# Gathering Agents on a Lattice by Coupling Reaction-Diffusion and Chemotaxis

Nazim Fatès

# Gathering Agents on a Lattice by Coupling Reaction-Diffusion and Chemotaxis

Nazim Fatès
LORIA, Campus Scientifique B.P. 239
54 506 Vandoeuvre-lès-Nancy, France.
Nazim.Fates@loria.fr

September 15, 2008

**Abstract**

We address the question as to which are the minimal ingredients to obtain a decentralised gathering of agents that move on a lattice. The agents and their environment are described with a stochastic model inspired from biology: the aggregation of the *Dictyostelium discoideum* cellular slime mold. The environment is an active lattice, of which cells transmit information according to a reaction-diffusion mechanism. The agents trigger excitations randomly; they move by following excitation fronts. We show that despite its simplicity this model exhibits interesting properties of self-organisation and allows to achieve the decentralised gathering. Moreover, observations show that the system has interesting robustness properties, as being able to resist to the presence of obstacles on the lattice and to resist to the addition of noise on the moves on the agents.

**keywords:**

decentralised gathering problem ; pattern formation ; bio-inspired modelling ; cellular automata ; multi-agent systems ; self-organisation ; reaction-diffusion-chemotaxis ; Dictyostelium discoideum ; phase transitions

**foreword:**

Animations showing the experiments described in this article can be viewed at:
`http://www.loria.fr/~fates/Amybia/expe.html`
The simulations presented in this paper were made with *FiatLux* [11].

# 1 Introduction

**Decentralised Gathering Problem** Let us consider the problem where agents are initially randomly scattered on a lattice and have to group to form a compact cluster. The agents, all identical, have no idea of their own position, nor do they have on the position of the other agents. All they can do is to send messages that can be relayed, possibly with errors, by the cells of the lattice. The agents have only a rudimentary level of perception and a limited repertoire of actions: they can perceive the state of only their neighbouring cells and the only actions they can undertake is to move to these cells or change the state of the cell on which they are located.

The main question is to determine what are the simplest ingredients involved to achieve a decentralised gathering with these constraints. More precisely, we wish to determine how a simple model can be to solve the decentralised gathering problem, where "simple" means using a small number of states for the propagation of messages and a small number of rules for controlling the motion of the agents.

An early reference that addressed the problem of controlling a swarm of robots to form simple shapes is the paper by Sugihara & Suzuki [19]. They proposed decentralised algorithms to form circular or polygonal shapes. One of their assumptions was that robots could perceive the positions of other robots without any limitation. This hypothesis is of course most demanding and further works followed where robots were considered as having a limited visibility. For example, authors exhibited an algorithm that could gather robots that are initially in the same "visibility component", *i.e.*, each robot of the component is linked to other robots by a path of "visibility" relationship [4]. Their algorithm uses a simple idea: each robot converges toward the "centre of gravity" and ensures that it does not break the visibility component. The moves of the robots were supposed to be instantaneous, an hypothesis which suppresses the risk that the computation of the robot is based on an outdated perception of the world.

Since then, several versions of the gathering problem were examined. For example, the problem was modified to demand that the robots, which are considered as points, not only gather in the same area but succeed to be all placed on the *same point* [13]. The robots are considered in an "asynchronous" framework: the delay between their starting point and their end point is explicitly taken into account. The solution proposed relies on the assumption that the robots share a common sense of orientation (x and y axis). This showed that there exists a form of equivalence between the model with instantaneous moves and no sense of orientation and a model with time-dependent moves and a sense of orientation. We refer to the work of Prencipe [18] for recent theoretical developments on the decentralised gathering problem.

In this paper, the problem of visibility of robots is not fundamental since messages are not exchanged directly between robots but are instead transmitted by the environment on arbitrarily long distances. This of course has advantages and drawbacks, which will be studied in the following of the article.

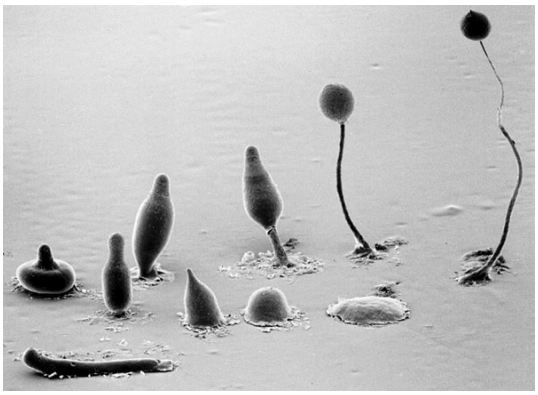


Figure 1: Life cycle of *Dictyostelium*. Courtesy of M.J. Grimson & R.L. Blanton, Biological Sciences Electron Microscopy Laboratory, Texas Tech University

**Other Related Problems** As it is frequently noted, the decentralised gathering problem is related to the Leader Election problem, initially introduced in [16], where all cells are initially in the same state and where the goal is to attain a configuration in which a single cell is in a distinguished state. In our problem, the objective is to form a cluster with the agents initially dispersed in random locations ; it is a form of symmetry breaking where some special location has to be chosen by consensus. By contrast with diffusion-limited aggregation (DLA), where fractal clusters are formed by random walks of particles, we want the clusters to be compact and efficiently generated. The *amorphous computing* paradigm [1], which uses dispersion of the agents and the introduction of noise in the system, also relates to our work. In such a context, solving the decentralised gathering problem may be used to aggregate components, which can constitute a first step before making computations.

The next section explains the biological inspiration of the model and draws a quick presentation of its features. Section 3 describes formally the model we use to achieve the decentralised gathering. In Section 4, the environmental layer of the model is studied as a source of interest *per se*. Section 5 presents the experiments that explore how the gathering occurs. In Section 6, we evaluate the robustness of the model by applying various perturbations. Finally, we conclude by discussing the results and their perspectives in the fields of biological modelling, computer science, or robotics.

## 2 Social Amoebae as a Source of Inspiration

The cellular slime mold *Dictyostelium discoideum* is a fascinating species whose individuals usually live as a mono-cellular organisms but may also transform into a multi-cellular organism when needed. In normal conditions, the cells live as single individuals by eating decaying logs, humus and bacteria (*e.g.*, [7]).

They reproduce by simple cellular division (mitosis). However, when the environment becomes depleted of food, a gathering process is triggered and single cells aggregate to form a complex organism that will move and react with coordination of its components. The transformation from a group of individual amoebae into a multi-cellular aggregate is a complex phenomenon that involves different stages. This article takes inspiration from the first stage of the multicellular organisation process, the *aggregation* stage, which consists in gathering all the cells in a compact mass called a *mound* (*e.g.*, see [20]).

Observations of *in vitro* experiments show that this aggregation is triggered by the spontaneous emergence of "pacemakers" or "signalling centres" (*e.g.*, [9]). These pacemakers are formed by one or several cells that attract other cells that are located in their vicinity. Once the first pacemakers are formed, they are in an unstable situation: under normal conditions, they struggle against each other and merge until only a few pacemakers remain ; these will attract other cells to them to form a group where cell differentiations will occur. From a quantitative point of view, the order of magnitude of the size of an amoeba is $10\,\mu m$, the size of the aggregates can be up to $10^5$ individual cells, the gathering can occur on a distance as far as 20 mm [3, 7].

The signalling occurs by transmission of waves, which follow typical evolving reaction-diffusion patterns. The waves are constituted of high-concentration profiles of cyclic adenosine monophosphate (cAMP), an extracellular messenger that guides the amoebae. This attraction phenomenon is called *chemotaxis* in the biological context and we will use it by analogy to qualify the moves of our virtual agents. The origin of these reaction-diffusion patterns resides in the concomitant realisation of four actions: (a) a cell synthesises cAMP internally until there is enough product to be emitted ; (b) when an amoeba detects a high increase in external cAMP concentration, it follows the concentration gradient (chemotaxis) and releases its own internal cAMP (exocytosis) (c) it then becomes insensitive to cAMP during a given refractory period , (d) in the meanwhile, the cAMP released diffuses and excites other sensitive cells, etc.

It is out of scope of this article to review the models that have been proposed to study the dynamics of *Dictyostelium*. Interested readers may refer to the works by Nagano [17] or by Deutsch & Dormann [8] as entry points to the literature. Problem solving by simulation of excitable media by reaction-diffusion models is studied in [2]. Our proposition is to take the essential ingredients of the aggregation mechanism of *Dictyostelium* to achieve the decentralised gathering of agents. The idea is to couple reaction-diffusion on the one hand and chemotaxis on the other hand with simple stochastic laws defined with three probabilities. The next section describes the mechanisms that define this novel aggregation scheme.

# 3 Coupling Reaction-Diffusion and Chemotaxis

We present the Reaction-Diffusion-Chemotaxis scheme as a stochastic dynamical system where time, space and state are discrete. We describe it at two levels:

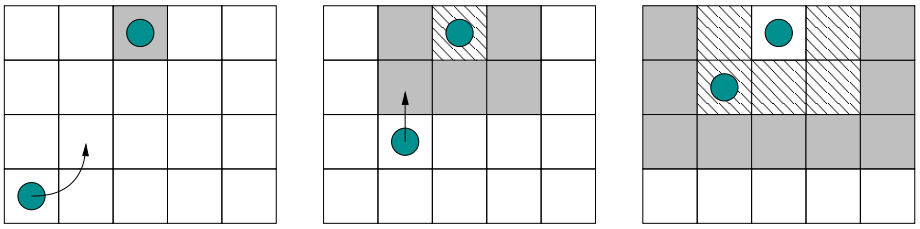


Figure 2: Three sucessive steps showing an evolution with two amoebae (circles). Excited, refractory, neutral cells are in grey, hatched, white, respectively. The amoeba on the left moves randomly (left diagram); and then follows an excitation front (middle diagram).

(1) an informal description of this scheme is given as a set of `instructions`; (2) *one particular* formalisation of these instructions, the *model*, is given with a mathematical description. It is possible to derive other models of the scheme; their study is left for future work. The model we study is meant to examine *sufficient* conditions under which decentralised gathering is possible. Two layers compose it: the *environmental* layer is a cellular automaton that models a reaction-diffusion process while the *particle* layer describes the moves of virtual amoebae (or simply *amoebae* in the following).

Space is modelled by a regular lattice $\mathcal{L} = \{1, \ldots, X\} \times \{1, \ldots, Y\}$ in which each cell $c = (c_x, c_y) \in \mathcal{L}$ is associated to a state. We denote by $\sigma_c^t$ the state of a cell $c$ at time $t$ and by $P_c^t$ the number of amoebae it contains. Our model of the Reaction-Diffusion-Chemotaxis scheme is expressed by giving $\sigma_c^{t+1}$ and $P_c^{t+1}$ as functions of $(\sigma_c^t)_{c\in\mathcal{L}}$ and $(P_c^t)_{c\in\mathcal{L}}$ . In this paper, we arbitrarily use the eight-cell neighbourhood, *i.e.*: $\mathcal{N}_c = \{c' \in \mathcal{L}, |c'_x - c_x| = 1 \text{ or } |c'_y - c_y| = 1\}$. Note that cells are excluded from their own neighbourhood and that cells in the border of the lattice $\mathcal{L}$ have a smaller neighbourhood. We now describe the model in three steps: (1) the environment, (2) the amoebae, (3) the interactions.

## 3.1 The Environmental Layer

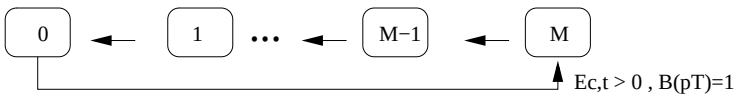



Figure 3: Transition rule of the environmental layer (simple reaction-diffusion)

The set of possible states for each cell is $\{0, \ldots, M\}$: the state 0 is the *neutral* state, the state $M$ is the *excited* state, the states 1 to $M-1$ are the *refractory states*. We call neutral, excited, or refractory, a cell of a given configuration that is in the neutral, excited, or refractory state, respectively.

The evolution of a cell is represented on Figure 3 ; it is described as follows:

- `A neutral cell stays neutral unless it has (at least) one excited neighbour. It then becomes excited with probability` $p_{\mathrm{T}}$`, the` *`transmission rate`*`.`
- `An excited cell becomes refractory (`$M-1$`) in one step.`
- `A refractory cell decrements its state until it becomes neutral.`

Formally, let $E_c^t$ be the set of excited cells in $\mathcal{N}_c$, the neighbourhood of $c$ at time $t$: $E_c^t = \{c' \in \mathcal{N}_c \,|\, \sigma_{c'}^t = M\}$. We denote by $\mathrm{card}\{X\}$ the cardinal of a set $X$ and by $B(\alpha)$ the Bernoulli random variable of parameter $\alpha$, *i.e.*, a random variable that equals 1 with probability $\alpha$ and equals 0 with probability $1-\alpha$. The local rule governing the evolution of each cell is a stochastic version of the classical Greenberg-Hastings model [14]:

$$\sigma_c^{t+1} = \begin{cases} M & \text{if } \sigma_c^t = 0 \text{ and } \mathrm{card}\{E_c^t\} > 0 \text{ and } \mathcal{B}(p_{\mathrm{T}}) = 1 \quad (R1) \\ \sigma_c^t - 1 & \text{if } \sigma_c^t \in \{1, \ldots, M\} \quad (R2) \\ 0 & \text{otherwise} \quad (R3) \end{cases}$$

## 3.2 The Amoebae Layer

The amoebae are supposed to be all identical, and in constant number as no birth or death process is considered. The movement of amoebae obeys the following rule: only one amoebae is allowed to move from a source cell to a target cell ; a cell that contains two (or more) amoebae will not accept further amoebae. This allows a trade-off between the need to limit the number of amoebae per cell (to observe clusters) and the need to keep the updating rules simple (dealing with simultaneous moves generally requires more elaborate procedures).

An *empty* cell is a cell which contains no amoeba; a *free* cell is a cell that contains less than *two* amoebae. Informally, the motion rules state that, at each time step, for each non-empty cell, one single amoeba obeys:

- `Move randomly to a free neighbour cell with probability` $p_{\mathrm{A}}$`, the` *`agitation rate`*`.`
- `or move randomly to a free` *`excited`* `neighbour cell,`
- `or stay on the same cell if this is not possible.`

Formally, let $\tilde{N}_c^t$ and $\tilde{E}_c^t$ be the set of *free* cells and *excited free* cells, respectively, in the neighbourhood of $c$ at time $t$. For a finite set $X$, we denote by $\mathcal{R}(X)$ the random variable that selects one element in $X$ with uniform probability; with the convention $\mathcal{R}(\emptyset) = \emptyset$. $\mathcal{R}$ is used to select a random neighbour for moving. A move of an amoeba from a non-empty source cell $c$ to a target cell $\Delta_c^t$ is given by ( $\Delta_c^t = \emptyset$ if no move occurs):

$$\begin{array}{lll} \text{if } \mathcal{B}(p_{\mathrm{A}}) = 1 \text{ then} & \Delta_c^t = \mathcal{R}[\tilde{N}_c^t] & (R4) \\ \text{else} & \text{if } \sigma_c^t = 0 \text{ and } \mathrm{card}\{\tilde{E}_c^t\} > 0 \text{ then } \Delta_c^t = \mathcal{R}[\tilde{E}_c^t] & (R5) \\ & \text{else } \Delta_c^t = \emptyset & (R6) \end{array}$$

The number of amoebae in a cell $c$ is then updated following:

$$P_c^{t+1} = P_c^t + \mathrm{card}\{c' \in \mathcal{L} \,|\, \Delta_{c'}^t = c\} - \mathrm{card}\{\Delta_c^t\}$$

### 3.3 Coupling the Environment and the Amoebae

How do amoebae and the environment interact? The interaction is given with the single law:

- `A non-empty` *`neutral`* `cell becomes excited with probability` $p_{\mathrm{E}}$`,` `the` *`emission rate`*.

Formally, we model the interaction between amoebae of a non-empty cell $c$ and the environment by:

$$\sigma_c^{t+1} = M \quad \text{if } \sigma_c^t = 0 \text{ and } P_c^t > 0 \text{ and } \mathcal{B}(p_{\mathrm{E}}) = 1 \quad (R7)$$

What is important to note is that, as a result of the rules R1 to R7, an amoeba that emits an excitation wave is not attracted by its own wave. Also remark that rules R1 and R7 may interfere. To clear this ambiguity, we choose here to combine rule R1 and R7 into a single rule R1':

$$\begin{aligned} \sigma_c^{t+1} = M \quad & \text{if } \mathcal{B}(p_{\mathrm{T}}) = 1 \text{ and } \sigma_c^t = 0 \\ & \text{and } [\ \mathrm{card}\{E_c^t\} > 0 \text{ or } (P_c^t > 0 \text{ and } \mathcal{B}'(p_{\mathrm{E}}) = 1)\ ] \quad (R1') \end{aligned}$$

The new rule R1' states that a cell becomes excited only if it is neutral, with probability $p_{\mathrm{T}}$. The excitation is either received from a neighbouring cell or, with probability $p_{\mathrm{E}}$, from the amoebae it contains.

There are of course many other ways to formulate the rules of the model. Here, we deliberately consider only the difference between empty and non-empty cells for the emission rule (R7). This is meant for facilitating the coding of the model and, if needed, its implementation on massively parallel devices. We may of course consider other models of the scheme where excitations are triggered by each amoeba independently but such models are harder to control. Programmers should note that: (a) we adopted a cellular-automaton point of view rather than a multi-agent one, *i.e.*, actions are cell-centred, amoebae are represented by an attribute of cells rather by an independent list of agents ; (b) As our formulation does not forbid simultaneous moves of amoebae to the same cell, up to nine amoebae are allowed to share the same cell in the unlikely event where all the neighbours of a cell move *simultaneously* to a target cell with *one* amoeba on it.

To finish the presentation of the model, as our goal is to study the *simplest* model in terms of states and rules, we set the excitation level to $M = 2$ ; the set of states is thus $\{0, 1, 2\}$.

## 4 Experimental Study of the Environment

We begin our study with the analysis of the environmental layer with simulations. We describe the properties of reaction-diffusion dynamics under various conditions and show that a second-order phase transition occurs when the transmission rate $p_{\mathrm{T}}$ varies.

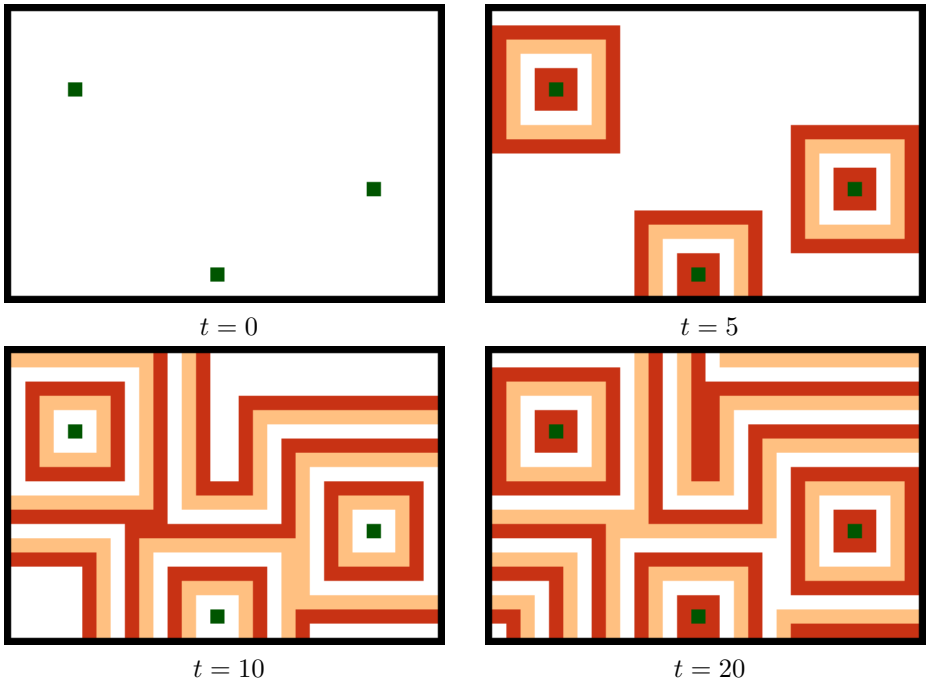



Figure 4: Four views of the evolution of the model with two amoebae in the fully deterministic case for a grid size $(30, 20)$ and $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 1, 0)$. Amoebae are represented by black squares, white squares are neutral cells, darkest brown/grey squares are excited cells, lighter brown/grey squares are refractory cells. This colour code is kept in the following.

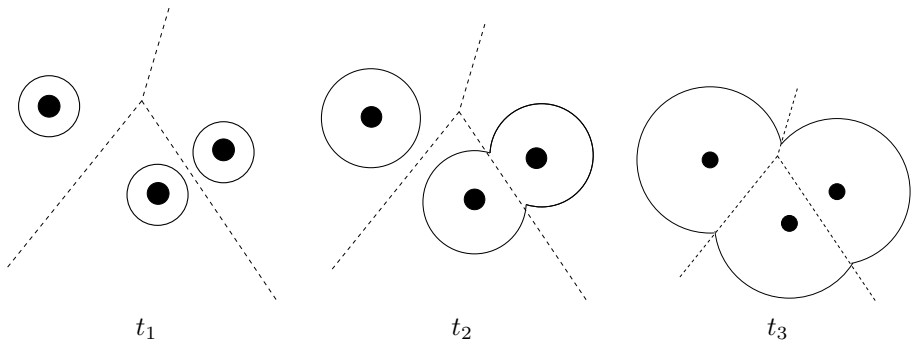



Figure 5: Abstract view of the propagation of fronts in an Euclidian space. Dashed lines represent frontiers between influence regions. $(t_1 < t_2 < t_3)$

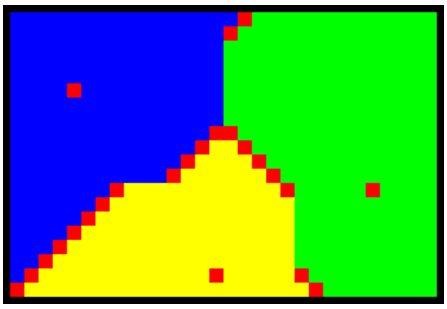


Figure 6: The corresponding influence regions for each amoebae (see text). Diagonal frontiers are special cells that belong to two or three influence regions.

### 4.1 The Fully Deterministic Case is Static

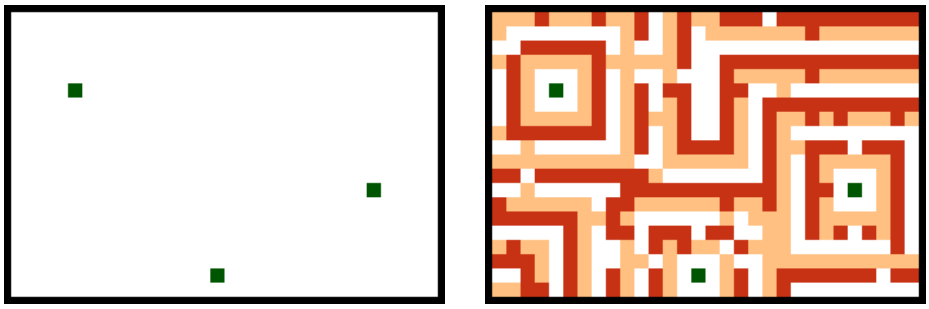
**Experiment** First, let us examine the system in the fully deterministic case: the transmission rate and the emission rate are set to 1, the agitation rate is 0: $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 1, 0)$. Figure 4 shows the evolution of a system that contains only three amoebae on a small lattice size $(30, 20)$.

We call a set of adjacent cells that are all in the excited state $M$ an *excitation front*. Excitations fronts propagate radially from the amoebae at the speed of one cell per unit of time, they annihilate when they meet frontally and merge when they meet perpendicularly. As a result, the gathering is impossible: no chemotaxis ever occurs as excitation fronts do not hit the amoebae.

**Interpretation** Contrarily to classical diffusion waves, it is a well-known phenomenon that reaction-diffusion fronts annihilate when they meet in opposite directions and merge otherwise (see Fig. 5 for an abstract view). This property is respected by the model although it is discrete. An important consequence is the limitation of the transfer of information from an amoeba to the others. Let us define informally an *influence region* of an emitting cell as the set of cells that will receive the excitation wave emitted by this cell. Intuitively, we see that in the fully deterministic case, the influence regions of the amoebae correspond to the discrete Voronoi diagram of the lattice with amoebae as centre points (see Fig. 6 for an illustration and [2] for a more detailed analysis). It is important to note that communication between amoebae via the environment can not occur in the fully deterministic case. Indirect communication happens only when an amoeba enters the influence region of another amoeba.

### 4.2 The Non-coherent Regime

**Experiment** Will the gathering become possible if we take $p_{\mathrm{T}} < 1$ ? To observe the effects of transmission errors in the environment, we set the transmission rate to $p_{\mathrm{T}} = 0.99$, *i.e.*, we introduce a 1% chance that a cell fails to receive an excitation from its neighbours. Figure 7 shows an evolution of the



$t = 0$ $t = 40$

Figure 7: Loss of coherence occuring with non-perfect transmission rate $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (0.99, 1, 0)$ on a small lattice of $(30, 20)$.

system for these settings. We observe that for small simulation times the system behaves qualitatively as for the non-perturbed case: waves are initiated by amoebae, they propagate until they collide and annihilate. However, as transmission errors accumulate, the waves progressively loose their coherent shape. Additional sources of waves appear: these are spiral waves whose behaviour is well-studied in reaction-diffusion media, whether discrete or continuous [14]. As time advances, more and more of these persistent spiral waves appear. Finally, when the coherence is totally lost, amoebae start moving erratically as they sometime receive excitation fronts.

**Interpretation** Small transmission errors in the environment create new sources of excitation waves, independently of the amoebae. The multiplication of such parasitic waves causes a confusion that does not allow the amoebae to group into clusters. We call this behaviour where persistent waves develop independently of the position of the amoebae, the *non-coherent regime*.

### 4.3 The Extinction Regime

For smaller values of $p_{\mathrm{T}}$ the loss of coherence is observed even more rapidly. Interestingly, when the transmission rate becomes small enough, waves are no longer persistent. We call this new qualitative behaviour, where waves spontaneously disappear, the *extinction regime*. We observed that the transition from the non-coherent regime to the extinction regime is sharp and occurs for $p_{\mathrm{T}} \sim 0.20$.

**Experiment** In order to understand the origin of this abrupt change, we considered a system depleted of amoebae, where some cells were randomly set *excited* with probability 10% and left *neutral* otherwise. To separate the non-coherent regime from the extinct regime, we monitored the evolution of the density of excited cells: $e(t) = \mathrm{card}\{c \in \mathcal{L} \,|\, \sigma_c^t = M\}/X.Y$

We expect this quantity to reach zero quickly for the extinction regime and to remain strictly positive for the non-coherent regime. To test this hypothesis,

we varied the transmission rate $p_T$ by 1% steps from 0.01 to 1, for a lattice size $(100, 100)$, and measured the evolution of the $e(t)$ during 10 000 time steps. We computed an approximation of the asymptotic density of excited cells by taking the average value of $e(t)$ for $t \in [5\,000, 10\,000]$.

Figure 8 shows this average as a function of $p_T$. It confirms the presence of a qualitative change for $p_T \sim 0.20$. The shape of the curve suggests that the transition between the two regimes is a second-order phase transition that occurs for $p_T \sim 0.20$. We can also note that for $p_T$ close to 1, we observe a non-regular behaviour that is due to finite-size effects: because of the limitation of the lattice, the probability of apparition of persistent waves is small and the non-coherent regime may not be reached. This means that for values of $p_T$ close to 1, the system oscillates between the static regime and the non-coherent regime.

**Refining the previous experiment** To analyse the nature of the change near criticality, we monitored the evolution of $e(t)$ for $p_T \sim 0.20$. For the transmission rates $p_T \in \{0.2044, 0.2045, 0.2046\}$, for different lattices sizes up to $(400, 400)$, the average value of $e(t)$ was measured with 50 random initial conditions. We observed that this small variation of $p_T$, of the order of $10^{-4}$, separated the extinction regime from the non-coherent regime (in which excitations survive arbitrary long periods of time).

Plotting $e(t)$ in a log-log scale shows that the curve closely follows a power-law for $t \gtrsim 100$. It is well-known in statistical physics that the power-laws observed in phase transitions are not arbitrary: particular sets of exponents characterise the evolution of the system near criticality. The different models that display the same sets of exponents is called a *universality class*. By analogy with previous observations made in asynchronous cellular automata, we tested whether the phase transition belonged to the universality class of *directed percolation* [5, 10]. The evolution of excited cells density should then follow $e(t) \sim t^{-\delta}$ near criticality, with $\delta = 0.451$ for two-dimensional lattices (this number is known only experimentally [15]). Figure 9 shows the evolution of the average value of $e(t)$ for a lattice size $(400, 400)$; we see that, as expected, for $p_T = 0.2045$ the slope of $e(t)$ follows the predicted value, which confirms the directed percolation hypothesis.

**Synthesis** The existence of different regimes in the environmental layer underscores a strong condition for the amoebae to achieve the gathering: the medium which implements the reaction-diffusion has to relay the excitations without errors. For $p_T \to 1$, we observed that the system became more robust as the value of $M$ is increased, *i.e.*, the probability of falling in the non-coherent regime becomes smaller for large values of $M$. We believe that this question is strongly related to the metastability problem in the Greenberg-Hastings model studied in [12]. Understanding this robustness is a problem that arises from these observations and is left for further studies. In the following, we will simply consider that the environmental layer is perfect by taking $p_T = 1$.

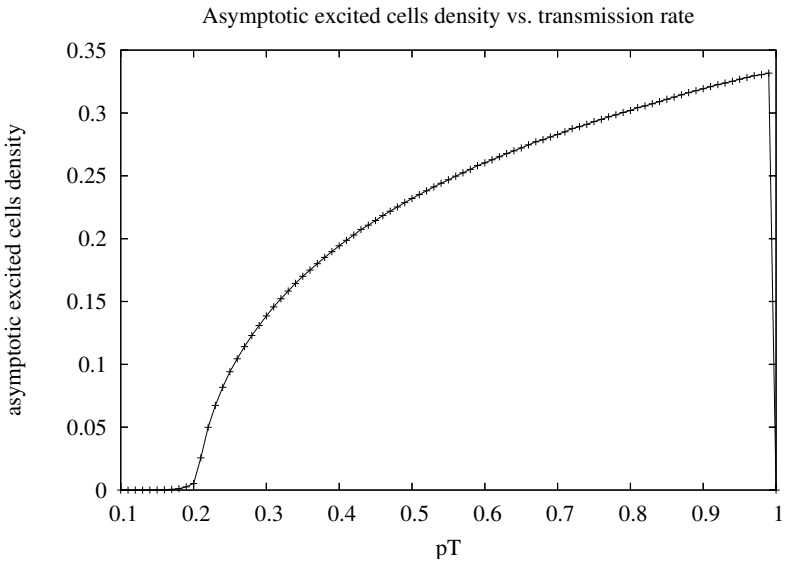



Figure 8: Average density of excited states as a function of $p_{\mathrm{T}}$ (see text). Lattice size is $(100, 100)$, the environment is depleted of amoebae, 10% of the cells were intially set to the excited state.

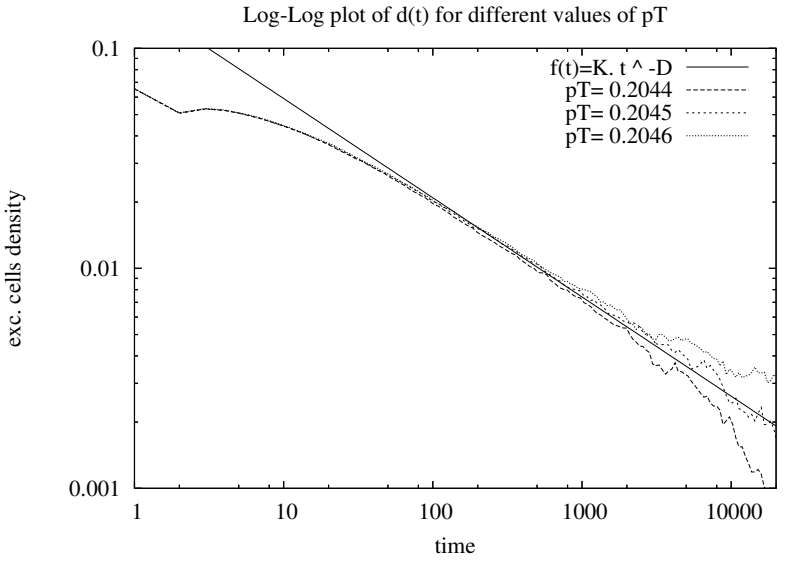



Figure 9: Evolution of the density of excited cells $e(t)$ around the critical value transmission rate $p_{\mathrm{T}}$ (log-log scale). Lattice size is $(400, 400)$. The straight line has slope $-\delta_{\mathrm{DP}} = -0.451$, which is predicted by the directed percolation theory.

# 5 Obtaining the Decentralised Gathering

We now examine the gathering behaviour both from a qualitative and quantitative point of view. Our method consists in varying the probability of emission $p_{\mathrm{E}}$ and to observe whether the gathering is successful. We begin our study by simulating the system on a small lattice and then we examine what happens on a larger lattice to analyse the scaling properties of the system.

## 5.1 Gathering in Clusters with the Pacemaker Effect

**Experiment** What happens to the system when the amoebae emit their excitation at random times? We examined the dynamics of the system for a wide range of values $p_{\mathrm{E}} \in \{0.01, 0.10, 0.50, 0.80\}$, keeping $p_{\mathrm{T}} = 1$ and $p_{\mathrm{A}} = 0$. We initially assigned to each cell a 10% probability to contain an amoeba. Unless otherwise mentioned, we keep this random initial condition for all the following experiments ; this is compatible with experiments conducted by other authors in the biological modelling context (*e.g.*, [3]). We observed that the gathering of amoebae occurs for all the values of $p_{\mathrm{E}}$ considered. Moreover, the smaller $p_{\mathrm{E}}$ was, the quicker the gathering occurred. Figure 10 shows the evolution of the system for $p_{\mathrm{E}} = 0.10$: a compact cluster emerges in a few hundred steps ; it then emits waves with a good regularity. The gathering is achieved in less than 3 000 steps, which is impressively quick.

**Quantifying the gathering** To quantify more precisely the gathering of amoebae, we propose to examine the temporal evolution of the *bounding box ratio* (BBR), defined as the ratio of the surface of the largest rectangle containing all the amoebae over the total surface of the lattice. Formally, let $C = \{c \,|\, , P_c^t > 0\}$ be the set of non-empty cells at time $t$, let $x_{\min}, x_{\max}$ (respectively $y_{\min}, y_{\max}$) be the minimal and maximal value of $c_x$ (respectively $c_y$ ) such that $c = (c_x, c_y) \in C$. We define:

$$\mathrm{BBR} = \frac{(x_{\max} - x_{\min}).(y_{\max} - y_{min})}{X.Y}$$

This parameter is rather simplistic since it captures only a small part of the system's organisation into clusters. However, note that it is a "strong" criterion in the sense that a low values of BBR is attained only if no amoeba is forgotten from the gathering.

Figure 11 shows evolution of the BBR for different values of $p_{\mathrm{E}}$. The first plot (top) displays the average evolution of for 50 independent samples. We see that this evolution is regular; statistical measures confirm the counter-intuitive fact that the gathering process is accelerated when $p_{\mathrm{E}}$ is decreased. However, for very small values of the emission rate ($p_{\mathrm{E}} < 10^{-3}$), the gathering process is slowed as the waves are not emitted frequently. It is an open problem to determine the optimal value of $p_{\mathrm{E}}$ as a function of the lattice size.

Also note that the smoothness of the curves comes from the averaging and does not well describe the evolution of a single sample. The bottom plot of

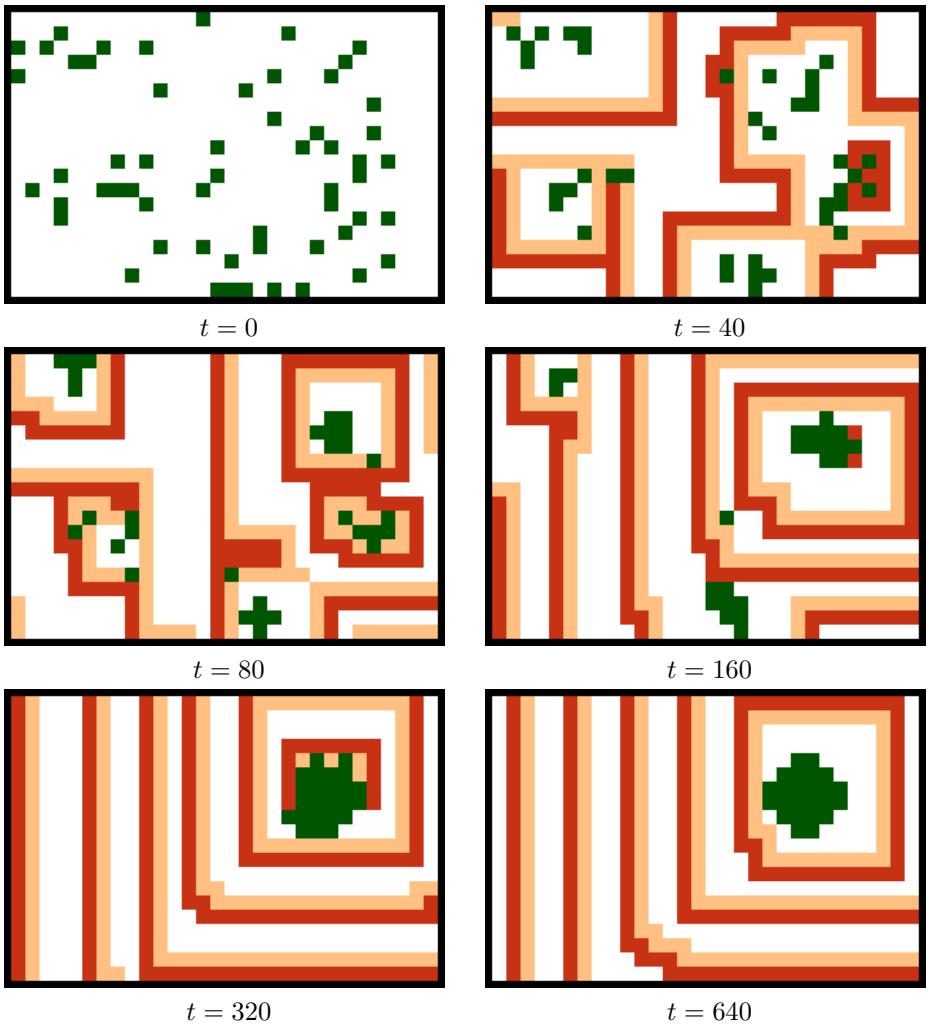



Figure 10: Sequence showing the formation of a pacemaker with $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 0.10, 0)$ and $(X, Y) = (30, 20)$.

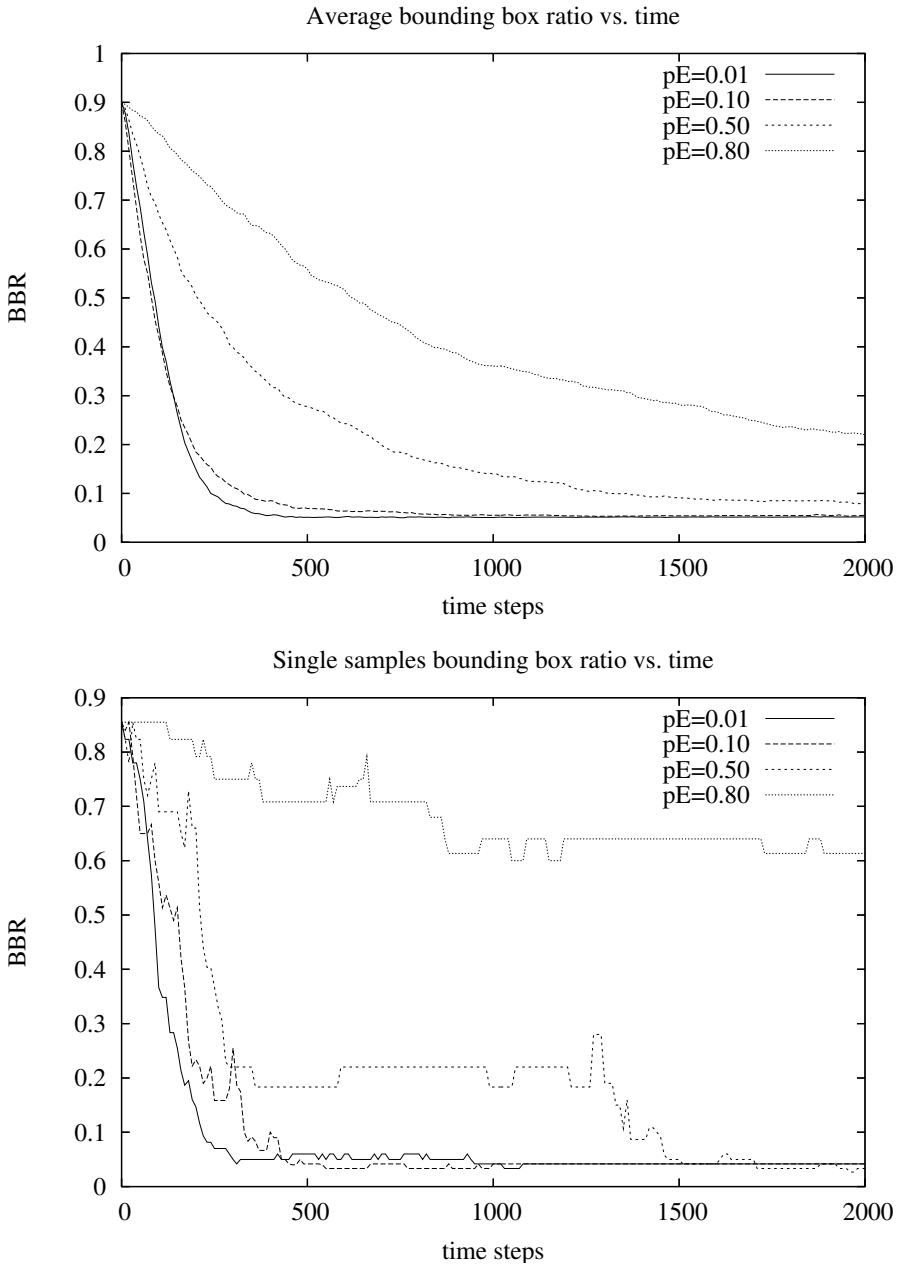



Figure 11: Evolution of the bounding box rate (BBR) as a function of time for a grid size $(30, 20)$, $p_{\mathrm{T}} = 1$, $p_{\mathrm{A}} = 0$, and different values of $p_{\mathrm{E}}$. Average evolution for 50 samples (top) Example of a single sample evolution (bottom).

Fig. 11 shows the evolution of a single sample for three values of $p_{\mathrm{E}}$. The curve obtained for $p_{\mathrm{E}} = 0.5$ shows three stages, which correspond to:

- for $t \in [0, \sim 300]$ amoebae gather until they form two unstable clusters,
- for $t \in [\sim 300, \sim 1200]$ the two clusters emit excitations fronts with an irregular pace, during this stage, influence regions of the two clusters shrink or extend due to the irregularities of emissions,
- for $t \gtrsim 1200$ one of the two clusters, the "winning" cluster, have its influence region touch the other cluster and "captures"' its amoebae.

**Interpretation** For small lattices, it is sufficient to decrease $p_{\mathrm{E}}$ to allow the system to achieve the decentralised gathering in a few thousand steps. The process starts when randomly chosen amoebae emit an excitation wave which will attract non-emitting amoebae to it. The repetition of the process creates more populated regions of the lattice. These dense regions, the *clusters*, can be considered as a whole object, which emits excitation waves and attracts the cells that are in its influence region. The more a cluster increases in size, the more regularly and frequently it emits excitations. This correlation between the size of a cluster and its frequency of emission creates a positive feedback ; asymptotically, all the amoebae should gather in the same cluster. When a cluster has a reasonable size, it emits waves with a good regularity, close to the maximal frequency $1/M + 1$. By analogy with the biological phenomenon, we call this the *pacemaker* effect (*e.g.*, [9]).

## 5.2 Gathering on Larger Lattices

**Experiment** How does this self-organising system behave for larger lattices and for a great number of amoebae? Keeping $p_{\mathrm{T}} = 1$ and $p_{\mathrm{A}} = 0$, for a lattice size $(150, 100)$ and for an observation time of 20 000 time steps, we measured the evolution of the BBR with $p_{\mathrm{E}} \in \{0.01, 0.10, 0.50, 0.80\}$ on 100 random samples . Figure 12 shows one evolution of the system for these settings. Statistical data is displayed on Fig. 14 p. 20 for $p_{\mathrm{E}} = 0.01$ and $p_{\mathrm{E}} = 0.10$ ("no noise" curves).

We see that the system stabilises to small BBR values (below 15%). By looking at numerous simulations, we observed that such value correspond to the formation of a single cluster and thus to the achievement of the decentralised gathering task. To compare with the previous experiment, we can remark that the lattice size and the number of amoebae was multiplied by 25, the time scale was multiplied by a factor smaller than 10. To determine analytically the scaling laws of the system is an open problem that arises from these observations.

Moreover, observing some simulations, we noticed surprising effects that can only be observed on large lattices. For example, it happened that clusters were destroyed and spontaneously reformed when, by "chance", the attracting cluster shrank its influence region. We also observed rare cases where a cluster divided into two parts, each part being attracted by a different cluster.

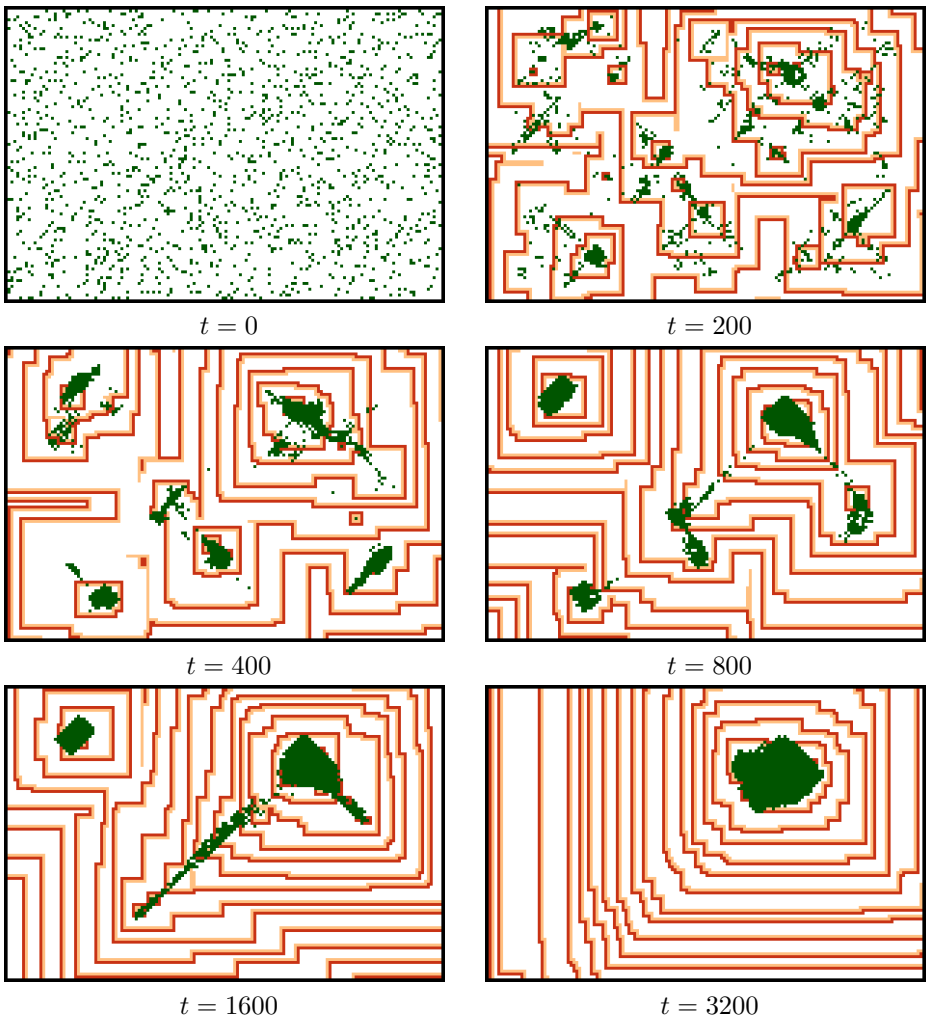



Figure 12: Evolution of the model with perfect transmission rate and no agitation: $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 0.01, 0)$. Lattice size is $(150, 100)$. Note that the streams follow the diagonal axes only with the current model that uses the eight-cell neighbourhood. In the cases where the model is defined with four-cell neighbourhood (N,E,S,W), the streams follow the $X$ and $Y$ orthogonal axes.

**Interpretation** The self-organisation of amoebae in clusters is a phenomenon that emerges from the competition between clusters. When to clusters compete for extending their influence region, we observe that the cluster that contains more amoebae has a tendency to win. Indeed, if, in a given period of time, a cluster A emits a wave and a cluster B does not emit a wave, as excitation fronts travel at constant speed, the influence region of cluster A will extend and the influence region of cluster B will shrink. Frontiers between influence regions perform a biased random walk where the bias is favourable to the cluster with the highest emitting frequency. Once a cluster sees its influence region totally shrunk, its amoebae come under the influence of the winning cluster and they move towards the attracting pacemaker by forming streams. Noteworthy is the analogy with the biological phenomenon where these streams are also observed, at least in *in vitro* experiments (see *e.g.*, [21]).

# 6 Robustness to Perturbations

Now that we have established that the gathering is possible simply by decreasing the value of $p_{\mathrm{E}}$, we examine how the model resists to two types of perturbations: randomness imposed on the moves of the amoebae ($p_{\mathrm{A}}$) and introduction of obstacles on the lattice. Studying robustness is particularly important if we are to apply the Reaction-Diffusion-Chemotaxis scheme to control robots aggregation.

## 6.1 Self-organisation with Agitation

**Experiment** Is the system robust to noise superimposed on the moves of the amoebae? We set the agitation rate to 20%, keeping the two other parameters unchanged: $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 0.01, 0.20)$. Figure 13 shows one evolution of the system for these settings and Fig. 14 shows statistical measures obtained with 100 samples. It is remarkable that the addition of $p_{\mathrm{A}} = 20\%$ of random move does not slow much the gathering process. For higher values of $p_{\mathrm{A}}$ the perturbations are too important to allow compact and stable clusters to form ; however, a form of gathering can still be observed, at least for $p_{\mathrm{A}}$ smaller than 0.75 on large lattices.

**Interpretation** The gathering process shows good robustness to the introduction of noise on the moves of the amoebae. The most noticeable difference in the dynamics is that groups of small size do not appear. This suggests that there exists a link between the stability of groups of a given size and the parameter $p_{\mathrm{A}}$. Intuitively, the bigger a cluster, the more robust to noise it will be. Finding a relationship between the minimal group size (if such a property exists) and the quantity of noise is another interesting question that arises from these observations.

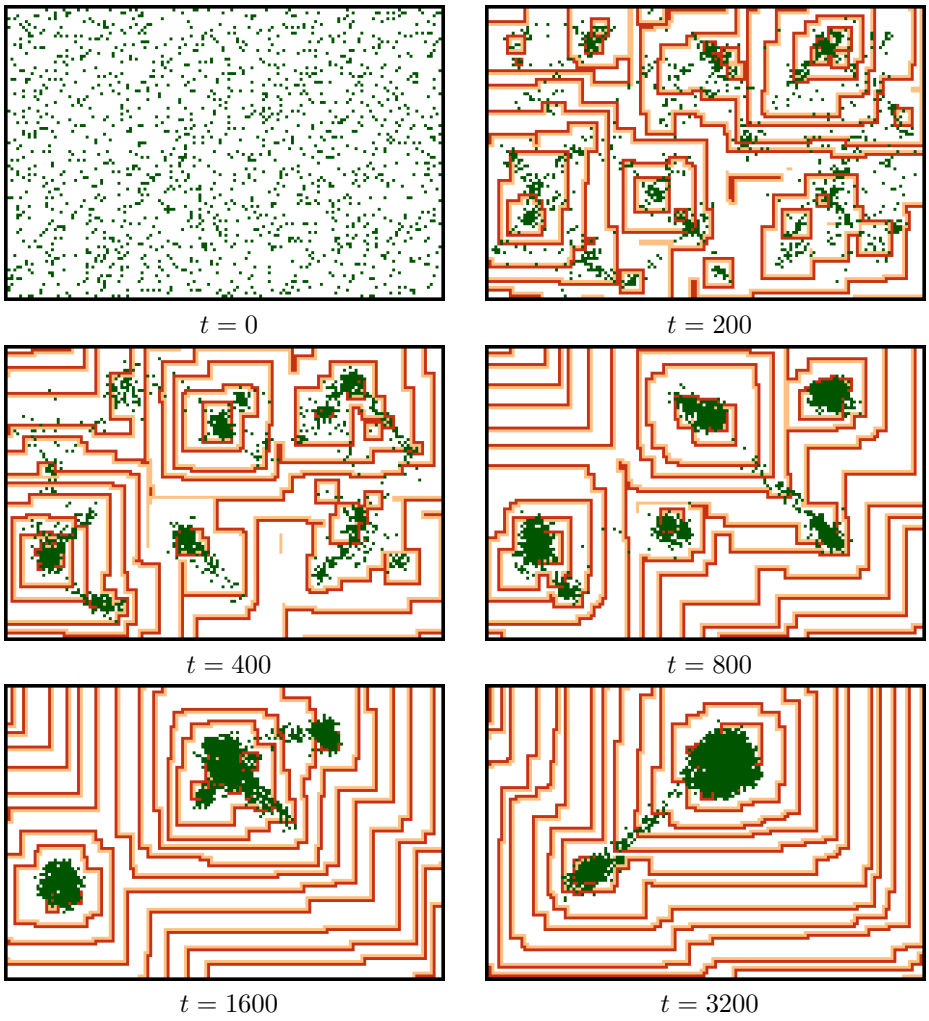



Figure 13: Evolution of the model with perfect transmission rate and small agitation: $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 0.01, 0.1)$.

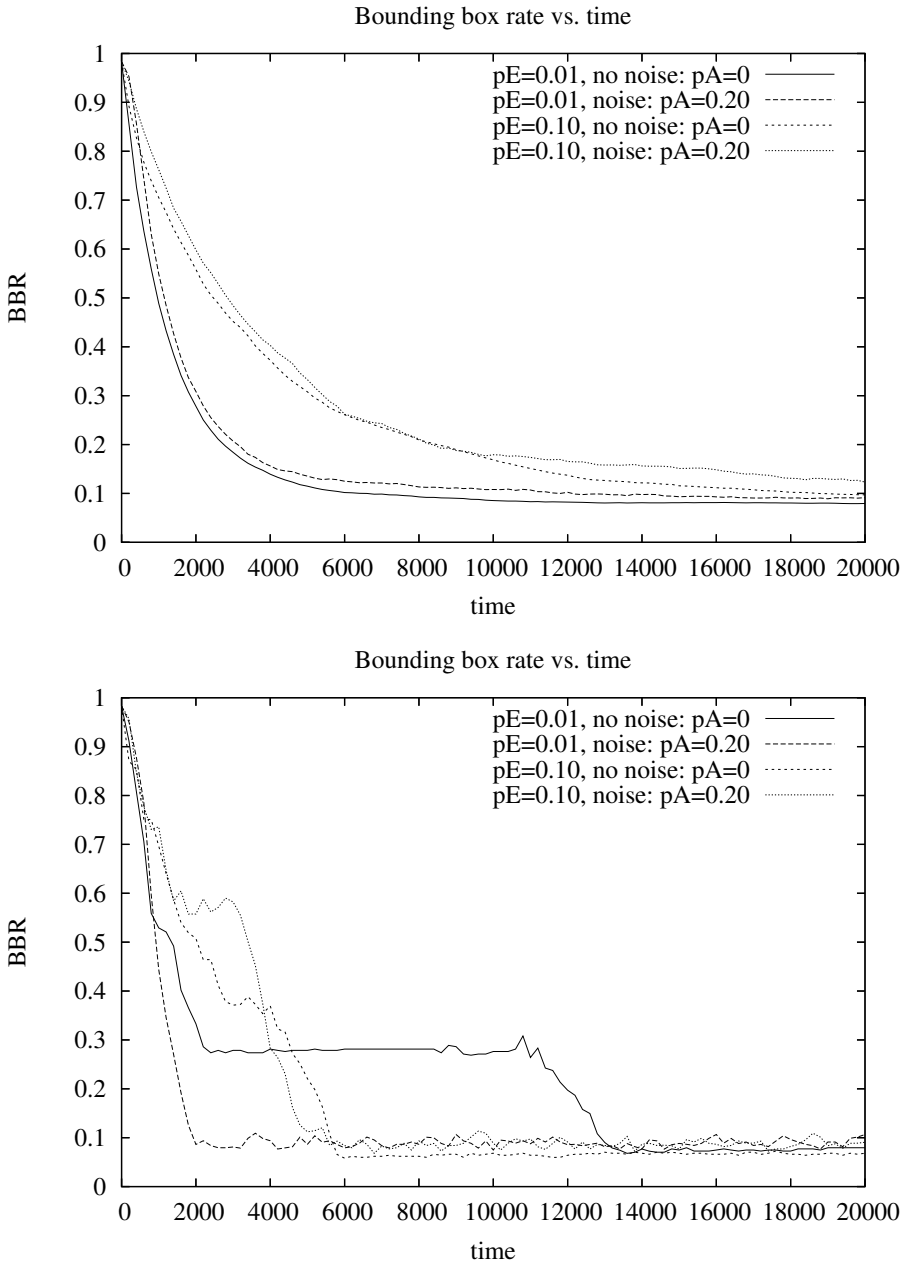



Figure 14: Evolution of the BBR as a function of time for a grid size $(150, 100)$, $p_{\mathrm{T}} = 1$, and different values of $p_{\mathrm{E}}$ and $p_{\mathrm{A}}$. Average evolution for 100 samples (top) Example of a single sample evolution (bottom).

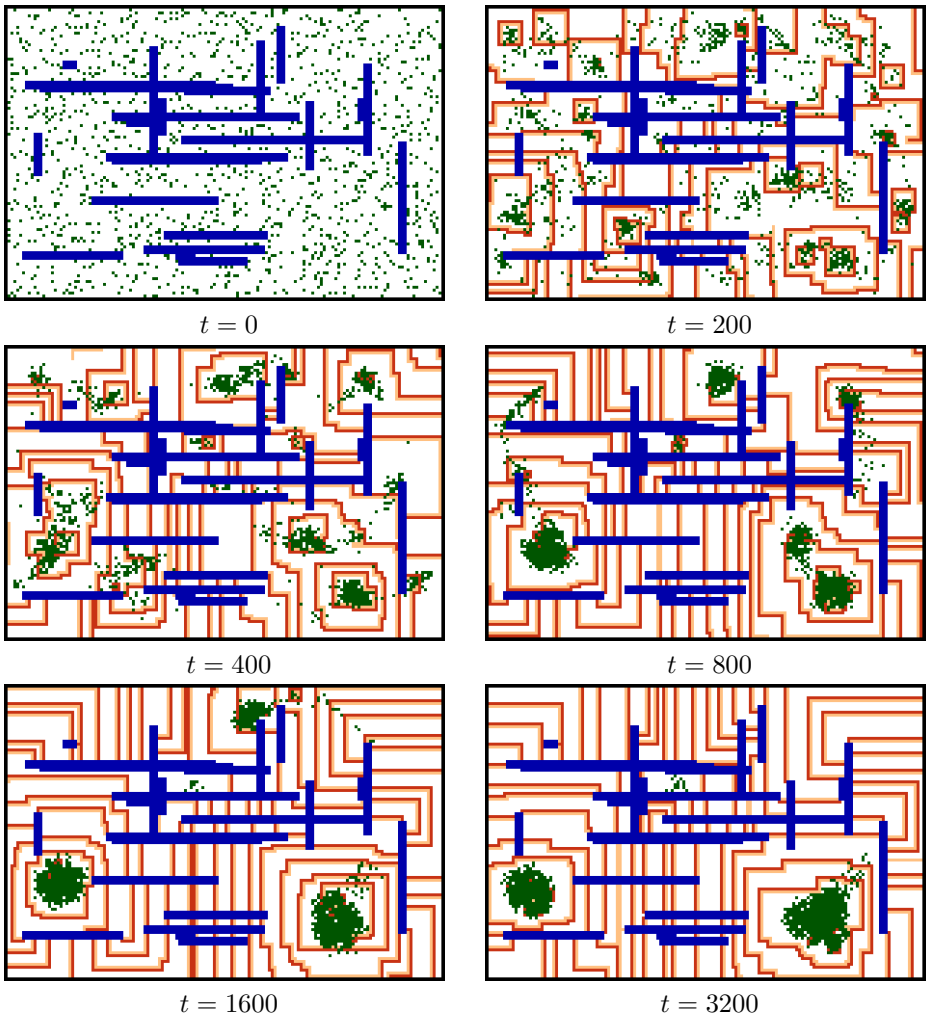



Figure 15: Evolution of the model with obstacles, perfect transmission rate and small agitation: $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 0.01, 0.1)$.

Table 1: Synthetic view of the qualitative behaviours oberved by simulating the aggregation model

| $p_E$ | $p_T$ | $p_A$ | qualitative behaviour |
|---|---|---|---|
| 1 | 1 | 0 | static |
| any | $[0.2, 1[$ | 0 | non-coherent |
| any | $< 0.2$ | 0 | extinct |
| 0.10 | 1 | 0 | self-organising (slow) |
| 0.01 | 1 | 0 | self-organising (quick) |
| 0.01 | 1 | 0.2 | self-organising (quick) |

### 6.2 Obstacles on the Lattice

**Experiment** Finally, what happens to the system when the topology of the lattice is modified? In particular, how do amoebae operate when obstacles are introduced? Real amoebae evolve in very inhomogeneous media and also need to achieve the gathering despite the presence of numerous obstacles. Figure 15 presents the evolution of the system where obstacles are placed randomly on the lattice. Obstacles do not allow information nor amoebae to cross.

**Interpretation** We observe that the system is robust to the presence of obstacles. In the parts that are totally disconnected from the rest of the lattice, isolated groups are formed. In the others parts of the lattice, the aggregation process is not perturbed by the presence of obstacles. In particular, in some parts of the lattice, it is possible to observe the amoebae taking narrow spaces to converge to a pacemaker. In this case, the streams formed are thin and they usually split and re-form several times before reaching the pacemaker. We underline that this type of robustness is somewhat obtained "for free": it can be seen as a an emergent property since at no time it was explicitly coded in the local rules governing the system.

## 7 Discussion

**Synthesis** The Reaction-Diffusion-Chemotaxis scheme provides an original solution to the decentralised gathering problem in the case where the agents have limited abilities and move on a lattice. The discrete model we studied is inspired from a biological example, the aggregation process of *Dictyostelium*. The rules that define it are simple: only three states are needed for propagating messages in the lattice and only three rules guide the actions of the virtual amoebae. One single parameter, the emission rate $p_E$, controls the interaction between the agents and their environment. The experiments showed that this parameter was easy to tune and produced the gathering for a wide range of values. The only constraint is that it should not be too low, otherwise excitation waves are not frequently emitted, and not too high, otherwise excitation waves annihilate and do not reach the virtual amoebae. To give an image, all happens as if the agents had to find the right compromise between "speaking" and "hearing".

**A Hierarchical Process** The exploration of the properties of the system showed four different qualitative behaviours: the static regime, the non-coherent regime, the extinction regime and, last but not least, the self-organised regime where the gathering could be observed in a few thousand steps (see Table 1). We showed that the dynamics of the system, far from trivial, allowed the system to achieve the gathering even when the agents' motion was subject to a high level of noise. We presented our analysis with an "ascendant" view, *i.e.*, from the local rule to the global behaviour. Conversely, we may also look at the gathering phenomenon with a top-down view. We propose the following hierarchical observation, where we put into brackets the objects that appear at different levels of description:

1. The gathering phenomenon results from a competition between clusters; the bigger a cluster, the higher its probability to <capture> other clusters.
2. A cluster captures another cluster when its <influence region> touches it.
3. The extension or shrinking of the influence regions depends on the average frequency of emission of the <pacemakers> of each cluster,
4. The pacemaker effect of a cluster results from independent emissions of <excitation waves> by the amoebae it contains and the diffusion of this waves out of the cluster.
5. Excitation waves <propagate> at the speed of one cell per unit of time without attenuation.
6. The propagation of waves results from the application of simple <local rules>. These rules stipulate how to update the state of the cells and how to move the amoebae they contain to the neighbouring cells.

Among interesting properties observed in the system, it was shown that the gathering could also occur in the presence of obstacles on the lattice. The gathering process was not much perturbed as the virtual amoebae could take advantage of narrow corridors to find their way to a pacemaker.

**Robotics Perspectives** The main advantage of our approach to group robots is the simplicity of programming: robots should only follow the excitations in the right direction (from neutral to excited). Moreover, as the intensity of the excitations is constant, there is no problem of signal attenuation with the distance to the emitting robot. The difficulty is to implement an active environment that receives the excitations from the robots and propagate them across the space without creating any new parasitic waves. This could be realised by small components regularly placed in space that would for example emit and receive light signals. Note that as we do not use any memory or specialisation in the robots, the system is naturally self-stabilising: an unwanted displacement of robots or a temporary or definitive failure should not perturb the gathering process.

**Computer Science Perspectives** Our amoebae model should now be compared with other bio-inspired models such as virtual ants. Virtual ants use simple diffusion and chemotaxis to realise complex tasks in a decentralised way. They were used to solve various complex problems such as the Travelling Salesman Problem [6]. However, to our knowledge, the use of simple diffusion and chemotaxis is delicate in the cases where the ants are not always moving. The difficulty is to tune the diffusion and evaporation to prevent agents to be "trapped" by their own emission of pheromones. Furthermore, as the intensity of pheromones decreases exponentially in space, the aggregation process might become difficult to achieve on arbitrary large distances.

The advantage of reaction-diffusion over classical diffusion is that waves propagate without attenuation and over arbitrary large distances; the drawback as seen in Section 4, is that messages have to be relayed synchronously and perfectly in order to prevent the creation of self-entertained excitation waves. How to make the environment more robust without complicating too much the local rules? Biology is a possible source of inspiration as spiral waves are also used by real amoebae to complete the aggregation stage.

**Back to Biological Modelling** There is a significant difference between our model and the real amoebae: in nature, the reaction-diffusion process is not implemented by the environment but by the amoebae themselves (see Section 2). Despite this common simplification, our model reproduces qualitatively the competition between the pacemakers to form an aggregate. The model also shows that the formation of streams of amoebae does not necessitate much communication between cells but might be a mere consequence of the global dynamics of the system.

To conclude, we recall that the propagation of excitation waves in the environment is subject to a phase transition, which we characterised as belonging to the directed percolation universality class. This class is known to be robust to small variations of the model. Using similar models, statistical physics may shed light on how real amoebae collectively decide to start the gathering stage. With its simplicity, the Reaction-Diffusion-Chemotaxis scheme appears as a good starting point to examine whether phase transitions may create a form of consensus in amoebae-like societies.

# 8 Acknowledgements

The author is grateful to anonymous referees who commented a first version of this paper and to the many persons that read the manuscript with attention, among whom H. Berry, S. Contassot-Vivier, B. Girau and P. Flocchini. Part of the research was supported by the ARC AMYBIA grant by INRIA,see:
`http://www.loria.fr/~fates/Amybia/project.html`

# References


[1] Harold Abelson, Don Allen, Daniel Coore, Chris Hanson, George Homsy, Jr. Thomas F. Knight, Radhika Nagpal, Erik Rauch, Gerald Jay Sussman, and Ron Weiss, *Amorphous computing*, Commun. ACM **43** (2000), no. 5, 74–82.

[2] Andrew Adamatzky, *Computing in nonlinear media and automata collectives*, ISBN 075030751X, Institute of Physics Publishing, 2001.

[3] Pankaj Agarwal, *Simulation of aggregation in* Dictyostelium *using the cell programming language*, Computer Applications in the Biosciences **10** (1994), no. 6, 647–655.

[4] Hidekei Ando, Yoshinobu Oasa, Ichiro Suzuki, and Masafumi Yamashita, *Distributed memoryless point convergence algorithm for mobilerobots with limited visibility*, IEEE Transactions on Robotics and Automation, 1999.

[5] Hendrik J. Blok and Birger Bergersen, *Synchronous versus asynchronous updating in the "game of life"*, Physical Review E **59** (1999), 3876–9.

[6] Eric Bonabeau, Marco Dorigo, and Guy Theraulaz, *Swarm intelligence - from natural to artificial systems*, Oxford University Press, 1999.

[7] John C. Dallon and Hans G. Othmer, *A discrete cell model with adaptive signalling for aggregation of* Dictyostelium Discoideum, Philosohical Transactions of the Royal Society B **352** (1997), 391–417.

[8] Andreas Deutsch and Sabine Dormann, *Cellular automaton modeling of biological pattern formation characterization, applications, and analysis*, Modeling and Simulation in Science, Engineering and Technology, Birkhäuser, 2005, ISBN: 978-0-8176-4281-5.

[9] G. DeYoung, P. B. Monk, and H. G. Othmer, *Pacemakers in aggregation fields of dictyostelium discoideum: does a single cell suffice?*, Journal of Mathematical Biology **26** (1988), no. 5, 487–517, http://www.springerlink.com/content/h096p3nnhr8w4631.

[10] Nazim Fatès, *Asynchronism induces second order phase transitions in elementary cellular automata*, To appear in *Journal of Cellular Automata* `http://hal.inria.fr/inria-00138051`.

[11] ———, *Fiatlux CA simulator in Java*, See `http://nazim.fates.free.fr` for downloading.

[12] Robert Fisch, Janko Gravner, and David Griffeath, *Metastability in the Greenberg-Hastings model*, The Annals of Applied Probability (1993), no. 4, 935–967.

[13] Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, and Peter Widmayer, *Gathering of asynchronous robots with limited visibility*, Theor. Comput. Sci. **337** (2005), no. 1-3, 147–168.

[14] J. M. Greenberg, B. D. Hassard, and S. P. Hastings, *Pattern formation and periodic structures in systems modeled by reaction-diffusion equations*, Bulletin of the American Methematical Society **84** (1978), no. 6, 1296–1327.

[15] Haye Hinrichsen, *Nonequilibrium critical phenomena and phase transitions into absorbing states*, Advances in Physics **49** (2000), 815–958.

[16] Alvy Ray Smith III, *Two-dimensional formal languages and pattern recognition by cellular automata*, FOCS, IEEE, 1971, pp. 144–152.

[17] Seido Nagano, *Modeling the model organism* Dictyostelium discoideum, Development Growth and Differentiation **42** (2000), no. 6, 541–550.

[18] Giuseppe Prencipe, *Impossibility of gathering by a set of autonomous mobile robots*, Theor. Comput. Sci. **384** (2007), no. 2-3, 222–231.

[19] Kazuo Sugihara and Ichiro Suzuki, *Distributed motion coordination of multiple mobile robots*, 5th IEEE International Symposium on Intelligent Control, 1990, pp. 138–143.

[20] Bakhtier Vasiev Till Bretschneider and Cornelis J. Weijer, *A model for cell movement during* Dictyostelium *mound formation*, Journal of Theoretical Biology **189** (1997), no. 1, 41–51.

[21] O. O. Vasieva, B. N. Vasiev, V. A. Karpov, and A.N. Zaikin, *A model of* Dictyostelium discoideum *aggregation*, Journal of Theoretical Biology **171** (1994), no. 4, 361–367.