Single product scheduling and transportation optimization in a supply chain

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1 Introduction

Supply chain management (SCM) has risen to prominence over the past ten years [1]. Supply chain main functions are: To buy, to move, to transform and to sell. Scheduling and transportation optimization are central issues for effectiveness of supply chain management systems. Most operations research models consider transportation optimization and scheduling optimization separately. This work is a modest alternative to consider them jointly, beginning with the simplest model and adding progressively some more realistic constraints.

Until now, there are relatively few papers dealing directly with scheduling and transportation problems in a supply chain. Hall and Potts [2] considered various scheduling, batching and delivery problems in supply chains with the objective of minimizing the overall scheduling and delivery cost. They got some complexity results including pseudo-polynomial dynamic programming approaches. Selvarajah and Steiner [4] proposed a polynomial algorithm for the problem of minimizing the sum of inventory holding and delivery costs from the point of view of the supplier. Kreipl and Pinedo [3] described how planning and scheduling models can be used for the design of decision support systems in a supply chain.

In this paper we consider a basic supply chain composed of one or several suppliers that provide components needed by a manufacturer. The manufacturer has to deliver finished products to only one customer before given delivery dates. We first verify if the associated decision problem is feasible or not with unlimited transportation capacities. At this decision level we assume that for the components, the delivery dates and the associated quantities are known. We further integrate progressively to the model more realistic transportation constraints and costs to be optimized : number of travels with unlimited number of trucks with unlimited capacity, number of travels with unlimited number of trucks with limited capacity and finally we consider limited number of physical trucks as a perspective.

2 Problems description

The considered elementary supply chain contains one or several component suppliers, one products manufacturer and one customer. The suppliers deliver components to the manufacturer at given arrival dates with known quantities. The customer has to be delivered at given delivery dates with known quantities. Through this paper we use the following assumptions.

There is only one finished product (mono-product production) and we need P time units to manufacture one item on a single machine. The machine is continuously available and can process only one job at a time. In this model, we do not consider inventory holding costs, in consequence, left shifted schedule (as soon as possible relatively to the components arrival) is optimal and is used by the manufacturer. It can be noted that in this very particular case, the optimal left shifted schedule on several identical parallel machines can be obviously polynomially obtained. For simplicity, and without lack of generality, we present the single sequential machine case. We naturally assume we stop the production when we attain the sum of the asked finished products.

We consider a transportation time between the manufacturer and the customer of t time units. These t transportation time units are supposed to be subtracted from the delivery dates. The finished product quantities can be delivered in advance (but probably paid by the customer only at the required date) but never late, the delivery dates are considered here as imperative dead lines.

In the section 3, we compute the left shifted production schedule and we check at every delivery date if we have produced enough items of the finished product (production feasibility verification). In the section 4, we deal with a first transportation problem. The production feasibility is supposed to be verified, we make the assumptions that we have unlimited number of trucks with unlimited capacity. This first transportation problem is then always feasible. Our objective is to reduce transportation costs which is here achieved by minimizing the number of travels done between the manufacturer and the customer. In the section 5, we consider a second transportation problem, which consists in minimizing the number of travels, but with a limited capacity common to each truck (homogeneous set of trucks). Nevertheless, we assume the number of vehicles is not limited and we do not optimize their movements. The considered simple problems are all polynomial.

3 Left shifted production schedule and down-stream necessary condition

3.1 Notations

We use the following notations :

NC : number of arrivals of components.

QC_j : quantity of the j^{th} arrival of components.

DC_j : arrival date associated to QC_j.

ND : number of delivery dates of the finished product.

QD_i : i^{th} quantity to be delivered.

DD_i : delivery date associated to QD_i (the t time units necessary for the transportation are already deleted)

CQP_i : left shifted cumulative produced quantity between instant 0 and the i^{th} delivery date DD_i.

CQD_i : cumulative demanded quantity until the i^{th} delivery date DD_i (included).

ΔPQ_i : produced quantity between DD_{i-1} and DD_i.

ND, NC, QD_i, DD_i, QC_j and DC_j are the data of the problem. CQP_i, CQD_i and ΔPQ_i are computed by algorithm 1.

3.2 Algorithm 1 and first feasibility check

When the transportation capacities are unlimited, the problem is feasible if, and only if, at each delivery date the cumulative asked quantity until this date is less than or equal to the cumulative produced quantity at this delivery date. More formally, this condition can be written as : $CQP_i \geq CQD_i$ for $i = 1$ to ND.

Algorithm 1 computes the values of CQP_i and ΔPQ_i by producing any product as soon as possible.

Algorithm 1

j=T=inventory=Qtity=0

for $i = 1,..,ND$ *do*

  Qtity=inventory

  While $DC_j \leq DD_i$ and T$\leq DD_i$ *do*

    Qtity= Qtity+QC_j ; T=max (T, DC_j)+QC_j*P ; j=j+1 *endWhile*

if T> DD_i *then* inventory= $\left\lceil \frac{T - DD_i}{P} \right\rceil$; Qtity= Qtity -inventory e*ndif*

ΔPQ_i =Qtity

****end of the iteration*** *(to be completed in the other sections)*

endfor

The complexity of algorithm 1 depends on the dates at which there is components arrival or products delivery. Thus the complexity of algorithm 1 is O(ND+NC).

4 Transportation organization with number of travels minimization

In order to optimize the transportation costs by minimizing the number of travels between the manufacturer and the customer we use a right shifted transportation policy, making the minimum number of travels as late as possible. We obviously assume that the first necessary condition is verified (i.e. $CQP_i \geq CQD_i$ for $i = 1$ to ND).

This transportation problem can be modelled by the following equations:

First we define the decision variables X_i such that :

$$X_i = \begin{cases} 1 \;\; \textit{if a travel is done at the } i^{\text{th}} \textit{ product delivery date} \\ 0 \;\; \textit{otherwise} \end{cases}$$

Let CQT_i denote the cumulative transported quantity between instant 0 and the i^{th} delivery date DD_i including potential departure on DD_i, when the realized travels are defined by the unknown X_i.

$CQT_i = \sum_{j=1}^{i} \Delta PQ_j . X_j$ for $i = 1$ to ND.

$CQT_i \geq CQD_i$ for $i = 1$ to ND (satisfaction of the customer cumulative demands).

Minimize $\sum_{i=1}^{ND} X_i$

Property : *The optimal number of travels between the manufacturer and the customer is given by :*

$$X = \sum_{i=1}^{ND} X_i \;\; \textit{such that if } CQT_{i-1} \geq CQD_i \textit{ then } X_i = 0 \textit{ else } X_i = 1.$$

Proof. We suppose that our solution is not optimal. This means that we can remove at least one travel. Let this travel be the i^{th} one, we will have $X_i = 0$ which implies that $CQT_i = CQT_{i-1}$ and as $CQT_{i-1} < CQD_i$ we will have $CQT_i < CQD_i$ which makes the problem infeasible which is absurd.

Algorithm 2 is similar to algorithm 1. In fact, we only replace the comment at the **end of the iteration** by the following operations:

If $CQT_{i-1} \geq CQD_i$ *then* $X_i = 0$

else $X_i = 1$ *endif*

$CQT_i = CQT_{i-1} + \Delta PQ_i$

As these instructions do not affect the algorithm cost time, complexity of algorithm 2 is O(ND+NC).

5 Transportation organization with limited capacity of the trucks

This problem is similar to the problem of section 4 with a constraint on trucks capacity. Algorithm 3, that we propose for optimizing travels number under this capacity constraint, is an extension of algorithm 1.

Algorithm 3 complexity is also O(ND+NC, because we only replace in Algorithm 1 the comment at the **end of the iteration** by the following operations:

$$X_i = \left\lfloor \frac{\Delta PQ_i}{C} \right\rfloor; \; CQT_i = CQT_{i-1} + X_i.C$$

If $CQT_i < CQD_i$ *then*

$X_i = X_i + 1;$

$CQT_i = CQT_i + \Delta PQ_i \, mod \; C$

else

inventory= $\Delta PQ_i \;$ *mod* C *endif*

Theorem. *The optimal travels number for problem 3 is given by algorithm 3.*

Proof. We suppose that our solution is not optimal. This means that we can remove at least one travel. Let this travel be in the i^{th} iteration. We will have two cases :

I. We remove a full truck : in this case as the sum of moved products is equal to the sum of asked products, this travel must be done in another iteration k, such that $i < k \le ND$ and the number of travels remains constant.

II. We remove a non full truck: we are in the case $CQT_i < CQD_i$ and we choose not to transport, so CQT_i remains unchanged and we still have $CQT_i < CQD_i$ which makes the problem infeasible.

6 Conclusion

In this paper, we address three models of scheduling and transportation problems in a simple single product supply chain. We begin with a simple model to which we progressively integrate more realistic transportation constraints. We have proposed polynomial time algorithms for these problems and have verified they are optimal. As a perspective to this modest work, we will consider the problem of limited number of physical vehicles. We will probably hybridize algorithms for optimizing flows in networks with procedures developed in this paper. In next future we will also consider supply chain with multiple products demands, an interesting challenge for transportation and scheduling optimization in a supply chain.

7 Bibliography

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