BW}_l$
- QoS: $\forall i \in \mathcal{C}, \forall j \in \mathsf{Ancestors}(i), \mathsf{dist}(i,j) y_{i,j} \leq \mathsf{q}_i y_{i,j}$

Altogether, we have fully characterized the linear program for the *Multiple* policy.

5.3 An ILP-based lower bound

The previous linear programs contain boolean or integer variables, because it does not make sense to assign half a request or to place one third of a replica on a node. However, we can still relax the constraints and solve the linear program assuming that all variables take rational values. The optimal solution of the relaxed program can be obtained in polynomial time (in theory using the ellipsoid method [11], in practice using standard software packages [1, 4]), and the value of its objective function provides an absolute lower bound on the cost of any valid (integer) solution. Of course the relaxation makes the most sense for the *Multiple* policy, because several fractions of servers are assigned by the rational program. While not likely to be achievable, this lower bound will provide an absolute reference for the performance of the polynomial heuristics described in Section 6.

6 Heuristics for the Replica Cost problem

In this section several heuristics for the *Closest*, *Upwards* and *Multiple* policies are presented. As previously stated, our main objective is to provide an experimental assessment of the relative performance of the three access policies. Our first attempt targets heterogenous trees without QoS nor bandwidth constraints, thus considering the Replica Cost problem, but further work will be devoted to analyzing the impact of the additional constraints (and in particular of the QoS constraints) on the replica costs achieved by each policy.

All the eight heuristics described below have polynomial, and even worst case quadratic complexity $O(s^2)$, where $s = |\mathcal{C}| + |\mathcal{N}|$ is the problem size. Indeed, all heuristics proceed by traversing the tree, and the number of traversals is bounded by the number of internal nodes (and is much lower in practice).

We assume that each node $k \in \mathcal{N} \cup \mathcal{C} \setminus \{root\}$ knows its $\mathsf{parent}(k)$. Additionally, an internal node $j \in \mathcal{N}$ knows its $\mathsf{children}(j)$, and the set $\mathsf{clients}(j)$ of the clients in its subtree $\mathsf{subtree}(j)$. At any step of the heuristics, we denote by inreq_j the number of requests in $\mathsf{subtree}(j)$ reaching j with the current replicas already placed (initially, with no replica, $\mathsf{inreq}_j = \sum_{i \in \mathsf{clients}(j)} r_i$). We use a boolean variable $\mathsf{treated}_j$ to mark if a node j has been treated during a tree traversal. The set of replicas is initialized by $\mathsf{replica} = \emptyset$.

6.1 *Closest*

The first two heuristics enforce the *Closest* policy through a top-down approach, whereas the third heuristic uses a bottom-up approach.

Closest Top Down All (CTDA) – The basic idea is to perform a breadth-first traversal of the tree. Every time a node is able to process the requests of all the clients in its subtree, the node is chosen as a server, and we do not explore further that subtree. The procedure *ClosestTopDownAll (CTDA)* is presented in Algorithm 4. It is called until no more servers are added in a tree traversal.

```
procedure CTDA (root, replica)
Fifo fifo;
fifo.push(root);
while fifo ≠ ∅ do
    s = fifo.pop();
    if s ∉ replica then
        if W_s ≥ inreq_s & inreq_s > 0 then
            replica = replica ∪ {s};
            foreach a ∈ Ancestors(s) do inreq_a = inreq_a − inreq_s;
        else
            foreach i ∈ children(s) do
                if i ∈ N then fifo.push(i);
            end
        end
    end
end
```

Algorithm 4: Procedure CTDA

Closest Top Down Largest First (CTDLF) – The tree is traversed in breadth-first manner as in CTDA. However, we treat the subtree which contains the most requests first when considering the children of the tree (we sort the children by increasing number of requests inreq to perform the "fifo.push(i)"). Also, instead of adding all possible servers in a single step, the tree traversal is stopped as soon as a server that can process all the requests in its subtree has been found. This is done by adding an instruction *return* each time a server has been found in the procedure CTDA (Algorithm 4), just after the update of the inreq values of the server's ancestors. As for the previous heuristic, the procedure is called until no more server is chosen. In fact CTDLF is called exactly $|R|$ times, where R is the final set of replica.

Closest Bottom Up (CBU) – The last heuristic for the *Closest* policy performs a bottom-up traversal of the tree. A node is chosen as a server if it can process all the requests of the clients in its subtree. Algorithm 5 describes a recursive implementation of *ClosestBottomUp (CBU)*. The procedure is initially called with the root of the tree; while we do not reach the bottom of the tree, we go down. Once arrived at the bottom, i.e. when the current node s has only clients as children (test $atBottom(s)$) or when all its children have already been treated (test $allChildrenTreated(s)$), the node is marked as treated and added to the set replica if $W_s \geq \mathsf{inreq}_s$. Then we go up in the tree until all nodes are treated, performing recursive calls.

Each of these three heuristics is placing a number of replicas, but none is ensuring whether a valid solution has been found or not. We need to check the final value of inreq_{root}. If there still are some pending requests at the root, there is no valid solution. However, if $\mathsf{inreq}_{root} = 0$, the heuristic has found a solution.

```
procedure CBU (s ∈ N, replica)
if atBottom(s) || allChildrenTreated(s) then
    treated_s = true;
    if W_s ≥ inreq_s & inreq_s > 0 then
        /* node can treat all children's requests */
        replica = replica ∪ {s};
        foreach a ∈ Ancestors(s) do inreq_a = inreq_a − inreq_s;
    else
        /* node cannot treat all children's requests, go up in the tree */
        if Ancestors(s) ≠ ∅ then  call CBU (parent(s), replica);
    end
else
    foreach i ∈ children(s) do
        /* not yet at the bottom of the tree, go down */
        if i ∈ N & ¬treated_i then  call CBU (i, replica);
    end
end
```

Algorithm 5: Procedure CBU

6.2 *Upwards*

We propose two heuristics for the *Upwards* policy, the first one using a top-down approach, the other considering the clients one by one, by non-increasing order of their number of requests.

Upwards Top Down (UTD) – The top down approach works in two passes. In the first pass (see Algorithm 7), each node $s \in \mathcal{N}$ whose capacity is exhausted by the number of requests in its subtree ($W_s \leq \mathsf{inreq}_s$) is chosen by traversing the tree in depth-first manner. When a server is chosen, we delete as much clients as possible in non-increasing order of their number of requests r_i, until the server capacity is reached or no other client can be deleted. This delete procedure is described in Algorithm 6. If not all requests can be treated by the chosen servers, a second pass is started. In this *UTDSecondPass*-procedure (see Algorithm 8) servers with remaining requests are added. Note that all these servers are non-exhausted by the remaining requests ($\mathsf{inreq}_s < W_s$). These two procedures are each called only once, with $s = root$ as a parameter.

Similarly to the *Closest* heuristics, we need to check that $\mathsf{inreq}_{root} = 0$ at the end of UTD to find out whether a valid solution has been found.

```
procedure deleteRequests (s ∈ N, numToDelete)
clientList = sortDecreasing(clients(s));
foreach i ∈ clientList do
    if r_i ≤ numToDelete then
        numToDelete = numToDelete - r_i;
        foreach a ∈ Ancestors(i) do inreq_a = inreq_a − r_i;
        children(parent(i)) = children(parent(i)) \ {i};
        if numToDelete == 0 then  return;
    end
end
```

Algorithm 6: Procedure deleteRequests

Upwards Big Client First (UBCF) – The second heuristic for the *Upwards* policy works in a completely different way than all the other heuristics. The basic idea here is to treat all clients in non-increasing order of their r_i values. For each client we identify the server with minimal current capacity (in the path from the client to the root) that can treat all its requests. The capacity of a server is decreased each time it is assigned some requests to process. If there is no valid server to

procedure **UTDFirstPass** ($s \in \mathcal{N}$, replica)
if $\mathit{inreq}_s \geq W_s$ & $\mathit{inreq}_s > 0$ **then**
  replica = replica $\cup \{s\}$;
  treated$_s$ = true;
  deleteRequests(s, W_s);
end
foreach $i \in \mathit{children}(s)$ **do**
  if $i \in \mathcal{N}$ **then** UTDFirstPass (i, replica);
end

Algorithm 7: Procedure UTDFirstPass

procedure **UTDSecondPass** ($s \in \mathcal{N}$, replica)
if $s \notin \mathit{replica}$ & $\mathit{inreq}_s > 0$ **then**
  replica = replica $\cup \{s\}$;
  deleteRequests(s, inreq$_s$);
else
  foreach $i \in \mathit{children}(s)$ **do**
    if $i \in \mathcal{N}$ & $\mathit{inreq}_i > 0$ **then** UTDSecondPass (i, replica);
  end
end

Algorithm 8: Procedure UTDSecondPass

assign to a given client, the heuristic has failed to find a valid solution. Please refer to Algorithm 9 for details.

procedure **UBCF** ($s \in \mathcal{N}$, replica)
$clientList$ = sortDecreasing(clients(s));
foreach $i \in clientList$ **do**
  $ValidAncests = \{a \in \mathsf{Ancestors}(i) | W_a \geq r_i\}$;
  if $ValidAncests \neq \emptyset$ **then**
    $a = Min_{W_j}\{j \in ValidAncests\}$;
    if $a \notin \mathit{replica}$ **then** replica = replica $\cup \{a\}$;
    $W_a = W_a - r_i$;
  end
  else return *no solution*;
end

Algorithm 9: Procedure UBCF

6.3 *Multiple*

We propose three heuristics for the *Multiple* policy. The first one uses a top-down approach, the second one a bottom-up approach. The last one performs a greedy bottom-up traversal of the tree.

Multiple Top Down (MTD) – The top-down approach for the *Multiple* policy is similar to the top-down approach for *Upwards*, with one significant difference: the delete procedure. For *Upwards*, requests of a client have to be treated by a single server, and it may occur that after the delete procedure a server still has some capacity left to treat more requests, but all remaining clients have a higher amount of requests than this leftover capacity. For *Multiple*, requests of a client can be treated by multiple servers. So if at the end of the delete procedure the server still has some capacity, we delete this amount of requests from the client with the largest r_i. This modified delete procedure is described in Algorithm 10.

```
procedure deleteRequestsInMTD (s ∈ N, numToDelete)
clientList = sortDecreasing(clients(s));
foreach i ∈ clientList do
    if r_i ≤ numToDelete then
        numToDelete = numToDelete - r_i;
        foreach a ∈ Ancestors(i) do inreq_a = inreq_a − r_i;
        children(parent(i)) = children(parent(i)) \ {i};
    else
        r_i = r_i - numToDelete;
        foreach a ∈ Ancestors(i) do inreq_a = inreq_a − r_i;
        return;
    end
end
```

Algorithm 10: Procedure deleteRequestsInMTD

Multiple Bottom Up (MBU) – The first pass of this heuristic performs a bottom-up traversal of the tree, as in CBU. During this traversal, nodes $s \in \mathcal{N}$ are added to the set replica if their capacity is exhausted ($W_s \leq \mathsf{inreq}_s$), similarly to the first pass of the MTD procedure. The delete procedure is identical to the MTD delete procedure (Algorithm 10), except that clients are deleted in non-decreasing order of their r_i values (instead of the non-increasing order). Intuitively, we aim at deleting many small clients rather than fewer demanding ones. The *MBUFirstPass* is described in Algorithm 11, and the *MBUSecondPass*, which adds extra servers if required (similarly to the second pass of MTD), is described in Algorithm 12.

```
procedure MBUFirstPass (s ∈ N, replica)
if atBottom(s) || allChildrenTreated(s) then
    treated_s = true;
    if W_s ≤ inreq_s & inreq_s > 0 then
        /* node is exhausted by the requests of its clients */
        replica = replica ∪ {s};
        deleteRequestsInMBU(s, W_s);
    else
        /* node is not exhausted, go up the tree */
        if Ancestors(s) ≠ ∅ then  call MBU (parent(s), replica);
    end
else
    /* not yet at the bottom of the tree, go down */
    foreach i ∈ children(s) do
        if i ∈ N & ¬treated_i then  call MBU (i, replica);
    end
end
```

Algorithm 11: Procedure MBUFirstPass

Multiple Greedy (MG) – The last heuristic performs a greedy bottom-up assignment of requests, similarly to Pass 3 of the optimal algorithm for the homogeneous case (see Algorithm 3 in Section 4.1). We add a replica whenever there are some requests affected to a server. For heterogeneous platforms, we may often return a cost far from the optimal, but we ensure that we always find a solution to the problem if there exists one.

It might be particularly interesting to use MG only for problem instances for which MBU or MTD fail to find a solution.

procedure **MBUSecondPass** ($s \in \mathcal{N}$, replica)
if $s \notin$ *replica* & *inreq*$_s > 0$ **then**
  replica = replica $\cup \{s\}$;
  deleteRequestsInMBU(s, inreq$_s$);
else
  foreach $i \in$ *children*(s) **do**
    if $i \in \mathcal{N}$ & *inreq*$_i > 0$ **then** UTDSecondPass (i, replica);
  end
end

Algorithm 12: Procedure MBUSecondPass

7 Experiments: comparisons of different access policies

We have done some experiments to assess the impact of the different access policies, and the performance of the polynomial heuristics described in Section 6. We obtain an absolute lower bound of the solution for each tree platform with a linear program similar to those of Section 5, but modified so as to solve larger problems. Section 7.1 details how we compute this lower bound. We outline the experimental plan in Section 7.2. Results are given and commented in Section 7.3. In the following, we denote by s the problem size: $s = |\mathcal{C}| + |\mathcal{N}|$.

7.1 Obtaining a lower bound

The linear programs exposed in Section 5 must be solved in integer values if we wish to obtain an exact solution to an instance of the problem. This can be done for each access policy, but due to the large number of variables, the problem cannot be solved for platforms of size $s > 50$. Thus we cannot use this approach for large-scale problems.

For all practical values of the problem size, the rational linear program returns a solution in a few minutes. We tested up to several thousands of nodes and clients, and we always found a solution within ten seconds.

However, we can obtain a more precise lower bound for trees with up to $s = 400$ nodes and clients by using a rational solution of the *Multiple* instance of the linear program with fewer integer variables. We treat the $y_{i,j}$ and $z_{i,l}$ as rational variables, and only require the x_j to be integer variables. These variables are set to 1 if and only if there is a replica on the corresponding node. Thus, forbidding to set $0 < x_j < 1$ allows us to get a realistic value of the cost of a solution of the problem. For instance, a server might be used only at 50% of its capacity, thus setting $x = 0.5$ would be enough to ensure that all requests are processed; but in this case, the cost of placing the replica at this node is halved, which is incorrect: while we can place a replica or not but it is impossible to place half of a replica.

In practice, this lower bound provides a drastic improvement over the unreachable lower bound provided by the fully rational linear program. The good news is that we can compute the refined lower bound for problem sizes up to $s = 400$, using GLPK [4]. We used the refined bound for all our experiments.

7.2 Experimental plan

The important parameter in our tree networks is the load, i.e. the total number of requests compared to the total processing power:

$$\lambda = \frac{\sum_{i \in \mathcal{C}} r_i}{\sum_{j \in \mathcal{N}} \mathsf{W}_i}$$

We have performed experiments on 30 trees for each of the nine values of λ selected ($\lambda = 0.1, 0.2, ..., 0.9$). The trees have been randomly generated, with a problem size $15 \leq s \leq 400$.

When λ is small, the tree has a light request load, while large values of λ implies a heavy load on the servers. We then expect the problem to have a solution less frequently.

We have computed the number of solutions for each lambda and each heuristic. The number of solutions obtained by the linear program indicates which problems are solvable. Of course we cannot expect a result with our heuristics for those intractable problems.

To assess the relative cost of each heuristic, we have studied the distance of the result (in terms of replica cost) of the heuristic to the lower bound. This allows to compare the cost of the different heuristics, and thus to compare the different access policies. For each λ, the cost is computed on the trees for which the linear program has a solution. Let T_λ be the subset of trees with a solution. Then, the relative cost for the heuristic h is obtained by:

$$rcost = \frac{1}{|T_\lambda|} \sum_{t \in T_\lambda} \frac{cost_{LP}(t)}{cost_h(t)}$$

where $cost_{LP(t)}$ is the lower bound cost returned by the linear program on tree t, and $cost_h(t)$ is the cost involved by the solution proposed by heuristic h. In order to be fair versus heuristics who have a higher success rate, we set $cost_h(t) = +\infty$ if the heuristic did not find any solution.

Experiments have been conducted both on homogeneous networks (Replica Counting problem) and on heterogeneous ones (Replica Cost problem).

7.3 Results

A solution computed by a *Closest* or *Upwards* heuristic always is a solution for the *Multiple* policy, since the latter is less constrained. Therefore, we can mix results into a new heuristic for the *Multiple* policy, called MixedBest (**MB**), which selects for each tree the best cost returned by the previous eight heuristics for this particular problem instance. Since MG never fails to find a solution if there is one, MB will neither fail either.

Figure 9 shows the percentage of success of each heuristic for homogeneous platforms. The upper curve corresponds to the result of the linear program, and to the cost of the MG and MB heuristics, which confirms that they always find a solution when there is one. The UBCF heuristic seems very efficient, since it finds a solution more often than MTD and MBU, the other two *Multiple* policies. On the contrary, UTD, which works in a similar way to MTD and MBU, finds less solutions than these two heuristics, since it is further constrained by the *Upwards* policy. As expected, all the *Closest* heuristics find fewer solutions as soon as λ reaches higher values: the bottom curve of the plot corresponds to CTDA, CTDLF and CBU, which all find the same solutions. This is inherent to the limitation of the *Closest* policy: when the number of requests is high compared to the total processing power in the tree, there is little chance that a server can process all the requests coming from its subtree, and requests cannot traverse this server to be served higher in the tree. These results confirm that the new policies have a striking impact on the existence of a solution to the Replica Counting problem.

Figure 10 represents the relative cost of the heuristics compared to the LP-based lower bound. As expected, the hierarchy between the policies is respected, i.e. *Multiple* is better than *Upwards* which in turn is better than *Closest*. For small values of λ, it happens that some *Closest* heuristics give a better solution than those for *Upwards* or *Multiple*, due to the fact that the latter heuristics are not well optimized for small values of λ. Also, UBCF is better than all the *Multiple* heuristics for $\lambda = 0.6$. Altogether, the use of the MixedBest heuristic MB allows to always pick up the best result, thereby resulting in a very satisfying relative cost for the *Multiple* instance of the problem. The greedy MG should not be used for small values of λ, but proves to be very efficient for large values, since it is the only heuristic to find a solution for such instances. To conclude, we point out that MB always achieves a relative cost of at least 85%, thus returning a replica cost within 17% of that of the LP-based lower bound. This is a very satisfactory result for the absolute performance of our heuristics.

The heterogeneous results (see Figure 11 and Figure 12) are very similar to the homogeneous ones, which clearly shows that our heuristics are not much sensitive to the heterogeneity of the platform. Therefore, we have an efficient way to find in polynomial time a good solution to all the NP-hard problems stated in Section 4.

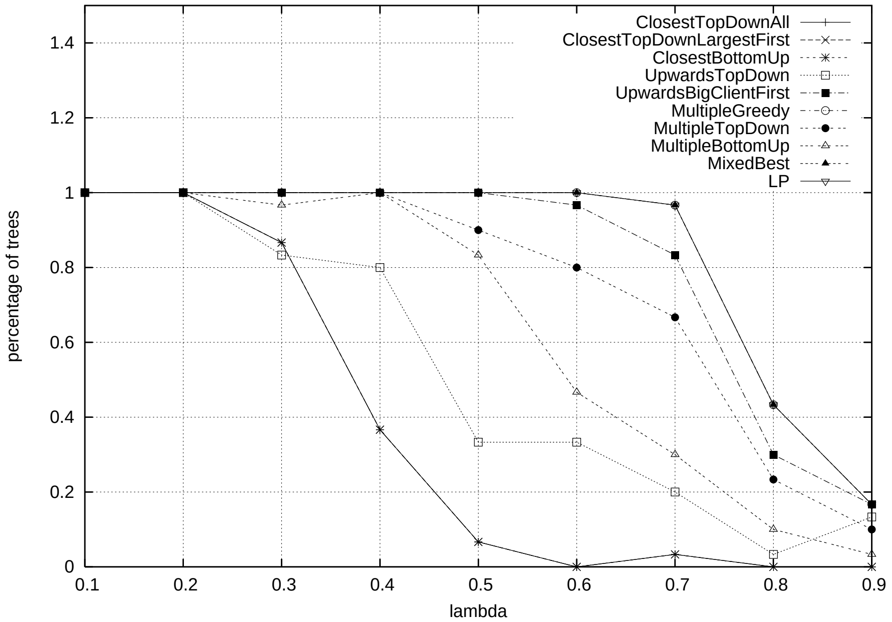


Figure 9: Homogeneous case - Percentage of success.

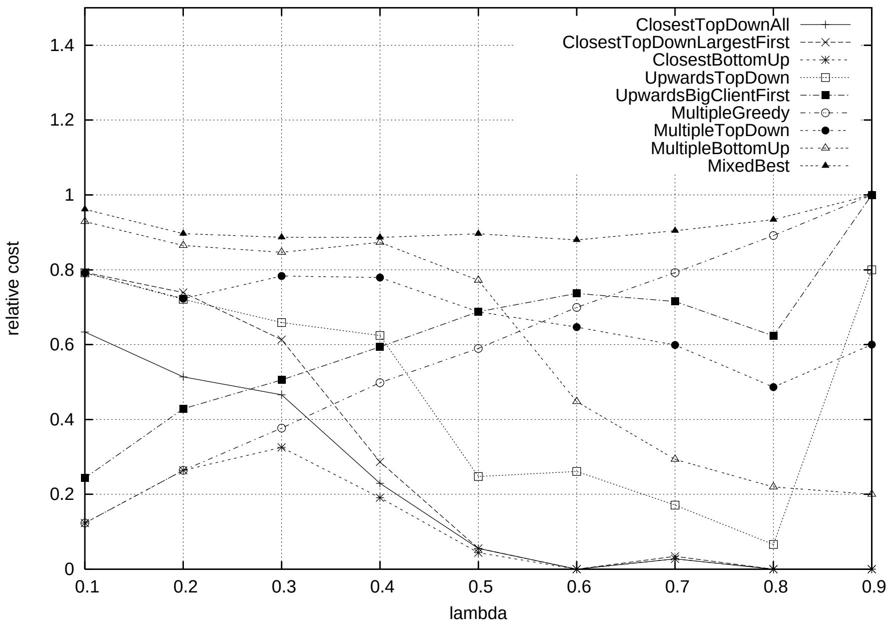


Figure 10: Homogeneous case - Relative cost.

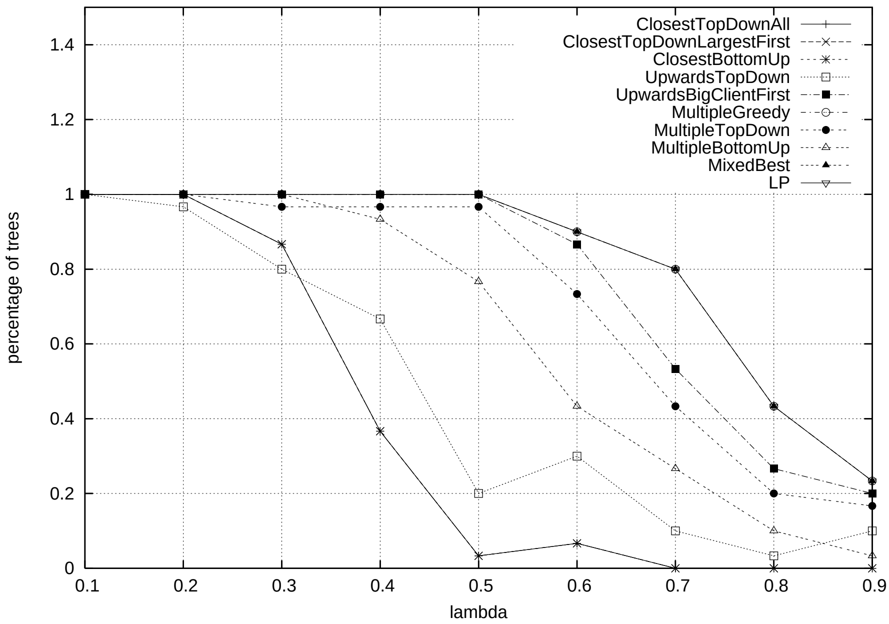


Figure 11: Heterogeneous case - Percentage of success.

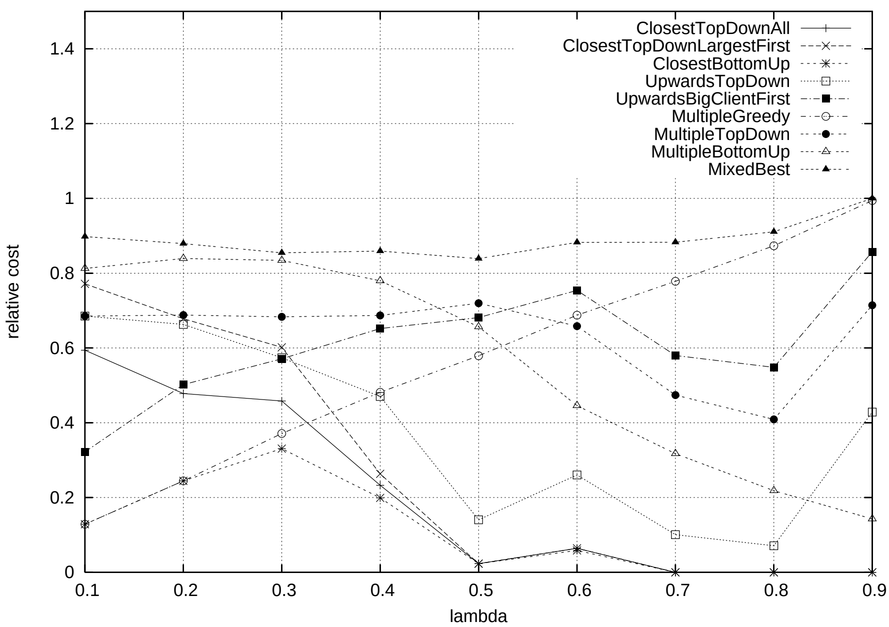


Figure 12: Heterogeneous case - Relative cost.

8 Extensions

In this paper we have considered a simplified instance of the replica problem. In this section, we outline two important generalizations, namely dealing with several objects, and changing the objective function.

8.1 With several objects

In this paper, we have restricted the study of the problem to a single object, which means that all replicas are identical (of the same type). We can envision a system in which different types of objects need to be accessed. The clients are then having requests of different types, which can be served only by an appropriate replica. Thus, for an object of type k, client $i \in \mathcal{C}$ issues $r_i^{(k)}$ requests for this object. To serve a request of type k, a node must be provided with a replica of that type. Nodes can be provided with several replica types. A given client is likely to have different servers for different objects. The QoS may also be object-dependent ($\mathsf{q}_i^{(k)}$).

To refine further, new parameters can be introduce such as the size of object k and the computation time involved for this object. Nodes parameters become object-dependent too, in particular the storage cost and the time required to answer a request.

The server capacity constraint must then be a sum on all the object types, while the QoS must be satisfied for each object type. The link capacity also is a sum on the different object types, taking into account the size of each object.

There remains to modify the objective function: we simply aim at minimizing the cost of all replicas of different types that have been assigned to the nodes in the solution to get the extended *replica cost* for several objects.

Because the constraints add up linearly for different objects, it is not difficult to extend the linear programming formulation of Section 5 to deal with several objects. Also, the three access policies *Closest*, *Upwards* and *Multiple* could naturally be extended to handle several objects. However, designing efficient heuristics for various object types, especially with different communication to computation ratios and different QoS constraints for each type, is a challenging algorithmic problem.

8.2 More complex objective functions

Several important extensions of the problem consist in having a more complex objective function. In fact, either with on or with several objects, we have restricted so far to minimizing the cost of the replicas (and even their number in the homogeneous case). However, several other factors can be introduced in the objective function:

Communication cost – This cost is the *read* cost, *i.e.* the communication cost required to access the replicas to answer requests. It is thus a sum on all objects and all clients of the communication time required to access the replica. If we take this criteria into account in the objective function, we may prefer a solution in which replicas are close to the clients.

Update cost – The *write* cost is the extra cost due to an update of the replicas. An update must be performed when one of the clients is modifying (writing) some of the data. In this case, to ensure the consistency of the data, we need to propagate the modification to all other replicas of the modified object. Usually, this cost is directly related to the communication costs on the minimum spanning tree of the replica, since the replica which has been modified sends the information to all the other replicas.

Linear combination – A quite general objective function can be obtained by a linear combination of the three different costs, namely replica cost, read cost and write cost. Informally, such an objective function would write

$$\alpha \sum_{\text{servers, objects}} \textit{replica cost} + \beta \sum_{\text{requests}} \textit{read cost} + \gamma \sum_{\text{updates}} \textit{write cost}$$

where the application-dependent parameters α, β and γ would be used to give priorities to the different costs.

Again, designing efficient heuristics for such general objective functions, especially in the context of heterogeneous resources, is a challenging algorithmic problem.

9 Related work

Early work on replica placement by Wolfson and Milo [13] has shown the impact of the write cost and motivated the use of a minimum spanning tree to perform updates between the replicas. In this work, they prove that the replica placement problem in a general graph is NP-complete, even without taking into account storage costs. Thus they address the case of special topologies, and in particular tree networks. They give a polynomial solution in a fully homogeneous case and a simple model with no QoS and no server capacity. Their work uses the closest server access policy (single server) to access the data.

Using this *Closest* policy, Cidon et al [2] studied an instance of the problem with multiple objects. In this work, the objective function has no update cost, but integrates a communication cost. Communication cost in the objective function can be seen as a substitute for QoS. Thus, they minimize the average communication cost for all the clients rather than ensuring a given QoS for each client. They target fully homogeneous platforms since there are no server capacity constraints in their approach. A similar instance of the problem has been studied by Liu et al [9], adding a QoS in terms of a range limit (QoS=distance), and the objective being the Replica Counting problem. In this latter approach, the servers are homogeneous, and their capacity is bounded.

Cidon et al [2] and Liu et al [9] both use the *Closest* access policy. In each case, the optimization problems are shown to have polynomial complexity. However, the variant with bidirectional links is shown NP-complete by Kalpakis et al [5]. Indeed in [5], requests can be served by any node in the tree, not just the nodes located in the path from the client to the root. The simple problem of minimizing the number of replicas with identical servers of fixed capacity, without any communication cost nor QoS contraints, directly reduces to the clasical bin packing problem.

Kalpakis et al [5] show that a special instance of the problem is polynomial, when considering no server capacities, but with a general objective function taking into account read, write and storage costs. In their work, a minimum spanning tree is used to propagate the writes, as was done in [13]. Different methods can however be used, such as a minimum cost Steiner tree, in order to further optimize the write strategy [6].

All papers listed above consider the *Closest* access policy. As already stated, most problems are NP-complete, except for some very simplified instances. Karlsson et al [8, 7] compare different objective functions and several heuristics to solve these complex problems. They do not take QoS constraints into account, but instead integrate a communication cost in the objective function as was done in [2]. Integrating the communication cost into the objective function can be viewed as a Lagrangian relaxation of QoS constraints.

Tang and Xu [12] have been one of the first authors to introduce actual QoS constraints in the problem formalization. In their approach, the QoS corresponds to the latency requirements of each client. Different access policies are considered. First, a replica-aware policy in a general graph is proven to be NP-complete. When the clients do not know where the replicas are (replica-blind policy), the graph is simplified to a tree (fixed routing scheme) with the *Closest* policy, and in this case again it is possible to find a polynomial algorithm using dynamic programming.

To the best of our knowledge, there is no related work comparing different access policies, either on tree networks or on general graphs. Most previous works impose the *Closest* policy. The *Multiple* policy is enforced by Rodolakis et al [10] but in a very different context. In fact, they consider general graphs instead of trees, so they face the combinatorial complexity of finding good routing paths. Also, they assume an unlimited capacity at each node, since they can add numerous servers of different kinds on a single node. Finally, they include some QoS constraints in their problem formulation, based on the round trip time (in the graph) required to serve the client requests. In such a context, this (very particular) instance of the *Multiple* problem is shown to be NP-hard.

10 Conclusion

In this paper, we have introduced and extensively analyzed two important new policies for the replica placement problem. The *Upwards* and *Multiple* policies are natural variants of the standard *Closest* approach, and it may seem surprising that they have not already been considered in the published literature.

On the theoretical side, we have fully assessed the complexity of the *Closest*, *Upwards* and *Multiple* policies, both for homogeneous and heterogeneous platforms. The polynomial complexity of the *Multiple* policy in the homogeneous case is quite unexpected, and we have provided an elegant algorithm to compute the optimal cost for this policy. Not surprisingly, all three policies turn out to be NP-complete for heterogeneous nodes, which provides yet another example of the additional difficulties induced by resource heterogeneity.

On the practical side, we have designed several heuristics for the *Closest*, *Upwards* and *Multiple* policies, and we have compared their performance for a simple instance of the problem, without QoS constraints nor bandwidth limitations. In the experiments, the constraints were only related to server capacities, and the total cost was the sum of the server capacities (or their number in the homogeneous case). Even in this simple setting, the impact of the new policies is impressive: the number of trees which admit a solution is much higher with the *Upwards* and *Multiple* policies than with the *Closest* policy. Finally, we point out that the absolute performance of the heuristics is quite good, since their cost is close to the lower bound based upon the solution of the integer linear program.

There remains much work to extend the results of this paper, in several important directions. In the short term, we need to conduct more simulations for the Replica Cost problem, varying the shape of the trees, the distribution law of the requests and the degree of heterogeneity of the platforms. We also aim at designing efficient heuristics for more general instances of the Replica Placement problem, taking QoS and bandwidth constraints into account. It will be instructive to see whether the superiority of the new *Upwards* and *Multiple* policies over *Closest* remains so important in the presence of QoS constraints. Also, including bandwidth constraints may require a better global load-balancing along the tree, thereby favoring *Multiple* over *Upwards*.

In the longer term, designing efficient heuristics for the problem with various object types, all with different communication to computation ratios and different QoS constraints is a demanding algorithmic problem. Also, we would like to extend this work so as to handle more complex objective functions, including communication costs and update costs as well as replica costs; this seems to be a very difficult challenge to tackle, especially in the context of heterogeneous resources.

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