Design of a pressure control system with dead band and time delay

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***Abstract*— This paper investigates the control of pressure in a hydraulic circuit containing a dead band and a time varying delay. The dead band is considered as a linear term and a perturbation. A sliding mode controller is designed. Stability conditions are established by making use of Lyapunov Krasovskii functionals, non-perfect time delay estimation is studied and a condition for the effect of uncertainties on the dead zone on stability is derived. Also the effect of different LMI formulations on conservativeness is studied. The control law is tested in practice.**

***Index Terms*— Dead zone, time varying delay, pressure control, stability, LMI**

I. Introduction

In applications, many systems exist with dead zones. The same applies for agricultural equipment where parts need to be reliable and cheap. Dead bands or zones are also encountered in, for example, robots and machine tools [4], [5], [7], hydraulic and pneumatic actuators [9], [6], servo systems [18], [8], thermal generating power units [11], automobile parts such as valves in cars [10], [12] etc. They can be introduced deliberately as is the case in the so called "overlap" hydraulic or pneumatic valves, to ensure closure. The latter kind of valves are used in mobile applications such as earth moving equipment and farm machinery [1]. As these dead zones are known and fixed, they can be overcome by adding or subtracting a fixed voltage to the control voltage of the valve, to overcome the overlap. Very often dead zones are introduced by friction phenomena [2] and degenerate system performance. In the latter case, which is dealt with in this paper, the dead zone is mostly introduced by non-linear friction, more specifically stiction. Several methods exist to handle friction in control systems of which an overview is given by Armstrong *et. al* [2] and Olsson *et. al* [3]. They range from friction compensation based on accurate determined models through robust control methods like the sliding mode e.g. [13], [14], [15] or adaptive algorithms to identifying the friction on-line e.g. [16], [17].

In this paper, a pressure control system, which is typically used on agricultural spray booms, to distribute pesticides across the field is studied. In addition to the dead zone, the pressure regulating valve contains a variable time delay. For ease of assembly, the struts of the electrical motor, actuating the valve, contain a kind of compliance with respect to the housing, resulting in the time delay which changes by the direction of rotation. In current pesticide application practice, the objective is to distribute the liquid as homogeneously as possible and the pressure is adjusted to the speed of the tractor. Recently, the effects of variable rate application and site-specific spraying *i.e.* spraying where the weeds are, were introduced by research labs [19]. With respect to site-specific spraying, sensors to discriminate between weeds and plants have been developed [20], [21], [22]. In site-specific spraying, nozzles are opened and closed continuously, resulting in considerable pressure changes. The objective of this paper is to study a control law, which can handle dead zones and variable time delays in order to minimise the pressure stabilisation time for hydraulic systems used for pesticide application.

There are several methods of analysing systems with variable time delays. The Lyapunov-Razumikhin approach is known to be conservative [31] but can deal with time varying delays without any restriction on the derivative of the delay. With the Lyapunov-Krasovskii method, a functional is sought, which allows to prove stability of time-delay systems where the delay parameters are bounded both in length and time variation. Recently, Lyapunov-Krasovkii functionals have been proposed which enable to prove stability of systems with arbitrarily fast time-varying delays ([30], [29]). As these results are promising, this approach is followed in the paper.

First, a description of the system is given, followed by a detailed mathematical description of the governing phenomena. Based on these equations, a reduced model is derived on which a controller is designed. Initially, perfect knowledge of the time delay and dead zone is considered and stability is analysed by making use of the Lyapunov-Krasovkii approach. Several stability conditions, based on different LMI formulations are compared. The effect of imperfect knowledge of the time delay and the dead zone are investigated. Finally, the controller is implemented on a sprayer.

II. DESCRIPTION OF THE SYSTEM

Figure (1) shows the lay out of the system. It is actually one section of an agricultural spray boom for application of herbicides and fertilizer. A pump, contain-

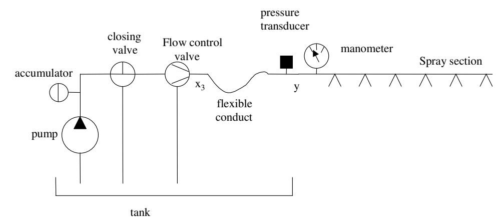

Fig. 1. Schematic representation of the pressure control system

ing two pistons, operating in anti-phase, feeds the circuit. Pressure peaks, resulting from fast activation of valves or originating from the pulsating flow of the pump, are attenuated by the accumulator. The closing valve allows to rapidly switch off the spraying without turning off the pump. The pressure at the nozzles is regulated by a flow control valve by adjusting the opening to the return. A long flexible duct links the pressure control valve with the metal duct, on which the nozzles are mounted. An electronic transducer measures the pressure at the entrance of the metal duct. This is the pressure of interest which is measured and should be controlled. The system is secured by a check valve, limiting the pressure to $7bar$.

III. MODELLING OF THE SYSTEM

The flow control valve is operated by an electrical $12V$ dc motor of which the electrical behavior is governed by:

$$L\frac{di}{dt} + Ri + Bl\frac{dx_1}{dt} = u \tag{1}$$

in which L is the inductance, i the current, R the resistance of the wires, Bl the torque or electromotive force constant, x_1 the angular position of the valve and u the input voltage. The flow control valve is actually a ball valve of which the equations of motion of the ball are described by:

$$I\ddot{x}_1 + C\dot{x}_1 = f_{fric}(i, \dot{x}_1), \tag{2}$$

where $\dot{x}_1$ denotes the time derivative of x_1. Equations (1) and (2) can be found in standard works (e.g. [23]) where equation (1) represents the electrical rotor dynamics and equation (2), the mechanical rotor dynamics. The latter equation contains the standard inertial term, with I the moment of inertia of the rotor and the ball, a viscous friction term, with C the viscous friction constant, and a more elaborate friction model $f_{fric}(i, \dot{x}_1)$:

$$\begin{array}{rcl} f_{fric}(i,\dot{x}_1) = Bli - F_c sign(\dot{x}_1) & if & |\dot{x}_1| > 0 \\ f_{fric}(i,\dot{x}_1) = Bli - F_s sign(i) & if & \dot{x}_1 = 0\& \\ & & |Bli| \geq F_s \\ f_{fric}(i,\dot{x}_1) = 0 & if & \dot{x}_1 = 0\& \\ & & |Bli| < F_s. \end{array} \tag{3}$$

F_c is the constant force counteracting the motion of the valve when it is moving and F_s the static friction or stiction force with $F_c \leq F_s$. The magnetic torque is represented by Bli with Bl again the electromotive force constant and i the current. These kind of models can be found back in literature [2], [3]. In the friction model (3), discrimination is made between motion and no motion (stiction). During motion, a constant force, independent of the speed, counteracts the motion. If there is stiction, the applied magnetic torque is counteracted perfectly by the friction force as long as it is below a certain threshold level F_s.

The electric motor operates the ball valve which regulates the resistance to the return. Flows through restrictions, in this circuit, can be considered turbulent and proportional to the square root of the pressure drop p [25]:

$$\sqrt{p} = R_{hyd} q, \tag{4}$$

where p is the pressure drop over and q the flow through the restriction. Parameter R_{hyd} represents the hydraulic resistance. For the flow to the return, the hydraulic resistance, $f_{return}(x_1(t-h))$, is a function of the delayed valve angle x_1 such that q_{return} can be computed from:

$$q_{return} = \frac{\sqrt{x_3}}{f_{return}(x_1(t-h))}. \tag{5}$$

For ease of installation, the motor support of the valve is connected to the housing through pins encapsulated by rubber, resulting in some compliance between the motor support and the housing. This causes a variable but bounded time delay h, the value of which changes whenever the motor switches direction. For one direction, the time delay equals approximately $0.23s$ and for the other 0.15.

Normally, the behavior of a duct should be described by a partial differential equation. The dynamics of the metal duct is negligible to the dynamics of the flexible one. A description by a set of linear ordinary differential equations provides a reasonable approximation, for the flexible duct behavior. Such descriptions can be obtained quite easily by, for example, linear black box identification methodologies:

$$\dot{X}_2 = A_l X_2 + B_l x_3 \tag{6}$$

where X_2 are the states of the duct (having no physical meaning), x_3 the pressure directly after the flow control valve (Figure 1), A_l and B_l constant system matrices. The pressure y at the end of the flexible duct (Figure 1) is calculated by:

$$y = C_l X_2 + D_l x_3 \tag{7}$$

in which C_l and D_l constant system matrices. Pressure y has to be controlled as stated in the previous section.

The accumulator maintains a pressure equilibrium between the fluid pressure and air pressure which are separated by a diaphragm. The behavior of the air can be described by a polytropic process [24]:

$$(10^5 + x_3)V^{\kappa} = constant \tag{8}$$

where V is the volume occupied by the air and the term 10^5 is added as the absolute pressure is required in the formula. For relatively low pressures, as is the case here, air behaves like an ideal gas such that in case of slow increasing pressure, the change of the state can be considered isothermal with $\kappa = 1$. For fast fluctuating pressures around an equilibrium pressure, there is no time for the fluid to exchange heat, such that the change of the state of the air can be considered adiabatic or isentropic with κ equal to the specific heat ratio of air, which equals approximately 1.4. By deriving equation (8), the fluid flow entering the accumulator q_{acc} can be computed, which is also the reduction in the air volume V or $q_{acc} = -\dot{V}$ such that:

$$\dot{x}_3 = \frac{\kappa(10^5 + x_3)}{V} q_{acc}. \tag{9}$$

With equation (8), V can be eliminated from equation (9):

$$\dot{x}_3 = K_{acc}(10^5 + x_3)^{(\frac{1}{\kappa}+1)} q_{acc}, \tag{10}$$

and $K_{acc} = \frac{\kappa}{(constant)^{\frac{1}{\kappa}}}$.

State equation (10) requires q_{acc}, the flow to the accumulator, which can be computed from the conservation of mass. The flow delivered by the pump q_{pump} equals the flow to the nozzles $q_{nozzles}$ plus the flow to the return q_{return} and the accumulator q_{acc}:

$$q_{acc} = q_{pump} - q_{return} - q_{nozzles} \tag{11}$$

The pump is a two piston pump such that the flow rate can be modelled as:

$$q_{pump} = K_p \frac{\pi}{2} |\sin(\omega t + \varphi)|, \tag{12}$$

where K_p is the nominal flow of the pump, ω the rotation frequency of the pump and φ some phase angle.

In case of the nozzles, the hydraulic resistance R_{hyd} is constant and called R_n and $q_{nozzles}$ can be written as:

$$q_{nozzles} = \frac{\sqrt{y}}{R_n}. \tag{13}$$

Substituting equation (7) in (13) and (5), (12), (13), in equation (11) and (11) in (10) results in:

$$\dot{x}_3 = K_{acc}(10^5 + x_3)^{(\frac{1}{\kappa}+1)} \left(\frac{-\sqrt{x_3}}{f_{return}(x_1(t-h))} - \frac{\sqrt{C_l X_2 + D_l x_3}}{R_n} + K_p \frac{\pi}{2} |\sin(\omega t + \varphi)| \right). \tag{14}$$

The entire model, consisting of equations (1), (2), (6) and (14) with output (7) has been validated and is used as an evaluation model.

For controller design, a simpler model is derived. The system described by equations (1), (2), (6) and (14) is a singular perturbed system [26] and the quasi-steady-state or slow model is derived.

A small inductance L and a small inertia I are assumed. From equations (1) and (2), the current i is eliminated and with respect to the friction model (3), the case in which $|\dot{x}_1| > 0$ is considered:

$$\dot{x}_1 = \frac{Bl}{RC + Bl^2} \left(u - \frac{F_c R}{Bl} sign(\dot{x}_1) \right). \tag{15}$$

In case $u > \frac{F_c R}{Bl}$, the factor $sign(\dot{x}_1)$ can be replaced by $sign(u)$. The latter condition can be made more restrictive and replaced by $u > \frac{F_s R}{Bl}$, by assuming a large damping coefficient C such that when the voltage u is put to zero, the valve almost immediately stalls. In this way, equations (1) and (2) reduce to one equation:

$$\dot{x}_1 = K_m f_d(u) \tag{16}$$

where $K_m = \frac{Bl}{RC+Bl^2}$ with a dead zone $f_d(u)$:

$$\begin{array}{lll} f_d(u) = u - c_0 sign(u) & if & c_1 \leq |u|, \\ f_d(u) = 0 & if & c_1 > |u|, \\ and & 0 \leq c_0 \leq c_1. & \end{array} \tag{17}$$

where $c_0 = F_c R/Bl$ and $c_1 = F_s R/Bl$.

The accumulator and flexible duct dynamics can be considered fast. Putting $\dot{x}_3$ to zero, implies that the third term of the right hand side of equation (14) equals zero. As the duct dynamics is fast, $\dot{X}_2$ can be set to zero and X_2 can be expressed as a function of x_3 by equation (6). In this way the third term of the right hand side of equation (14) can be rewritten:

$$\frac{-\sqrt{x_3}}{f_{return}\left(x_1(t-h)\right)} - \frac{\sqrt{D_l - C_l A_l B_l}\sqrt{x_3}}{R_n} + K_p \tag{18}$$

in which the average flow rate of the pump is considered and pump oscillations are discarded. Approximating $f_{return}(x_1(t-h))$ by a linear function:

$$f_{return}(x_1(t-h)) = b_{return} x_1(t-h) + c_{return} \tag{19}$$

with b_{return} and c_{return} regression constants, the output function $\sqrt{y}$, which is the square root of the measured pressure at the spray section, turns into:

$$\sqrt{y} = \frac{\alpha x_1(t-h)+\beta}{x_1(t-h)+\gamma} \quad 0 \leq h_{min} \leq h \leq h_{max} \tag{20}$$

Table (I) clarifies constants α, β and γ.

Based on the slow dynamics of the system, represented by equation (16), a control law will herein be designed. The stability of the control law on the slow model will be proved and its robustness assessed. Parameter K_m can be determined rather easily by regression techniques and remains constant during time. However, the dead zone parameters are more difficult to determine and are more subject to variations in time. The robustness with respect to the dead zone as well as the effect of a not perfectly known delay will also be investigated. No hard proof will be provided about the stability of the complete system, only some indications will be given, but practice proves its stability. About singular perturbed systems with time delay only some results are available for the linear case [27], [28]. For nonlinear systems, no results were found by the authors in literature.

TABLE I

CONSTANTS, α, β AND γ

α	$\frac{R_n K_p}{\sqrt{D_l - C_l A_l^{-1} B_l}}$
β	$\frac{R_n K_p c_{return}}{b_{return}\sqrt{D_l - C_l A_l^{-1} B_l}}$
γ	$\frac{R_n + \sqrt{D_l - C_l A_l^{-1} B_l}\, c_{return}}{\sqrt{D_l - C_l A_l^{-1} B_l}\, b_{return}}$

IV. STATE ESTIMATOR DESIGN

In order to stabilize the state equation in (16), the rotation angle of the valve should be known. The valve doesn't contain a measurement system to determine the rotation angle, such that only the delayed state $x_1(t-h)$ is available from the output $\sqrt{y}$. Applying directly the delayed state $x_1(t-h)$ in a control law leads to limit cycle behaviour. Therefore, a state estimator is constructed.

$$\dot{\hat{x}}_1(t) = K_m f_d(u) + E(x_1(t-h) - \hat{x}_1(t-\hat{h})) \tag{21}$$

where $\hat{x}_1$ is an estimate of the state x_1, $\hat{h}$ an estimate of the delay h and E the Kalman or observer gain. Actually, equation (21) can only have an interpretation of a Kalman filter if $h = \hat{h}$.

V. Perfectly known delay

Initially, it is assumed that the delay $\hat{h}$ is perfectly known.

Sliding mode control is applied. As an observer is used, the typical robustness properties of sliding mode control with respect to some model deviations are lost. This is because the estimator rather than the system is in sliding mode. However, as the system itself is discontinuous, it is the most natural to use a control law which is discontinuous. An obvious way to handle dead zones, is to compensate the dead zone, which implies the incorporation of a discontinuous term in the control law, which implicitly renders sliding mode control. Therefore at least stability assessment should be performed by using methodologies from sliding mode control.

A first choice in sliding mode control is the selection of the switching surface. As the actual state $x_1(t)$ is not available, the decision to switch, has to rely on the estimated state, giving rise to the following switching surface $s(t)$:

$$s(t) = \hat{x}_1(t) - \hat{x}_d \tag{22}$$

in which $\hat{x}_d$ is the desired state (rotation angle of the valve), which is calculated from the desired pressure p_d:

$$\hat{x}_d = \frac{\beta - \gamma\sqrt{p_d}}{\sqrt{p}_d - \alpha} \tag{23}$$

During sliding mode synthesis, it is favourable to transform the state equation (16) and the estimator (21) to the regular form. By defining the error state $e(t) = x_1(t) - \hat{x}_1(t)$, the dynamics of the error between the real and predicted state is obtained:

$$\dot{e}(t) = -Ee(t-h) \tag{24}$$

As $\hat{x}_d$ is piecewise constant, the dynamics of $s(t)$ are:

$$\dot{s}(t) = Ee(t-h) + K_m f_d(u) \tag{25}$$

The dead zone $f_d(u)$ can be considered as the input $u(t)$ and a disturbance $d(t)$ which leads to:

$$\dot{s}(t) = Ee(t-h) + K_m u(t) + K_m d(t) \tag{26}$$

From equation (17), it is easy to see that $d(t)$ is bounded:

$$|d(t)| < c_1 \tag{27}$$

Equation (26) involves a generalisation of expression (25). The system defined by equations (24), (26) and (27) is in regular form such that a control law can be sought. Richard *et. al* [29] studied a more general formulation of the foregoing equations and proposed the following control law:

$$u(t) = -\frac{1}{K_m}\left(u_d sign\left(s(t)\right) + Ee(t-h) + as(t)\right) \tag{28}$$

where

$$u_d = m_1 + K_m c_1 \tag{29}$$

and constants

$$a, m_1 > 0 \tag{30}$$

Mostly, in systems with dead zones, the control law is often selected such that $|u(t)| \geq c_1$, in order to assure that whenever a control action is desired, it really reaches the system and is not blocked by the dead zone. Note that this requirement is not fulfilled in the proposed control law (28). Nevertheless, asymptotic stability is achieved.

Theorem 1: A) [29] Assuming controllability of system (24), (26), the control law (28) makes the surface (22) attractive and reached in finite time.

B) The equilibrium $x_1 = \hat{x}_1 = \hat{x}_d$ is then globally asymptotically stable for $h_{min} \leq h \leq h_{max}$ and E is the solution of the following LMI

$$\begin{pmatrix} \frac{-2W}{h_{max}} & -\epsilon W & -W \\ -\epsilon W & -\epsilon S & 0 \\ -W & 0 & -\epsilon S \end{pmatrix} < 0 \qquad (31)$$

where $\epsilon, S > 0$, $W, S \in \mathbb{R}$ and $E = \frac{W}{S}$.

Proof:

See [29].

■

Comment 1: It is clear that the system (24), (26) is controllable.

Comment 2: Richard *et. al* [29], posed the problem slightly different. They didn't use a state estimator. In this paper, the regular form obtained, is a mixture of a state estimator and the state equation of the real system. In Richard *et. al* [29], it was assumed to have the full state available.

Comment 3: Richard *et. al* [29] considered more than one state and defined the following switching surface:

$$s(t) = z_2(t) + K z_1(t) \qquad (32)$$

where $K \in \mathbb{R}^{m \times (n-m)}$ is the regulator gain, n the number of states, m the number of inputs, and z_1 and z_2 the partitioning of the state of the regular form. Their formulation intents to maximise the delay h and finding the regulator gain K by solving a more generalised LMI then in equation (31). In this paper, there is no regulator gain K involved, h_{max} is known and the observer gain E needs to be sought.

Comment 4: Because of the simplicity of the LMI (31), it can be calculated by hand. The characteristic equation of the matrix in inequality (31) is:

$$\lambda^3 + \left(\frac{2W}{h_{max}} + 2\epsilon S\right)\lambda^2 + \left(\epsilon^2 S^2 + \frac{2\epsilon SW}{h_{max}}\right)\lambda + 2\epsilon SW\left(\frac{S}{h_{max}} - W\right) = 0 \qquad (33)$$

where λ is the eigenvalue parameter. In order to obtain eigenvalues with negative real parts, all coefficients of the characteristic polynomial need to be positive which, given the conditions of theorem 1, can only pose problems for the coefficient of λ^0. Requiring this coefficient to be strictly positive renders:

$$E < \frac{1}{h_{max}} \qquad (34)$$

In case the strict positiveness is not imposed, marginal stability may occur, resulting in limit cycle behaviour.

Comment 5: It turns out that the formulation (31) is rather unfortunate with respect to the numerical solution by the Matlab LMI toolbox. The numerically calculated observer gains E are much smaller than what is obtained by inequality (34). As part B) concerns the stability of the reduced system (24), when the sliding mode has been achieved, it can be replaced by other formulations found in literature on time delay systems e.g. lemma 2.1 of Fridman *et. al* [30], which is based on the descriptor form and which should render less conservative results. In the latter formulation E needs to be given and the feasibility of the LMI can be calculated. Similar to the formulation of theorem 1, it can be transformed to a form, by applying the Schur formula [32], in which E is computed from the positve definite matrices in the LMI's . Numerical solution of these LMI's provides a less conservative result as in the formulation of theorem 1, but is still far from the analytical result.

For this particular case, comparing to the analytical solution (34) of the LMI (31), the conservativity can even be reduced more.

Theorem 2: Let $E \geq 0$, if :

$$E < \frac{\pi}{2h_{max}}, \tag{35}$$

the system in equation (24) is stable with bounded delay: $h \leq h_{max}$

Proof: see appendix ■

VI. Imperfections on the delay estimation

In practice the delay h is never known exactly and an estimation $\hat{h}$ of the delay needs to be made. In that case, the equation of the error state (24), and the dynamics of the switching surface (26) change to:

$$\dot{e}(t) = -Ee(t-h) - Es(t-h) + Es(t-\hat{h}) \tag{36}$$

$$\dot{s}(t) = Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + K_m u(t) + K_m d(t) \tag{37}$$

The following control law is proposed:

$$u(t) = -\frac{1}{K_m}\big(u_d sign\left(s(t)\right) + Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + as(t)\big) \tag{38}$$

where constants u_d and a are defined in (29) and (30). Control law (38) is equivalent to:

$$u(t) = -\frac{1}{K_m}\left(u_d sign\left(s(t)\right) + E\left(x_1(t-h) - \hat{x}_1(t-\hat{h})\right) + as(t)\right) \tag{39}$$

This control law can easily be implemented as $x_1(t-h)$ is, through $\sqrt{y}$, readily available from the measurement. The stability properties presented for the case with perfect delay are preserved with this control law (38).

Theorem 3: A) The control law (38) makes the surface (22) attractive and reached in finite time for the state equations (36), (37).

B) The equilibrium $x_1 = \hat{x}_1 = \hat{x}_d$ is then globally asymptotically stable for $h_{min} \leq h \leq h_{max}$, bounded delay estimation $\hat{h}$ and for E satisfying inequality (35).

Proof: see appendix ■

Comment 1: Irrespective of the value of E, the sliding line $s(t)$ (22), is always attractive. This implies that even when the error $e(t)$ becomes unbounded, the system still remains on the sliding line and will not jump from it. In practice, this can never be true, as due to physical power limits, the control $u(t)$ is always bounded. If E is selected such that the reduced error equation (24) is unstable, in practical situations, there will be a time when the system leaves the sliding mode.

Comment 2: Theorem 3 requires only a bounded delay estimate $\hat{h}$. How to select $\hat{h}$ is discussed in the implementation section.

VII. Dead zone model deviations

As already highlighted in the section III above, the dead zone model $f_d(u)$ (17) originates from simplified complex friction phenomena. In reality the state equation of (16) is rather:

$$\dot{x}_1(t) = K_m f_r(u) \tag{40}$$

where

$$\begin{aligned} f_r(u) = u - (c_0 + \Delta_0)\, sign(u) \quad & if \quad c_2 \leq |u|, \\ f_r(u) = 0 \qquad\qquad & if \quad c_2 > |u| \\ and \quad 0 \leq c_0 + \Delta_0, c_2 \leq c_1 & \end{aligned} \tag{41}$$

in which Δ_0 may be a positive or negative deviation on c_0 and

$$|\Delta_0| \leq \Delta_{max} \leq c_1 - c_0 \tag{42}$$

Parameters Δ_0 and c_2 may be time dependent. For a specific time instant t, it is assumed that:

$$|c_0 + \Delta_0(t)| \leq c_2(t) \tag{43}$$

In equation (17), c_1 plays the role of the stiction parameter whereas in equation (41), c_1 represents an upper bound on the stiction and Coulomb friction parameters c_2 respectively $c_0 + \Delta_0$.

By the imperfect dead zone model $f_d(u)$, the error equation (36) gets an extra term:

$$\dot{e}(t) = -Ee(t-h) - Es(t-h) + Es(t-\hat{h}) + K_m\Delta_f(u) \tag{44}$$

where $\Delta_f(u)$:

$$\Delta_f(u) = f_r(u) - f_d(u) \tag{45}$$

Note that not inexact knowledge of the dead zone does not affect the attraction of the sliding line $s(t)$. Attraction of the sliding line $s(t)$ involves part A) of theorem 3. The proof of this part relies only on the time derivative of the sliding line $\dot{s}(t)$, (37). Imperfections of the dead zone model only change the error equation and not equation (37). Therefore the stability of the system on the sliding line needs to be investigated.

Due to finite time convergence to the sliding line, and bounded delays h and $\hat{h}$, $s(t-h)$ and $s(t-\hat{h})$ reach zero in finite time. The reduced system is then governed by the following differential equation:

$$\dot{e}(t) = -Ee(t-h) + K_m\Delta_f(u) \tag{46}$$

The stability of the sliding mode is then ensured by the following theorem:

Theorem 4: Selecting E such that

$$E < \frac{\pi(c_1 - c_0)}{2h_{max}(c_1 - c_0 + \Delta_{max})} \tag{47}$$

, the reduced state equation (46) is stable.

Proof: see appendix ■

Comment 1: As shown in the proof of the theorem, in case the $sign$ function is defined such that $sign(0) = 0$, model deviations have no influence.

Comment 2: The result of theorem 4 may be very conservative, which is proved by simulations.

VIII. IMPLEMENTATION

The control law of equation (39) and the predictor (21) were programmed on a digital controller (ADWIN Gold, Jaeger Gmbh.). In order to transform the equations (39) and (21) to discrete time, the zero order hold transformation rule was utilized. A sampling frequency of $1000Hz$ was selected, which is reasonably high for this application. This assures that the continuous phenomena are well approximated and that no low pass filter is required, which normally takes care that the Shannon principle is not violated. In this case, this is important as the two piston pump generates considerable harmonics with a base frequency of $23Hz$.

There is no need to approximate the $sign$ function of the control law of equation (39) by a smooth function. While observing the axle of the valve, no vibrations were noticed, even when the desired pressure was reached. The abscence of chattering can be explained from a theoretical point of view. The electrical motor has been modelled by an integrator but in reality it is rather a first or even second order system in series with an integrator. The first or higher order dynamics represent a singular perturbation. In [33] (chapter 3, pp. 66-69) it is shown that by taking into account the higher order dynamics, the effective relative degree is higher than one. As a consequence, during sliding, the system is performing a higher order sliding mode and if some conditions are fullfilled, which are satisfied for this application, the output of the system is chattering free. From a physical point of view, the dynamics of the electrical motor smoothen out the chattering.

The system parameters, used to design the control law, are listed in table II. Contrary to parameters c_0, parameter c_1 is very difficult to measure accurately in practice. Experiments reveal that it has approximately the same value as c_0. This makes the results of theorem 4, which provides a guideline for the selection of parameter E when uncertainties on the dead zone are present, irrelevant.

Based on the upper bound on the delay $h_{max} = 0.2$, the maximum allowable E is computed by using theorem 2, which is $E = 7.85$. Constant a of the control law (39) determines the speed of convergence to the sliding line and should therefore be selected as high as possible. On the other hand, the input voltage is limited to $14V$, the voltage available on a normal tractor. Taking into account a range of desired pressures between 0 and $5bars$, a good compromise is $a = 10$.

The state estimator (21) requires an estimate of the time delay $\hat{h}$. As the underlying mechanism, which determines the size of the time delay, is not understood, it is selected constant and initially the average of the minimum and the maximum time delay *i.e.* $\hat{h} = 0.15$. Experiments reveal that by decreasing $\hat{h}$, the performance increases. First of all the stability of the control law (39) is independent of the size of $\hat{h}$. Only a bounded $\hat{h}$ is required. Secondly, the delay estimation $\hat{h}$ affects the term $s(t - \hat{h})$ in the control law (38), reflecting the distance to the sliding line in the past. In order to react abruptly, it is clear that $\hat{h}$ is best as small as possible.

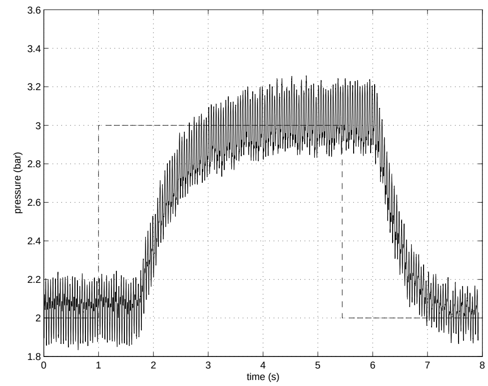

Fig. 2. Pressure evolution after set point changes from $2bar$ to $3bar$ and $3bar$ to $2bar$ for $a = 10$, $E = 7$, $\hat{h} = 0$, (full line: measured pressure, dashed line: set point)

Therefore, the estimation of the delay $\hat{h}$ should rather be considered as a design parameter instead of a delay estimate.

The controller was evaluated on the real system. Figure 2 shows the results, after a pressure change form $2bar$ to $3bar$ and from $3bar$ to $2bar$. The vibrations at $23Hz$ on the measured pressure are caused by the pump, which is a two piston pump, and not by chattering. These vibrations are the reason why such a high sampling frequency has been selected. They are not harmful for the spray distribution pattern of the liquid. Another important fact is the closed loop delay. From figure 2, it can be observed that the system starts reacting after $0.7s$, which is much higher than the open loop delay varying between $0.1s$ and $0.2s$. This implies that the control influences the delay. Unfortunately, this mechanism of influence could not be unravelled.

However, not including this effect results only in a model too conservative with respect to stability prediction. Experiments on the set-up show that the gain E could be increased to 30 without any limit cycle behavior or instability (figure 3). Nevertheless, the closed loop time delay is still around $0.15s$ and should theoretically

TABLE II

SYSTEM PARAMETERS

$u_d = 6.84$ (°/s)	$K_m = 1.71$ (°/(Vs))
$c_0 = c_1 = 2.72$ (V)	$h_{min} = 0.1$ (s)
$h_{max} = 0.2$ (s)	$\alpha = 1.5910^3$ ($\sqrt{Pa}$/°)
$\beta = -1.0410^5$ ($\sqrt{Pa}$)	$\gamma = -53.9$ (°)

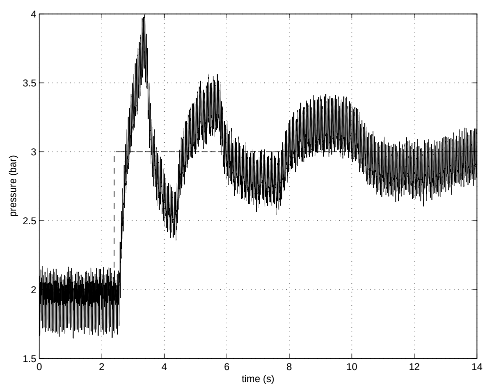

Fig. 3. Pressure evolution after set point changes from $2bar$ to $3bar$ for $a = 10$, $E = 30$, $\hat{h} = 0$, (full line: measured pressure, dashed line: set point)

result in instability. This implies that the phenomena which are actually described and observed as a time varying delay are in reality not delays. The theoretical conditions based on the simplified model still provide good guidelines for selecting E and a. A large gain E results in a smaller closed loop delay but pays off by a larger transient response. The best compromise is found by selecting E according to the conditions of the theorems presented in the paper *i.e.* $E = 7$. Therefore, the model used in this paper suits perfectly its purpose: controller design and not prediction.

Finally, the performance of the controller is com-

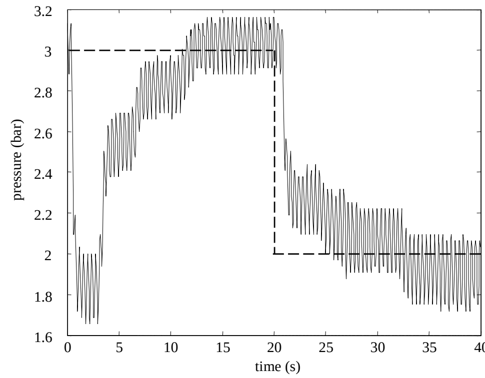

Fig. 4. Pressure evolution after set point changes from $2bar$ to $3bar$ for a PID controller, (full line: measured pressure, dashed line: set point)

pared to a PID. Comparing different control strategies is difficult as the designer has a large impact on the performance. Therefore it is difficult to judge whether the difference in performance between controllers originates from the difference in methodology or from the skills of the designer. As a starting point, a proportional feedback is introduced to stabilize the system. Around this closed loop, a PID controller is implemented with the Ziegler-Nichols rules. Afterwards the performance of the controller is increased by further hand tuning. The result is shown in Figure 4. At time zero the controller is switched on and some transient behavior is visible after which the pressure stabilizes to its set point value of $3bar$. After $20s$, the set point value is set to $2bar$. It is clear that the PID control has problems with the dead zone. The response of the system with this PID controller is much slower as with the developed control law.

IX. CONCLUSIONS

A pressure control system, used in agricultural applications, has been modelled. By considering only the most important dynamics, the system reduces to an integrator with a dead zone on the input, and an output which depends non-linearly on the delayed state. The dead zone is modelled as a linear control with a bounded input disturbance. Stability conditions, derived by using the Lyapunov-Krasovkii methods, lead to LMIs which introduce some conservativeness. Numerical solution of these LMIs even increase this conservativeness. By proper selection of the control law, imperfect knowledge of the varying time delay has no effect on system stability. Uncertainties on the dead band, may lead, depending on the definition of the $sign$ function, to changes in controller gain when the system is in sliding mode. A condition to adjust the controller gain has been derived. Implementation on a real spray section illustrate the validity of the followed appoach. The conditions derived in the paper allow to select the correct controller parameters, which confirms the appropriateness of the model. For output prediction, the model is not very suited.

Appendix

Proof of Theorem 2

Proof:

The poles σ of equation (24), determining the stability of the system, are obtained by solving following equation:

$$\sigma = -Ee^{-\sigma h} \tag{48}$$

Splitting σ in its real σ_r and its imaginary part σ_i and filling in in equation (48) delivers:

$$\sigma_r + j\sigma_i = -Ee^{-\sigma_r h}\left(\cos(\sigma_i h) - j\sin(\sigma_i h)\right) \tag{49}$$

Equating the real and imaginary parts of σ from equation (49) renders:

$$\sigma_r = -Ee^{-\sigma_r h}\cos(\sigma_i h) \tag{50}$$

$$\sigma_i = Ee^{-\sigma_r h}\sin(\sigma_i h) \tag{51}$$

The system is stable if and only if $\sigma_r < 0$ and marginally stable if and only if $\sigma_r = 0$. Marginal stability is obtained from equation (50) when $\cos(\sigma_i h) = 0$ or:

$$\sigma_i h = \pm\frac{\pi}{2} + k2\pi \tag{52}$$

where k is an integer number. The larger h, the smaller σ_i should be in order to preserve stability. So the worst case is when $h = h_{max}$. Filling in the pole with smallest imaginary part in absolute value in equation (51), enables to compute E, which gives rise to marginal stability:

$$E = \frac{\pi}{2h_{max}} \tag{53}$$

Therefore, in order to avoid marginal stability and assure stability, E should satisfy equation (35).

■

Appendix

Proof of Theorem 3

Proof:

For part A), the proof is entirely similar to the proof of theorem 1 [29]. The following Lyapunov function is selected:

$$V(t) = \frac{s(t)^2}{2} \tag{54}$$

Its derivative along the solution of (36), (37) is:

$$\begin{aligned}\dot{V}(t) = s(t)\Big(&Ee(t-h) + Es(t-h) - \\ &Es(t-\hat{h}) + K_m u(t) + K_m d(t)\Big)\end{aligned} \tag{55}$$

Inserting control law (38) renders:

$$\dot{V} < -2m_1\sqrt{V} - 2aV < -2m_1\sqrt{V} \qquad (56)$$

which proves that $s(t) = 0$ is a sliding surface, reached in finite time.

The proof of part B of the theorem relies on the attraction of the sliding line and by the fact that equation (56) is always valid such that once $s(t) = \dot{s}(t) = 0$, by the bounded delays h and $\hat{h}$, also $s(t-h)$ and $s(t-\hat{h})$ evolve in finite time to zero. Therefore in sliding mode, the reduced system corresponds to equation (24), of which the stability has already been proved by theorem 2.

■

APPENDIX
PROOF OF THEOREM 4

Proof:

Three cases need to be investigated.

***A** 1:* $|u| \leq c_2$

Is trivial as $\Delta_f(u) = 0$.

***A** 2:* $c_2 < |u| < c_1$

During sliding mode, assumption (A2) can never be met as equation (37) needs to be zero. The last two terms $K_m u(t) + K_m d(t)$ are actually a generalisation of the dead zone model $K_m f_d(u)$ (remember equations (25) and (26)). Performing this replacement on equation (37) results in :

$$\dot{s}(t) = Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + K_m f_d(u) \qquad (57)$$

By the assumption (A2) $f_d(u) = 0$, such that $Ee(t-h) + Es(t-h) - Es(t-\hat{h}) = 0$ needs to hold, which is not necessarily true and therefore assumption (A2) leads to a contradiction.

***A** 3:* $|u| \geq c_1$

By assumption $|u| \geq c_1$, equation (46) can be written as:

$$\dot{e}(t) = -Ee(t-h) - K_m \Delta_0 sign(u) \qquad (58)$$

In case of perfect sliding $sign(u)$ of expression (58) can be calculated from (38):

$$sign(u) = -sign\left(Ee(t-h) + u_d\right) sign\left(s(t)\right) \qquad (59)$$

Depending on how the function $sign$ is defined, conclusions can be drawn. In case $sign(0) = 0$, imperfect dead zone knowledge has no influence on the stability of the system as $sign(u) = 0$.

Very often $sign(0)$ is considered as undefined but belonging to the set $[-1, 1]$, which means that $sign(0)$ has a value but it is unknown. As $s(t) = 0$, the control u reduces to:

$$u(t) = -\frac{1}{K_m}\left(Ee(t-h) + u_d\right) sign(s(t)) \qquad (60)$$

by which it is clear that the second term of the right hand side $K_m \Delta_0 sign(u)$ introduces a deviation on the gain E, Δ_E, as $u(t)$ is dependent on $Ee(t-h)$. From the definition of $f_d(u)$ (17) and $f_r(u)$ (41), it is clear that the equivalent control of $K_m f_r(u)$ equals $-(E + \Delta_E)e(t-h)$. Only positive deviations Δ_E have a negative effect on the stability. Considering again the definition of $f_r(u)$ (41), it is clear that only negative Δ_0 decrease the stability of (58). The effect of Δ_0 on $f_r(u)$ is the largest when u is the smallest *i.e.* $|u| = c_1$. The worst case is when $\Delta_0 = -\Delta_{max}$. Take for example $u = c_1$ (for $u = -c_1$ the conclusions are the same), $f_d(u)$ and $f_r(u)$ equal:

$$f_d(u) = (-c_1 + c_0) sign(s(t)) \qquad (61)$$

$$f_r(u) = (-c_1 + c_0 - \Delta_{max})sign(s(t)) \tag{62}$$

Taking into account the equivalent control of $f_d(u)$ and $f_r(u)$, the following holds:

$$\frac{E}{c_1 - c_0} = \frac{E + \Delta_E}{c_1 - c_0 + \Delta_{max}} \tag{63}$$

As the stability of equation (58) is now governed by $-(E + \Delta_E)e(t-h)$, according to theorem 2, $E + \Delta_E$ needs to satisfy:

$$E + \Delta_E < \frac{\pi}{2h_{max}} \tag{64}$$

Combining equations (63) and (64) proves the theorem.

■

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