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Abstract: We investigate the performance of flooding in mobile ad hoc networks when the relay set is a set of nodes uniformly dispatched with density D. We give some bounds on the conditions under which a flooding will actually spread (or percolates) when the relay set density is above a certain threshold that depends on traffic density.

Key-words: wireless mesh networks, flooding, access points, performance and models

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Résultats théoriques sur les performances des inondations dans les réseaux mobiles ad hoc

Résumé : Nous nous intéressons aux performances de l'inondation dans les réseaux mobiles *ad hoc* quand l'ensemble des postes relais est uniformément distribué avec densité D. Nous donnons des bornes théoriques sur les conditions pour que l'inondation se propage (ou percole) quand la densité des relais est au dessus d'un seuil qui dépend dela densité de trafic.

Mots-clés : réseaux sans fil maillés, inondation, points d'accès, performances et modèles

1 Introduction

We investigate the performance of flooding in mobile ad hoc networks when the relay set is a set of nodes uniformly dispatched with density D. In [2] it has been shown that a flooding broadcast actually spread on the whole network when certain conditions are met, called the percolation point. In this paper we give some bounds on the percolation point in some rrealistic wireless networks. In particular we show that the relay set density can be made small so that some nodes may not be connected via unicast links to the relay node, but they can actually receive the broadcast via other nodes than neighbors. This comes from the fact that non neighbor nodes can receive packets from each other but with a success rate that would be acceptable for unicast routing. Therefore the neighbor radius that make optimal routing is in fact much smaller than the radius between relay set that would make the flooding percolating.

2 Preliminary results

2.1 Network and traffic model

The model is a network made of N mobile nodes on a large map of area $\mathcal{A}$ uniformly distributed with constant density $\nu = \frac{N}{\mathcal{A}}$. We take a slotted time model and a network on an infinite map. We assume that at each slot the density of transmitters is constant and uniform of λ per square unit with a Poisson distribution. Therefore the per node node traffic transmission is $\rho = \frac{\lambda}{\nu}$.

We assume that the attenuation coefficient is $\alpha > 2$, for instance wave propagation theory shows that $\alpha = 4$. We know [1, 3] that the Laplace transform of the signal level S(assuming all transmitter emitting at unit nominal power) is $w(\theta) = E(e^{-S\theta})$ and

$$w(\theta) = \exp(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})\theta^{\frac{2}{\alpha}}) \tag{1}$$

If we assume that the signal can be altered by a random fading factor $\exp(F)$ then the factor $\Gamma(1 - \frac{1}{\alpha})$ is to be simply replaced by $\Gamma(1 - \frac{1}{\alpha})E(e^{-\frac{2}{\alpha}F})$.

Taking $w(\theta) = e^{-\lambda C\theta^{\gamma}}$ with $\gamma = \frac{2}{\alpha}$, we get, via thorough computations we get the following expansion:

$$P(S < x) = \sum_{n\geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi}\Gamma(n\gamma)x^{-n\gamma}.$$

2.2 Unicast transmission optimization

If we assume that a packet is correctly received when the Signal over Noise Ratio is above a certain threshold K, for instance we will take $K = 4$ then one can evaluate the probability $p(r, \lambda)$ of correctly receiving a packet at distance r of the emitter. From easy scaling property we have $p(r,\lambda) = p(r\sqrt{\lambda}, 1)$. A failed packet will need to be retransmitted therefore a transmission at distance r must be transmitted in average $\frac{1}{p(r,\lambda)}$ times. It turns out that the optimal hop length r it the one that minimizes the global number of retransmissions: $\frac{\ell}{rp(r,\lambda)}$, *i.e.* that maximizes $rp(r,\lambda)$. The optimal value is $r = \frac{r_0}{\sqrt{\lambda}}$ where r_0 maximizes $rp(r,1)$. We denote $\sigma_0 = \pi r_0^2$ then the area of optimal neighborhood that needs to be managed by every node is $\frac{\sigma_0}{\lambda}$.

In the paper we will assume that $\alpha = 4$ and $K = 4$ which leads to $\sigma_0 \approx 0.181$ and $p_0 \approx 0.653$.

When a node A has packets to send to node B at distance $L \gg \frac{r_0}{\sqrt{\lambda}}$ (we consider that node density and traffic density are both large), the optimal route will be made of $\frac{L\sqrt{\lambda}}{r_0}$ relay nodes regularly spaced on a strait lines between A and B. Therefore the packet will be repeated in an average of $\frac{L}{r_0 p_0}$ times.

3 Flooding optimization

Our main goal mis to have single source and all nodes in the network as potential receivers.

3.1 Flooding model protocol

We assume that the relay set $\mathcal{M}$ is a random set of nodes with density D. For example nodes select themselves to be in the relay set with probability $\frac{D}{\nu}$.

A node A which have a broadcast packet to transmit broadcasts it to its neighbors. When a node belonging to the relay set $\mathcal{M}$ receives such broadcast packet, it retransmits it after a random jitter (in order to avoid collision with other relay nodes). Our aim is to analyze the performance of such a simple scheme.

3.2 Performance of flooding

Let assume that every relay node receives the broadcast with probability q. Since the transmissions are done under jitter we can consider that retransmissions occur at different time. Therefore a node B will retransmit the broadcast packet with probability $\frac{Dq}{\nu}$ which the combination of the probability to be in the relay set and the probability to have received the packet from other members of the relay set. An external node A will receive it from B with probability $p(|A-B|,\lambda)$ where $|A-B|$ is the distance between A and B. Our aim to compute the average number of reception of the broadcast packet by node A.

Every portion of area $d\mathcal{A}$ at distance r will contribute a correct transmissiom to node A with probability $p(r,\lambda)Dq$. Therefore the average number of correct reception by node A is given by $2\pi Dq\int_0^\infty p(r,\lambda)rdr$. This quantity is equal to $q\frac{D}{\lambda}\sigma_1$ with $\sigma_1 = 2\pi\int_0^\infty p(r,1)rdr$. Thorough computations gives $\sigma_1 = \frac{\sin(\pi\gamma)}{\pi\gamma}K^{-\gamma}\frac{\Gamma(1-\frac{2}{\alpha})}{C}$ with $\gamma = \frac{2}{\alpha}$. In absence of fading term we simply have $\sigma_1 = \frac{\sin(\pi\gamma)}{\pi\gamma}K^{-\gamma}$. With $\alpha = 4$ and $K = 4$ we get $\sigma_1 = \frac{1}{\pi} \approx 0.31831$.

Figure 1: succesful reception probability q_I with respect to relay set density D (expressed in $\frac{\lambda}{\sigma_1}$ unit).

Computing the average number of succesful reception of the broadcast gives the obvious upper bound: $P(A \text{ receives the packet}) \leq Dq\frac{\sigma_1}{\lambda}$.

When the flooding percolates, *i.e.* when it uniformly spread in the network, all nodes will get the same probability of correct reception. Therefore in percolation condition we will have $P(A \text{ receives the packet}) = q$ and the inequality

$$q \leq Dq\frac{\sigma_1}{\lambda}$$

Therefore the percolation condition implies that $\frac{D}{\lambda} \geq \sigma_1$. In passing we see that the lower bound for percolating flooding is a density of one relay node per $\frac{\sigma_1}{\lambda}$ which is larger than the neighbor area $\frac{\sigma_0}{\lambda}$. Therefore there can be nodes which have no neighbor in the relay set (but nevertheless can be reached from further nodes).

3.3 Refined flooding performance

Since radio signal naturally spread the correct reception of a packet are positively correlated between nodes (on the same ground, the uncorrect reception of a packet are also positively correlated). In other words when the nodes of the relay set which have received correctly the packet are in statistic more grouped than they would be as if they were receiving it independently. This suggests (but this will not be justified in detail in the present paper) that the probability of uncorrect reception of the packet by node A is in fact greater than it would be in the hypothesis that relay nodes would have received the packet independently. In the independence hypothesis, the number of correct reception would be a Poisson distribution of mean $\frac{D}{\lambda}q\sigma_1$ and therefore $1 - P(A \text{ receives the packet}) \geq \exp(-\frac{D}{\lambda}q\sigma_1)$. In percolating condtion this yields:

$$1 - q \geq \exp(-\frac{D}{\lambda}q\sigma_1)$$

Equating the above inequality will yield a fixed point value q_I which is actually an upper bound of the actual value q: $q_I \geq q$ with $1 - q_I = \exp(-\frac{D}{\lambda}q_I\sigma_1)$ (or more rigorously $\frac{D}{\lambda}\sigma_1 = -\frac{\log(1-q_I)}{q_I}$). Figure 1 displays quantity q_I with respect to relay node density D (expressed in $\frac{\lambda}{\sigma_1}$ unit).

In the same hypothesis $q_I\frac{D}{\lambda}$ is also an actual upper bound of the number of correct reception of the broadcast packet by an arbitrary node. The figure 2 displays this average number of actual reception with respect to relay set density D, this time the density D is expressed with respect to $\frac{\lambda}{\sigma_0}$ unit, the inverse of the average neighbor area. Notice that the average number of reception reach more than 1 even when $D < \frac{\lambda}{\sigma_0}$.

Figure 2: average number of correct reception in independence hypothesis with respect to relay node density D (expressed in inverse of average area $\frac{\lambda}{\sigma_0}$ unit).

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Appendix

Computation of σ_1. We use the identity $p(r,\lambda) = P(S < \frac{1}{Kr^\alpha})$ By integration by part:

$$\lambda\sigma_1 = \pi\alpha \int_0^\infty P(W = \frac{1}{Kr^\alpha}) \frac{r}{Kr^\alpha} dr$$

We use the fact that $P(W = x) = \frac{1}{2i\pi}\int_C w(\theta)e^{\theta x}d\theta$, where C is an integration path in the definition domain of $w(\theta)$, *i.e.* with $Re(\theta) > 0$parallel to the imaginary axis. We get by changing variable $x = (Kr^\alpha)^{-1}$ and inverting integrations ($\gamma = \frac{2}{\alpha}$).

$$\begin{aligned}
\frac{\sigma_1}{\lambda} &= \frac{1}{2i}\int_C w(\theta) \int_0^\infty e^{\theta x}(Kx)^{-\gamma}dx \\
&= \frac{1}{2i}K^{-\gamma}\Gamma(1-\gamma)\int_C w(\theta)(-\theta)^{\gamma-1}d\theta
\end{aligned}$$

We use $w(\theta) = \exp(-\lambda A\theta^\gamma)$ and now deforming the integration path to stick to the negative axis:

$$\begin{aligned}
\frac{\sigma_1}{\lambda} &= \frac{e^{i\pi\gamma} - e^{-i\pi\gamma}}{2i}K^{-\gamma}\Gamma(1-\gamma)\int_0^\infty \exp(-\lambda C\theta^\gamma)\theta^{\gamma-1}d\theta \\
&= \sin(\pi\gamma)K^{-\gamma}\frac{\Gamma(1-\gamma)}{\lambda C\gamma}
\end{aligned}$$

Therefore $\sigma_1 = \frac{\sin(\pi\gamma)}{\pi\gamma}K^{-\gamma}$.

Computation of function $\Psi(y) = 2\pi \int P(W < \frac{1}{Kr^\alpha}) r e^{-y\pi r^2} dr$.

By integration by part we get

$$\Psi(y) = \frac{\alpha}{y} \int P(W = \frac{1}{Kr^\alpha}) \frac{1}{Kr^{\alpha+1}} (1 - e^{-y\pi r^2}) dr$$

Using change of variable $x = (Kr^\alpha)^{-1}$:

$$\begin{aligned}
\Psi(y) &= \frac{1}{y} \int P(W = x)(1 - e^{-y\pi(Kx)^{-\gamma}}) dx \\
&= \frac{1}{2i\pi y} \int_C w(\theta) d\theta \int e^{\theta x} (1 - \exp(-y\pi(Kx)^{-\gamma})) dx
\end{aligned}$$

Using $1 - \exp(-y\pi(Kx)^{-\gamma}) = -\sum_{n>0} (-y\pi K^{-\gamma})^n \frac{x^n}{n!}$, we get

$$\Psi(y) = \frac{-1}{2i\pi y} \sum_{n>0} (-y\pi K^{-\gamma})^n \frac{\Gamma(1 - n\gamma)}{n!} \int_C w(\theta)(-\theta)^{n\gamma-1} d\theta$$

Using $w(\theta) = e^{-C\theta^\gamma}$:

$$\begin{aligned}
\Psi(y) &= \frac{-1}{2i\pi y} \sum_{n>0} (-y\pi K^{-\gamma})^n \frac{\Gamma(1 - n\gamma)}{n!} (e^{i\pi n\gamma} - e^{-i\pi n\gamma}) \frac{C^{-n}(n-1)!}{\gamma} \\
&= -\sum_{n>0} (-\pi \frac{K^{-\gamma}}{C})^n \Gamma(1 - n\gamma) \frac{\sin(n\pi\gamma)}{n\pi\gamma} y^{n-1}
\end{aligned}$$

Without fading it comes: $\Psi(y) = \sum_{n>0} (-\frac{K^{-\gamma}}{\Gamma(1-\gamma)})^n \Gamma(1 - n\gamma) \frac{\sin(n\pi\gamma)}{n\pi\gamma} y^{n-1}$. Notice that $\Psi(y) = \sigma_1 - (\frac{K^{-\gamma}}{\Gamma(1-\gamma)})^2 \Gamma(1 - 2\gamma) \frac{\sin(2\gamma\pi)}{2\pi\gamma} y + O(y^2)$.

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