

# The Performance of Broadcasting with Network Coding in Dense Wireless Networks

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*The Performance of Broadcasting with Network  
Coding in Dense Wireless Networks*

Cédric Adjih, Song Yean Cho, Philippe Jacquet

N° 6120

Février 2007

THÈME 1

A large blue rectangle occupies the lower half of the page. On the left side of this rectangle, there is a large, light grey, stylized letter 'R'. To the right of the 'R', the words 'Rapport de recherche' are written in a white, serif font. A horizontal white line is positioned below the text.

*Rapport  
de recherche*





## The Performance of Broadcasting with Network Coding in Dense Wireless Networks

Cédric Adjih, Song Yean Cho, Philippe Jacquet

Thème 1 — Réseaux et systèmes  
Projet HIPERCOM

Rapport de recherche n° 6120 — Février 2007 — 15 pages

**Abstract:** We present a protocol for wireless broadcast transmission based on network coding. The protocol does not need neighbor sensing and network topology monitoring. We give an analysis of the performance of the protocol in an unit graph wireless network model with uniform density, and in the case of a single source. We show that even with one source, due to density, network coding offers some gains.

In particular we show that in 1 dimensional (1D) network the performance of the simple protocol based on network coding is close to the optimal flooding (without network coding) by a factor arbitrary close to 1 when the network density increases. In 2 dimensional (2D) networks simulations show that the ratio to optimal is similar to outperforming MultiPoint Relay (MPR) flooding.

**Key-words:** wireless networks, network coding, broadcasting, multi-hop

## Performance de la diffusion par codage de réseau dans les réseaux sans fil denses

**Résumé :** Nous présentons un protocole pour la diffusion dans les réseaux sans fil basé sur le codage de réseau. Le protocole n'a pas besoin de découverte de voisins et ni de détection de la topologie du réseau. Nous donnons une analyse de la performance du protocole dans le cas d'une source simple et dans un modèle de réseau sans fil qui est un graphe disque unité avec une densité uniforme. Nous montrons que même avec une seule source, grâce à la densité des noeuds, le codage de réseau permet des gains. En particulier, nous prouvons que dans un réseau unidimensionnel (1D), la performance du protocole simple basé sur le codage de réseau est proche de la performance de l'inondation optimale (sans codage de réseau) par un facteur arbitrairement de proche de 1 quand la densité du réseau augmente. Dans réseaux en 2 dimensions, des simulations montrent que le rapport à l'optimal est similaire voire meilleur que celle de l'inondation par relais multipoints (MPR).

**Mots-clés :** réseaux sans fil, codage de réseau, diffusion, multi-sauts

## 1 Introduction

In traditional communication systems, nodes exchange data through relaying by intermediate nodes: the packets are forwarded by the intermediate routers unmodified for both traditional unicast and multicast communication. Recently, the pioneering work of Ahlswede et al. [1] has shown that *network coding*, where intermediate nodes mix information from different flows, can achieve higher throughput for multicasting than classical routing over networks represented as directed graphs. The works of [1, 2, 3, 4, 5, 6] set theoretical and practical foundations for the framework of many recent works concerning *wireless network coding* (see section 2). The wireless media is inherently a broadcast media which has crucial advantage since it allows overhearing. The inherent advantage could be exploited by wireless network coding, permitting gains by reducing the number of transmissions required to transmit the same amount information (*energy-efficiency*). Due to good matching between the inherent advantage and gaining, wireless network coding has been successfully applied to unicast traffic of 802.11 networks for instance in [8], and it has also been applied to multicast traffic, such as in [7], [5] and [10].

However, the focus has often been on multiple sources where the gain originates precisely from the existence of different directional flows, for instance in [8] for unicast, in [9] (with *packets* and *anti-packets* pairs), and in [5].

In contrast, in this article, we focus on broadcasting with a *single source*. Our goal is not so much to achieve maximum performance with wireless network coding, but rather to illustrate that wireless network coding can achieve gains even with a single source (as in the general, non-wireless, case of [1]), thank to *density*, and to give an insight on the dynamics of network coding in such an environment. Therefore our approach follows the spirit of the *algebraic gossip* [11], but focusing on the idiosyncrasies of the wireless medium.

To do so, we focus on a simple protocol for network coding, which does not need neighbor sensing or network topology monitoring. We initially consider its behavior in the one-dimensional (1D) case, i.e the 1D unit disk graph model. We describe how the packet flows from a source evolve in *waves*, and by studying the propagation of the waves, we are able to quantify the performance of the simple protocol in an unit graph wireless network model, with an asymptotic model. Our analysis shows that, in 1D, as the density increase the performance of the protocol is close to the optimal broadcast by a factor arbitrary close to 1. Simulations confirm the results of the asymptotic model.

Then we conjecture the behavior of the network with one source in 1D to that in 2D unit disk graph model. The behavior in 2D should be essentially the same as in 1D (with the same concept of waves, only with more complex waves), with similar performance as predicted by the model in 1D. Simulations results confirm this insight. The article is organized as follows: in section 2, we describe the simple network coding algorithm for wireless broadcasting. In section 3 we describe the wave propagation analysis that will be the basis of our performance analysis. In section 3 we develop the analytical models and our results. In section 5 we show the simulation results which confirm and extend our analytical results.

## 2 Related Work, Framework and Algorithms

### 2.1 General Framework of Wireless Network Coding

The starting point of network coding is the celebrated work from [1], showing that coding in networks could achieve maximum broadcast capacity (given by the *min-cut*), while in the general case, it is out of reach of traditional transmission methods (i.e. without network coding).

Subsequently, it has been shown that a simple form of coding, *linear coding* [2], (using linear combinations of data symbols belonging to Galois fields  $\mathcal{F}_p$ ), is sufficient to achieve the bounds of [1]. Furthermore, [3] presented one method which does not require coordination of (the coding at) the nodes, by introducing *random linear coding* and by showing that sufficient field size results in high probability of success. With random linear coding, the coding inside the network is no longer predetermined, since it uses random coefficients for the linear combinations. This approach was refined by [4], in the context of packet networks: the information is contained in individual packets and the randomized coefficients can then be included in a special additional packet header. Relying on the fundamental broadcast nature of the wireless media, [6] introduced the first practical implementation of network coding in a wireless environment, and demonstrated gains for crossing unicast data traffics.

These works set the path to practical foundations, which are described for instance in [6], and that we are using in this article.

**Generation:** first, the packets are supposed to be divided into *generations*. A generation is a set of packets (in general, from one or several sources), that could be mixed together by linear combination. In the rest of the paper, we will assume that all packets belong to one generation from a source and denote a generation size the *dimension*,  $D$ .

**Vectors:** second, the packets are equally sized and are divided into blocks of symbols over a field  $\mathcal{F}_p$ : content =  $(s_1, s_2, \dots, s_h)$ . As in [4], the packets include a header which is the list of coefficients. Hence the packet format is actually a vector of the format:  $(g_1, g_2, \dots, g_D; s_1, s_2, \dots, s_h)$ .

**Transmission:** at any point of time, a node of the network has a list of vectors, linear combinations of initial source packets. When the node transmits, it generates a random linear combination of the vectors  $v_0, v_1, \dots, v_k$  it currently has:  $\sum_i \alpha_i v_i$  (where the  $(\alpha_i)$  are random coefficients of  $\mathcal{F}_p$ ), and transmit it by wireless broadcasting.

**Decoding:** once a node has received  $D$  linearly independent vectors, it is able to decode the  $D$  packets of the generation.

Within this classical framework, the two final questions are: when does the source send a packet? and: when do the other nodes (non-source) make transmissions? This *transmission scheduling* is the heart of the algorithm.

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**Algorithm 1:** NCAL1

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- 1.1 **Source scheduling:** poisson source; the source transmits sequentially the  $D$  vectors (packets) of a generation with an exponential interarrival.
  - 1.2 **Nodes' start and stop conditions:** the nodes start transmitting when they receive the first vector. They continue transmitting until they have enough vectors to recover the  $D$  source packets. Then they still transmit one additional packet (after usual exponential delay).
  - 1.3 **Nodes' scheduling:** poisson retransmission; the nodes retransmit linear combinations of the vectors that they have, with an exponential interarrival
- 

## 2.2 Algorithms for Transmission Scheduling

### 2.2.1 Single source

The transmission scheduling used in our article is the algorithm NCAL1. Essentially, the nodes transmit random combinations of the vectors, with exponential interarrival, until they have sufficient vectors. This algorithm can be compared to the algorithms used in [5], [10] and [11] for instance. The algorithm 1 is a version of *Random Linear Coding (RLC) with push* for the wireless context (hence, with focus on *spatial gossip*). It is also a variant of *Blind-Forwarding with Network Coding (BF-NC)* from [10]. Finally, the algorithm 1 compares to the algorithm *NC3* of [5], except that we use a simple stop condition. By choice, in this article, we did not attempt to use opportunistic ([6]) mechanisms also presented in the cited works.

In algorithm 1, the intent of the “additional packet” transmission at 1.2 is to give neighbors one more chance to get the last transmission completing the  $D$  dimension space.

However, the stop condition in NCAL1 still does not guarantee that all information is disseminated to all nodes, hence for reference we used another algorithm, NCAL2 which assumes *reception reports* and stop transmitting only when all neighbors have all packets to restore all original packets.

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**Algorithm 2:** NCAL2

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- 2.1 **Source scheduling:** as in 1.1.
  - 2.2 **Nodes' start and stop conditions:** as in 1.2, the nodes start transmitting when they receive the first vector but they continue transmitting until themselves **and their neighbors** have enough vectors to recover the  $D$  source packets.
  - 2.3 **Nodes' scheduling:** as in 1.3
-



### 2.2.2 Extension to multiple sources

The two algorithm (NCAL1 and NCAL2) can be extended for multiple sources. Considering packets from different sources are in the same generation, synchronization is necessary and is not addressed here.

## 3 Cascading Wave Model

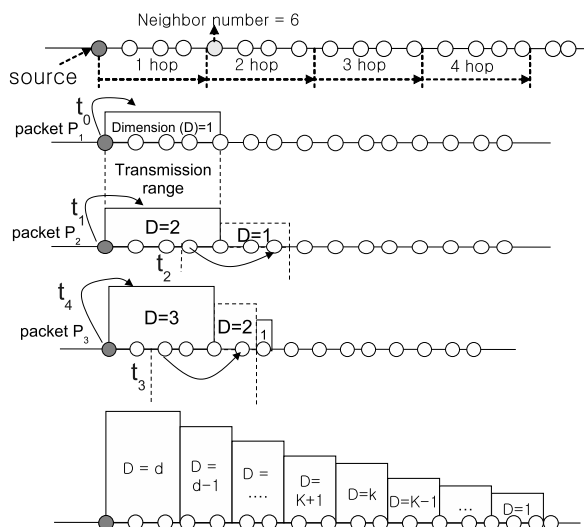


Figure 1: Wave model

The central focus of this article is the study the behavior of the above algorithms in wireless networks with are modeled with disk-unit graphs. The dynamic behavior of the network is represented in figure 1: the nodes (white disks) are randomly distributed in a line topology, the source is originating new vectors, (additional dimensions of the vector space) and these new dimensions propagate in the network as waves. We represented the effect of some successive transmissions  $t_0, t_1, t_2, t_3$  and  $t_4$ . The effect of one transmission can be summarized by how it increases (or does not increase) the dimension space of the nodes. Here, initial transmission  $t_0$  and the successive transmission  $t_1$  from the source, increase the dimension of the neighbors of the source (which became 1). Transmission  $t_2$  from a neighbor of the source, propagates further a linear combination of  $t_0$  and  $t_1$ . This process continues, and  $d$  transmission from the source in general propagates as seen in figure 1. Hence we have a *cascade* of waves. Note that this occurs in our case, because we have one source and a line topology, where any vector that a node receive must be a linear combination of the vectors already received at any left neighbor. Precise analysis of the propagation of the

wave is paramount to the characterization of the performance of algorithm 1. Such analysis is presented in section 4.

## 4 Analysis of performance

### 4.1 Wave propagation model

In this section, we provide an analysis of the propagation waves in the network coding scheme described in section 2.2.1. Each wave is defined by the border between areas where nodes have received same vector space combination of packets, as shown on figure 1. In particular the  $k$ th wave indicates the limit between the area where nodes have received a vector space of  $k$  dimension with the area of the nodes which have received a vector space of  $k - 1$  dimension.

In 1D, the wave positions are determined by the position of the border on the positive real axis:  $x_k(t)$  is the abscissa of the  $k$ th wave at time  $t$ , with origin  $x = 0$  at the location of the source. We also assume that the ordering  $x_k(t) > x_{k+1}(t)$  at all times. The model is the following:

- **Source transmission:** the source transmits packets at rate  $\lambda\nu$  where  $\nu$  is the node density and  $\lambda < 1$  (average neighbor size is  $M = 2\nu$ ).
- **Neighbors of the source:** note that the new created waves actually depart from position  $x = 1$ , since the all the direct neighbors of the source receive every transmission of the source.
- **Node retransmission:** each node retransmits at rate 1. Hence note that in a area of length unit and time unit, the relay nodes transmit at a Poisson rate of density  $\nu$ .
- **Wave propagation:** when a relay node at position  $y$  transmits a vector, two cases can occur. If there actually exists a closest wave border  $x_i$  such that  $x_i > y$  and  $x_i - y < 1$ , then the transmission moves the wave border at position  $y + 1$ . Otherwise transmission has no effect on existing waves.

Notice that wave propagation may break the order of the  $x_k$ , in which case the indexing would need to be adjusted.

### 4.2 Actual performance

From now we restrict our analysis to 1D unit disk graph model. Let consider a node at distance  $x$  from the source node. Let  $\Delta(x, Q)$  be the average time between the first wave and the last wave. For example if  $x \leq 1$  we have  $\Delta(x) = \frac{2Q}{\lambda}$ , since when neighboring the source, the node experience an average time of  $\frac{2}{\lambda M}$  between two wave generations (*i.e.* two data packet transmissions by the source) where the generation size  $D$  is  $MQ$ .

**Theorem 1** *The average number of packet transmission by a node at distance  $x$  to the source is  $\Delta(x, Q) + 1$ .*

**Proof:** The node starts to schedule encoded packet when it receives the first wave and continue as long as it has not received the last wave which arrives in average  $\Delta(x, Q)$  time unit later. Since the encoded packet generation is Poisson of rate 1, the average number of encoded packet transmitted during  $\Delta(x, Q)$  is exactly  $\Delta(x, Q)$ . Therefore the number of packet scheduled is  $\Delta(x, Q) + 1$ . The +1 denotes the last packet which is scheduled during the  $\Delta(x, Q)$  period but is actually transmitted after the period according to algorithm.

**Corollary 1** *The total average number of packets (data and encoded) transmitted by all the nodes between distance 0 and distance  $y$  is equal to  $QM + \frac{M}{2} \int_0^y (T(x, Q) + 1) dx$ .*

**Proof:** The term  $QM$  denotes the data packets transmitted by the source. The other packets are the encoded packet generated by the relay nodes where the density  $\nu = \frac{M}{2}$ .

When the node is not neighbor of the source, *i.e.*  $x > 1$ , we have the expression  $\Delta(x, Q) = \frac{2Q}{\lambda} + (x - 1)(s_l - s_f)$ . where  $s_f$  is the average slowness of first wave propagation and  $s_l$  is the average slowness of the last wave.

**Theorem 2** *We have  $s_f \geq \frac{2}{M}$  and  $s_l \leq \frac{16\lambda}{(1-\lambda^2)M}$*

Because  $\lambda < 1$  the time gap between the first wave and the last wave is proportional to  $\frac{\beta}{M}$ . When  $M \rightarrow \infty$  the time gap becomes zero, thus the wave propagation process does not bring much effect on the performance.

**Corollary 2** *The total average number of packet transmitted by nodes between distance 0 and distance  $y$  is equal to  $QM + (\frac{2Q}{\lambda} + 1)\frac{My}{2} + O(\frac{y-1}{1-\lambda})$  when  $M \rightarrow \infty$ .*

**Corollary 3** *The average number of retransmissions per data packet between distance 0 and distance  $y$  is equal to  $1 + (\frac{2}{\lambda} + \frac{1}{Q})\frac{y}{2} + O(\frac{y-1}{(1-\lambda)QM})$ .*

If the network is limited to distance  $y$  to the source:  $N = \frac{yM}{2}$  assuming the source is the leftmost node on the network, we have a retransmission ratio equal to  $1 + (\frac{2}{\lambda} + \frac{1}{Q})\frac{N}{M} + O(\frac{N}{(1-\lambda)QM^2})$  which is arbitrarily close to  $2\frac{N}{M}$  when  $N, M \rightarrow \infty$  and  $\lambda \rightarrow 1$ .

### 4.3 Wave propagation slowness

In this section we use  $\nu = \frac{M}{2}$  the uniform density of node per unit length in the 1D unit graph model.

#### 4.3.1 Propagation of the first wave

**Theorem 3** *The first wave moves at average speed  $v_f = \frac{y}{2}$  length unit per time unit.*

**Proof:** the relay nodes that move the first wave are those which are between  $x_1(t) - 1$  and  $x_1(t)$ . These nodes create a transmission rate of  $\nu$  transmissions per time unit. Each transmission moves the wave by an average translation of  $\frac{1}{2}$  unit length.

**Corollary 4** *The average slowness of the first wave satisfies  $s_f \geq \frac{2}{\nu}$  time unit per length unit.*

**Proof:** we have the inequality  $s_f \geq \frac{1}{v_f}$  since the inverse function is convex.

### 4.3.2 Propagation of the last wave

**Theorem 4** *The last wave propagates at an average slowness smaller than  $\frac{8\lambda\nu}{1-\lambda^2}$  time unit per length unit.*

The proof of this theorem is based on lemmas that study an upper bound of the actual system.

### 4.3.3 The upper-bound system

By upper-bound system we consider a system where wave positions  $y_k(t)$  are always smaller than actual wave position:  $y_k(t) < x_k(t)$  for all  $t$  and  $k$ . We describe such system.

Let  $\ell$  be a number such that  $\frac{1}{\ell} < 1 - \lambda$ . We cut the real axis in equal intervals of length  $\frac{1}{\ell}$ . We will force the wave positions  $y_k$  of the upper-bound system to be integer multiples of  $\frac{1}{\ell}$ .

- (unchanged) The source transmits packets at rate  $\lambda\nu$  where  $\frac{M}{2}$  is the node density and  $\lambda < 1$
- The new created waves are at abscissa  $1 - \frac{1}{\ell}$ .
- (unchanged) Relay nodes transmit at a Poisson rate of  $\nu$  per length unit and time unit.
- When a relay node transmits at abscissa  $y \in ]\frac{k-1}{\ell}, \frac{k}{\ell}]$  for  $k < \ell$ , it moves a wave at position  $\frac{\ell-1}{\ell}$  to position  $1 + \frac{k-1}{\ell}$ . If there was no wave at position  $\frac{\ell-1}{\ell}$ , then the relay node transmission has no effect on wave positions.
- When a relay node transmits at abscissa  $y \in ]\frac{k-1}{\ell}, \frac{k}{\ell}]$  for  $k \geq \ell$ , it moves a wave at position  $\frac{k}{\ell}$  to position  $1 + \frac{k-1}{\ell}$ . If there was no wave at position  $\frac{k}{\ell}$ , then the relay node transmission has no effect on wave positions.

Notice that the waves in the upper bound system move less frequently than in the actual system, and when they move, it is on a shorter distance. Therefore this is an upper-bound system.

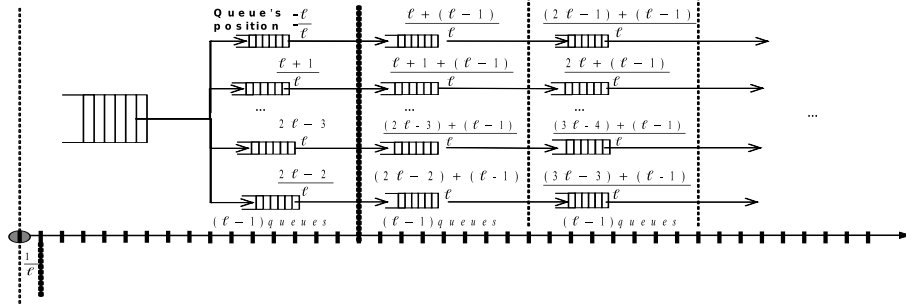


Figure 2: Queue network

**Lemma 1** *The number of wave at position  $\frac{\ell-1}{\ell}$  is the queue length of a M/M/1 system with arrival rate  $\nu\lambda$  and exit rate  $\nu\frac{\ell-1}{\ell}$ .*

Notice that  $\frac{\ell-1}{\ell} > \lambda$  satisfying stability condition of M/M/1 queue therefore the system is ergodic.

**Lemma 2** *The number of waves at position  $\frac{k}{\ell}$  for  $k \geq \ell$  is the queue length in a Jackson network where, as shown on figure 2:*

- the output of the queue at position  $\frac{\ell-1}{\ell}$  is equally distributed among the output of the  $\ell - 1$  queues ranging from position 1 to  $\frac{2\ell-2}{\ell}$ .
- Queues at position  $\frac{k}{\ell}$  for  $k \geq \ell$ , have respective service rate  $\frac{\nu}{\ell}$  and sends their respective output as input to queues at position  $\frac{k+\ell-1}{\ell}$ .

Notice that after the first queue, the queues are connected in  $\ell - 1$  parallel chains. Each queue in chains receives an input flow of  $\frac{\lambda\nu}{\ell-1}$  with service rate  $\frac{\nu}{\ell}$  satisfying the condition of Jackson's theorem because  $1 > \frac{\ell-1}{\ell} > \lambda$  and therefore are ergodic. In addition internal flows are poisson because waves moves not backward and forward but only toward the end of networks, thus satisfying Burke's theorem.

**Corollary 5** *The average number of waves in the upper bound system in each position  $\frac{k}{\ell}$  for  $k \geq \ell - 1$  is smaller than  $W = \frac{\lambda\frac{\ell-1}{\ell}}{1-\lambda\frac{\ell-1}{\ell}}$ . And the average number of waves per unit length is  $\ell W = \frac{\lambda\ell^2}{(1-\lambda)\ell-1}$*

**Proof:** This is the average stationary queue length as an application of Jackson theorem on queues with exponential service time connected in network with exponential arrivals. Since the queues are empty at time  $t = 0$  then the average queue length is always smaller than the stationary average queue length.

Notice that the value of  $\ell$  that minimizes the upper bound wave density is  $\frac{2}{1-\lambda}$  which gives a minimum density of  $\frac{4\lambda}{(1-\lambda)^2}$ .

**Lemma 3** *When the source stops sending packets average time needed to empty the first queue is  $\frac{\ell^2}{(\ell-1)\nu} \frac{\lambda}{(1-\lambda)\ell-1}$  and  $\frac{\ell^2}{\nu} \frac{\lambda}{(1-\lambda)\ell-1}$  for each of the next queues.*

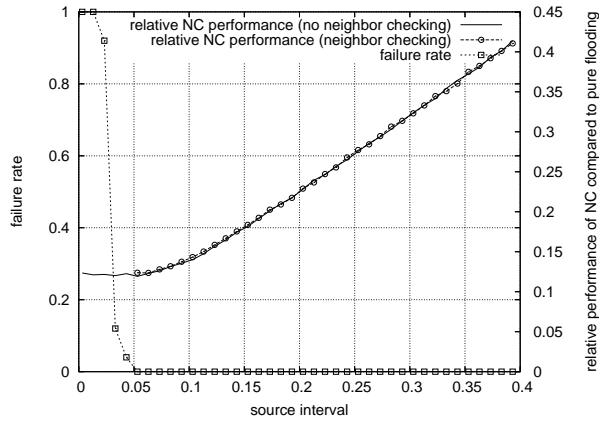
**Proof:** When the source stops transmitting its packets all the queues length decays in average. Therefore there are smaller in average than  $W = \frac{\lambda\ell}{(1-\lambda)\ell-1}$ . The first queue is emptied after an average time of  $\frac{\ell}{(\ell-1)\nu}W$ . If we stick to the upper bound model then then the second queue would be emptied after  $\frac{\ell}{\nu}W$ . In the worst case these times adds, therefore the speed at which queues empty is greater than  $\frac{\nu}{\ell^2W}$ .

We can improve this bound in changing the upper bound model when the first queue is empty: when the first queue is definitely empty, then the second queue is served as the first queue and is empty in an average time  $\frac{\ell}{(\ell-1)\nu}W$  instead of  $\frac{\ell}{\nu}W$ . When the second queue is definitely empty then the next queue is served as the previous one and so forth. This modified model is still an upper bound model and the bound of the speed at which queues empty is raised to  $\frac{(\ell-1)\nu}{\ell^2W}$ .

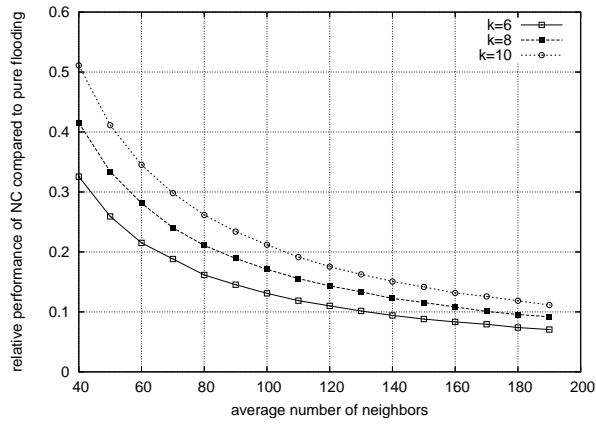
Therefore, and considering the optimal value  $\ell = \frac{2}{1-\lambda}$ , the last wave propagates at the average slowness as queues empty, *i.e.*  $\frac{8\lambda\nu}{1-\lambda^2}$

#### 4.4 Conjecture on 2D performance

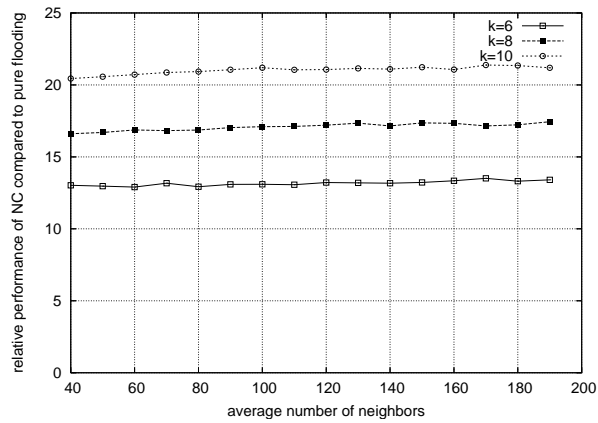
We conjecture a similar performance behavior of the protocol on 2D excepted that the retransmission ratio will tend to  $\alpha \frac{N}{M}$  for which we have no model performance, only simulations. However we strongly believe that the wave model presented can be adapted to 2D unit graph model but with significantly much harder problems since the wave are travelling like lines instead of points.



(a) 1D performance and failure rate

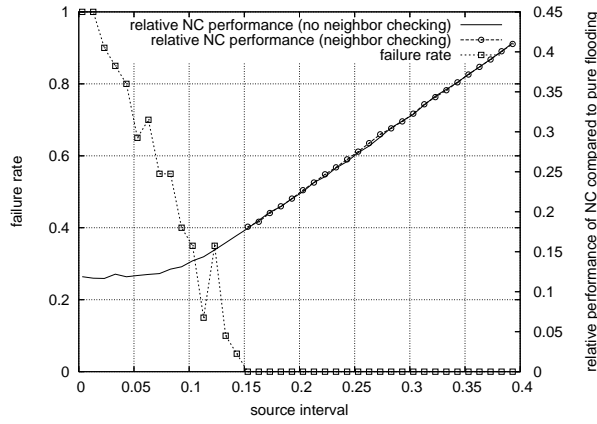


(b) 1D performance w.r.t. density (source rate  $\propto k$ )

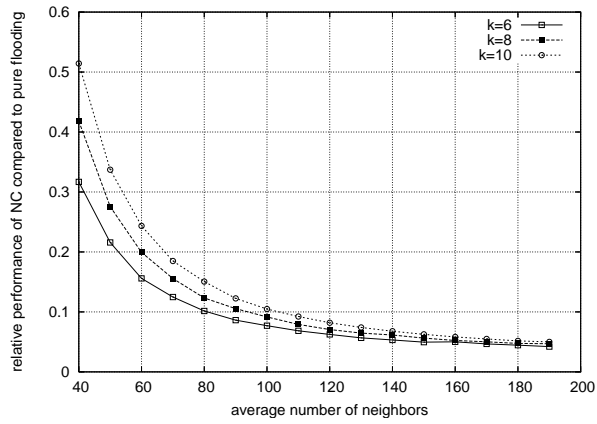


(c) 1D performance w.r.t. density (times  $M$ )

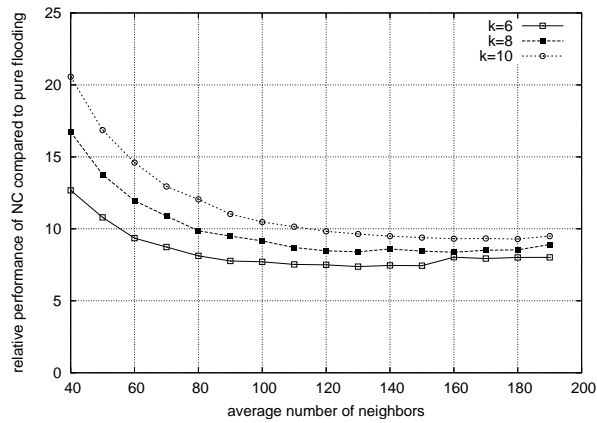
Figure 3: Simulation results



(a) 2D performance and failure rate



(b) 2D performance w.r.t. density (source rate  $\propto k$ )



(c) 2D performance w.r.t. density (times  $M$ )

Figure 4: Simulation results in 2D



## 5 Simulations

We performed simulations with a collisionless MAC assuming the unit disk radio range. The failure rate in figure 3(a) and figure 4(a) is the ratio between the number of nodes not receiving all dimensions (and thus enable to decode the data encoded within network coding) and the number of nodes receiving eventually all dimensions. The performance is measured by the ratio between the total number of retransmissions with network coding  $T_{nc}$  and the number of retransmissions  $T_w$  that would occur by executing  $D$  parallel classic floodings. As figure 3(a) and figure 4(a) shows, broadcasting with network coding fails without checking neighbor when the source transmitting interval  $\frac{1}{\alpha}$  becomes smaller than  $\frac{M}{2}$  in 1D and around  $\frac{M}{4}$  in 2D. Except these failure cases the increasing source transmitting interval  $\frac{1}{\alpha}$  also increases the ratio  $\frac{T_{nc}}{T_w}$  as analyzed. In addition graph a and b shows that performance becomes flat from the points where NCAL 1 fails. The flatten performance may be the upper bound, and is close to 1 in 1D as analyzed. When NCAL1 fails (source rate,  $\alpha > \frac{M}{2}$ ), we have an increasing failure rate as the source sends packets faster. It should be noted that this condition corresponds to  $\lambda > 1$  in the analytical model where the upper bound system is no longer ergodic. Figure 3(b) and figure 4(b) shows the performance as the average number of neighbors increases where the source transmitting intervals are  $\frac{2k}{M}$  ( $k = 6, 8, 10$ ) in 1D and 2D. As analyzed the peerformance  $\frac{T_{nc}}{T_w}$  is proportional to  $\frac{1}{M}$  and the proportion is maintained regardless of source rate ( $k = 6, 8, 10$ ). This feature is more clear in figure 3(c) and figure 4(c) because lines in graphs are flat. These simulation results confirm the analysis that the main factor affects the performance is source rate because the the last wave reaches at the end of network without much delay and the time gap between the first wave and the last wave is proportional to source transmitting interval.

## 6 Conclusion

We have introduced a simple network coding for wireless multihop networks. It is based on random linear coding and its performance is shown to be arbitrary close to optimal in 1D unit disk graph model. Similar performance is conjectured in 2D unit disk graph model. The proof in 1D is based on the analysis of wave propagation. We build a tractable upper bound system that allows to provide tight bounds on wave propagation. We show extensive simulation results that confirm the analytical results. The next objective is to find a similar bound in 2D unit disk graph model that confirms the very good performance shown by simulations. The network coding is very attractive because it provides an asymptotically very efficient broadcasting mechanism without the burden of tight neighbor sensing and two-hop neighbor management that is needed for building explicit connected dominating sets and multipoint relaying.

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