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Abstract

Classifying electroencephalography (EEG) signals is an important step for proceeding EEG-based brain computer interfaces (BCI). Currently, kernel based methods such as the Support Vector Machine (SVM) are considered the state-of-the-art methods for this problem. In this paper, we apply Gaussian Process (GP) classification to binary discrimination of motor imagery EEG data. Compared with the SVM, GP based methods naturally provide probability outputs for identifying a trusted prediction which can be used for post-processing in a BCI. Experimental results show that the classification methods based on a GP perform similarly to kernel logistic regression and probabilistic SVM in terms of predictive likelihood, but outperform SVM and K-Nearest Neighbor (KNN) in terms of 0-1 loss class prediction error.

Keywords: Gaussian Process; Brain Computer Interfaces; Support Vector Machine; EEG

1 Introduction

Brain Computer Interface (BCI) is a new technique that translates specific electrophysiological signals from mere reflections of central nervous system (CNS) into specific commands, aiming at accomplishing the intent of the people who lost their voluntary muscle control [21]. A variety of methods, such as electroencephalography (EEG), magnetoencephalography (MEG), electrocorticography (ECoG), positron emission tomography (PET), functional magnetic resonance imaging (fMRI) and optical imaging, could be used for monitoring those electrophysiological signals related to brain activities. However, at present it is likely that EEG and related methods are the most popular methods for offering a practical BCI. Classifying EEG is a main task in the translation algorithm step of an EEG-based BCI. Recent reviews have shown that most common classification methods which are largely used in BCI are non-probabilistic methods, and among which the Support Vector Machine (SVM) is likely to be an efficient one and has been popularly employed for classifying EEG in BCI [12, 8, 1]. It has been known that the class predictive probability outputs of a new feature vector are of importance in practical recognition circumstances [3, 15]. Unfortunately, the SVM does not naturally provide this quantity, and an additional technique for translating the SVM outputs into probabilities has been proposed [15]. However, this method may not really give a good approximation of the class predictive probabilities.

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Both kernel logistic regression (KLR) [17] and Gaussian Process (GP) [13, 20, 10, 16] methods can naturally give predictive probabilities for classification problems. Both methods require tuning of the kernel functions, which could be achieved by using, for example, k-fold cross validation. For KLR, the non-linear functions building the relationships between targets and feature vectors associate with some weights, which require to be estimated using the training data. So obviously KLR is a parametric method. From the Bayesian perspective those weights are assumed to follow some prior distributions such as a Gaussian prior with one variance parameter requiring to be tuned. In contrast, a GP is a non-parametric method and thus no weights are required to be estimated. For GP based methods, the non-linear functions are defined by Gaussian Process priors with associated covariance functions. So the hyper-parameters of the covariance functions are the main parameters which have to be tuned. Considering these advantages, the GP is considered for EEG classification problems in this paper. Exact inference methods are impossible for GP classification, and various approximation methods have recently been developed [13, 20, 5, 4]. Several approximation methods for Gaussian process classification are employed in this paper and the experimental results suggest that across all the data sets employed, all GP based approximation methods consistently give statistically similar results and outperform SVM and K-Nearest Neighbor (KNN) in terms of 0-1 loss class prediction error. It is known that KLR, the probabilistic SVM (pSVM) of Platt, and GP based methods, which can give probability outputs, are essentially kernel based methods. Hence for comparison purposes, both KLR and pSVM were also applied to the same EEG data sets. Despite the advantages of GP described, no significant differences in terms of predictive likelihood were shown between these probabilistic methods when applyed to the three EEG data sets.

2 Gaussian Process for Binary Classification

The GP model for binary classification is described in this section. Suppose a feature vector $\mathbf{x} \in \mathbf{R}^{D\times 1}$ corresponds to a binary class variable $t \in \{-1, 1\}$. We have N such observations denoting $\mathcal{D} = \{(\mathbf{x}_i, t_i)\}_{i=1}^N$, and conveniently denote $\mathbf{X} = (\mathbf{x}_1, \cdots, \mathbf{x}_N)^T$, $\mathbf{t} = (t_1, \cdots, t_N)^T$. The aim is to infer a classifier by using the observations and then assign a new feature vector $\mathbf{x}^*$ to one of the two classes with a certain agreement. In order to make prediction, we make a function transformation for the feature vectors such that $\mathbf{f} : \mathbf{X} \longrightarrow \mathbf{f}(\mathbf{X})$. Note that $\mathbf{f}(\mathbf{X}) = (f_1(\mathbf{x}_1), \cdots, f_N(\mathbf{x}_N))^T$ and for simplicity $f_i(\mathbf{x}_i)$ is denoted as f_i. Rather than specifying an explicit form for each of the functions f_i we assume that this nonlinear transformation corresponds to a GP prior such that $\mathbf{f}(\mathbf{X}) \sim \mathcal{N}_{\mathbf{f}}(\mathbf{0}, \mathbf{C}_{\boldsymbol{\theta}}(\mathbf{X}, \mathbf{X}))$ where $\mathbf{C}_{\boldsymbol{\theta}}(\mathbf{X}, \mathbf{X})$ is the covariance matrix defined by kernel functions which are related to a set of hyper-parameters $\boldsymbol{\theta}$. It should be noted that the ijth element of $\mathbf{C}_{\boldsymbol{\theta}}(\mathbf{X}, \mathbf{X})$ can be defined by some kernel functions, e.g. the Gaussian kernel $c(\mathbf{x}_i, \mathbf{x}_j) = \exp\{-\phi \sum_{d=1}^D (x_{id} - x_{jd})^2 + \lambda\}$ where we denote the hyper-parameters $\boldsymbol{\theta} = \{\phi, \lambda\}$. Other kernels could also be used, see e.g. [10]. Four methods can be employed for approximating the required posterior distribution over $\mathbf{f}$, i.e. the Gibbs sampler [4], variational Bayes [4], expectation propagation [11, 5] and the Laplace approximation [20]. These approximations correspond to different classifiers, which are shortly described as follows.

For the Gibbs sampler and variational Bayes approaches, we employ an auxiliary variable vector $\mathbf{y} = (y_1, \cdots, y_N)^T$ for the noise model such that $y_n = f_n(\mathbf{x}_n) + \mathcal{N}(0, 1)$ which defines a non-linear regression between $\mathbf{y}$ and $\mathbf{X}$. The relationship between y_n and t_n is as follows:

$$t_n = -1 \quad \text{if} \quad y_n < 0; \qquad t_n = 1 \quad \text{if} \quad y_n \geq 0. \tag{1}$$

The posterior over the hidden variables can be represented as follows using Bayes' rule

$$p(\mathbf{f}, \mathbf{y}|\mathcal{D}, \boldsymbol{\theta}) = \frac{P(\mathbf{t}|\mathbf{y})p(\mathbf{y}|\mathbf{f}, \mathbf{X})p(\mathbf{f}|\mathbf{X}, \boldsymbol{\theta})}{\int\int P(\mathbf{t}|\mathbf{y})p(\mathbf{y}|\mathbf{f}, \mathbf{X})p(\mathbf{f}|\mathbf{X}, \boldsymbol{\theta})d\mathbf{y}d\mathbf{f}} \tag{2}$$

The Gibbs sampler and variational Bayes have been developed for approximating this joint posterior by using an approximating ensemble of factored posteriors such that $p(\mathbf{f}, \mathbf{y}|\mathcal{D}, \boldsymbol{\theta}) \approx Q(\mathbf{f})Q(\mathbf{y})$, details of which can be found in [4]. It has been shown that given a new feature vector $\mathbf{x}^*$, the predictive probability of it belonging to class 1 can be represented as [4]

$$P(t^* = 1|\mathcal{D}, \boldsymbol{\theta}, \mathbf{x}^*) = \Phi\left(\frac{\widetilde{f}^*}{\sqrt{1 + \widetilde{\sigma}_*^2}}\right) \tag{3}$$

with $\widetilde{f}^* = \widetilde{\mathbf{y}}^T \left(\mathbf{I} + \mathbf{C}_{\boldsymbol{\theta}}\right)^{-1} \mathbf{c}^*$ where $\mathbf{c}^* = (c(\mathbf{x}_1, \mathbf{x}^*), \cdots, c(\mathbf{x}_N, \mathbf{x}^*))^T$ and $\widetilde{\mathbf{y}}^T$ is the expectation of $Q(\mathbf{y})$, and $\widetilde{\sigma}_*^2 = c^* - (\mathbf{c}^*)^T \left(\mathbf{I} + \mathbf{C}_{\boldsymbol{\theta}}\right)^{-1} \mathbf{c}^*$ where $c^* = c(\mathbf{x}^*, \mathbf{x}^*)$. Note that $\Phi(\cdot)$ denotes the cumulative function of the standard Normal distribution, i.e. the *probit* function.

As an alternative approach, the hidden vector $\mathbf{y}$ in (2) can be integrated out such that,

$$P(\mathbf{t}|\mathbf{f}) = \int P(\mathbf{t}|\mathbf{y})p(\mathbf{y}|\mathbf{f})d\mathbf{y} = \prod_{n=1}^{N} \int P(t_n|y_n)p(y_n|f_n)dy_n = \prod_{n=1}^{N} \Phi(t_n f_n) \tag{4}$$

Note that this *probit* function could be directly replaced by a *logistic* function such that,

$$P(\mathbf{t}|\mathbf{f}) = \prod_{n=1}^{N} \mu(t_n f_n) = \prod_{n=1}^{N} \frac{1}{1 + \exp(-t_n f_n)} \tag{5}$$

Therefore, the posterior distribution function over $\mathbf{f}$ given hyper-parameters can then be represented as follows

$$p(\mathbf{f}|\mathcal{D}, \boldsymbol{\theta}) = \frac{P(\mathbf{t}|\mathbf{f})p(\mathbf{f}|\mathbf{X}, \boldsymbol{\theta})}{\int P(\mathbf{t}|\mathbf{f})p(\mathbf{f}|\mathbf{X}, \boldsymbol{\theta})d\mathbf{f}} = \frac{\mathcal{N}_{\mathbf{f}}(\mathbf{0}, \mathbf{C}_{\boldsymbol{\theta}}) \prod_{n=1}^{N} \lambda(t_n f_n)}{p(\mathbf{t}|\boldsymbol{\theta})} \tag{6}$$

where $\lambda(\cdot)$ denotes the *probit* or *logistic* function. For a new feature vector $\mathbf{x}^*$, the predictive likelihood of it belonging to class 1 can be represented as follows

$$P(t^* = 1|\mathcal{D}, \boldsymbol{\theta}, \mathbf{x}^*) = \int P(t^* = 1|f^*)p(f^*|\mathcal{D}, \boldsymbol{\theta}, \mathbf{x}^*)df^* \tag{7}$$

where

$$p(f^*|\mathcal{D}, \boldsymbol{\theta}, \mathbf{x}^*) = \int p(f^*|\mathbf{f}, \mathbf{X}, \boldsymbol{\theta}, \mathbf{x}^*)p(\mathbf{f}|\mathcal{D}, \boldsymbol{\theta})d\mathbf{f} \tag{8}$$

Note that the posterior distribution $p(\mathbf{f}|\mathcal{D}, \boldsymbol{\theta})$ is non-Gaussian which makes the predictive distribution analytically intractable. Various approximations are required to be employed to represent it as a Gaussian form such that $p(\mathbf{f}|\mathcal{D}, \boldsymbol{\theta}) \approx \mathcal{N}_{\mathbf{f}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. These include the Laplace approximation [20] and expectation propagation (EP) [11, 5]. It should be noted that for the reason of analytic convenience the EP approximation could only be used for the *probit* model. In contrast, the Laplace approximation can be used for both noise models. So for the *probit* and *logistic* noise

models, the Laplace approximations are respectively denoted as probit and logistic Laplace approximations. Details of the derivations can be found in [16, 5]. Therefore, equation (8) could be represented as a gaussian form such that $p(f^*|\mathcal{D}, \boldsymbol{\theta}, \mathbf{x}^*) \approx \mathcal{N}(\widetilde{f}^*, \widetilde{\sigma}_*^2)$ where $\widetilde{f}^* = (\mathbf{c}^*)^T \mathbf{C}_{\boldsymbol{\theta}}^{-1} \boldsymbol{\mu}$ and $\widetilde{\sigma}_*^2 = c^* - (\mathbf{c}^*)^T \left(\mathbf{C}_{\boldsymbol{\theta}}^{-1} - \mathbf{C}_{\boldsymbol{\theta}}^{-1} \boldsymbol{\Sigma} \mathbf{C}_{\boldsymbol{\theta}}^{-1}\right) \mathbf{c}^*$ where c^* and $\mathbf{c}^*$ have the same meanings with those in equation (3). For the *probit* model, the predictive likelihood (7) can then be computed analytically, resulting in a closed form of (3). However, for the *logistic* noise model, we need to resort to sampling methods or analytical approximations to compute this predictive likelihood [20, 16]. These approximations will be used to infer classifiers for the EEG data sets described below.

3 Data Sets

The data used for this study correspond to the EEG data set IIIb of the BCI competition III [2]. This data set gathers the EEG signals recorded for three subjects who had to perform motor imagery, i.e. to imagine left or right hand movements. Hence, the two classes to be identified were "Left" and "Right".

The EEG were recorded by the Graz team, using bipolar electrodes C3 and C4 (that are located over the motor cortex area), and were filtered between 0.5 and 30 Hz. Subject 1 took part in a virtual reality experiment [6] where the detection of left or right imagined hand movements triggered a camera rotation towards the left or right respectively, in a virtual room. Subjects 2 and 3 took part in a "basket" experiment where the detection of left or right hand movements made a falling ball displayed on the screen, move towards the left or the right. The aim was to reach one of the two baskets located at the bottom left and bottom right of the screen [19].

For subject 1, 320 trials were available in the training set, whereas the test set was composed of 159 trials. For subjects 2 and 3, both the training and the test sets were composed of 540 trials. Each trial was 8 seconds long, and was divided as follows: during the first two seconds, a blank screen was presented to the subject. At second 3, a visual cue was presented to the subject in order to tell him which imagined hand movement he should start performing immediately. Finally, the data from second 4 to 8, for subject 1, or from second 4 to 7 for subjects 2 and 3, were used to provide feedback to the subject, according to the imagined hand movement detected. This feedback was either the rotation of the virtual environment, for subject 1, or the movement of the ball for subjects 2 and 3. More details about this data set can be found in [2].

4 Feature Extraction

For further classification, it is first necessary to extract features from these EEG signals. In order to do so, we chose to use Band Power (BP) features. Such features correspond to the power of the signal, in specific frequency bands. They are simply obtained by band-pass filtering the signal, squaring it and averaging it over a given time window [14]. Such features are very popular and efficient for motor imagery as imagination of hand movements is known to cause amplitude changes in the α ($\approx$ 8-13 Hz) and β ($\approx$ 16-24 Hz) rhythms, over the motor cortex areas [14].

The main drawback of such features is that subject-specific frequency bands, in which is to be

computed the BP, must be identified before use. Actually, the optimal frequencies for discriminating between left and right hand movements vary from subject to subject [14]. Moreover, and independently from the features used, it is necessary to identify, for each subject, the optimal time window in which to extract the features in order to achieve maximal discrimination. This time window is located, for each trial, after the start of the feedback presentation, i.e., after second 4. It is indeed the period in which the subject is performing motor imagery.

To achieve these two goals, we used a method based on statistical analysis which was successfully used in previous BCI studies [7, 9]. It should be noted that these calibration steps were performed before entering the classification procedures with the aim of identifying the frequency bands and the time window to be used. Once identified, these frequency bands and the time window will be used without modification in the classification procedures.

To identify the subject-specific frequency bands, we used a paired t-test which compared the BP means between both classes, for every 2 Hz wide frequency band between 1 Hz and 30 Hz, with a step of 1 Hz. As expected from the literature [14], the frequencies for which the BP achieved the best discrimination were found in the α and β bands, which supports the use of such features (see Fig. 1).

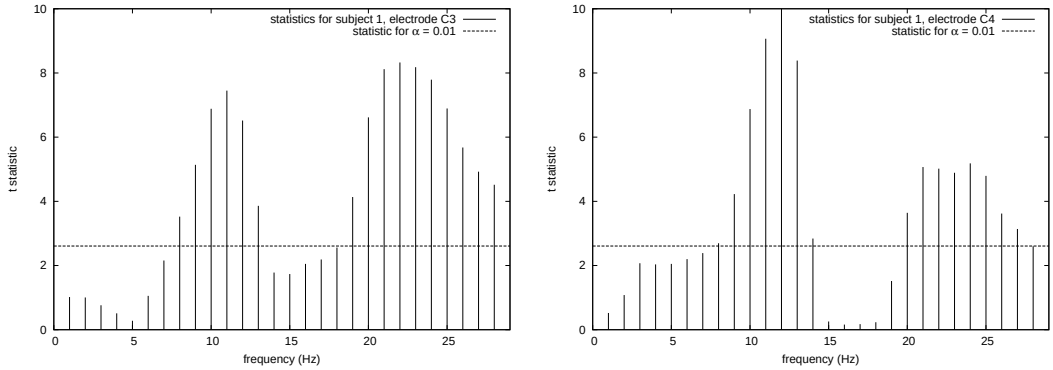

Figure 1: T statistics obtained with the BP features extracted for each frequency, for electrodes C3 (on the left) and C4 (on the right) for Subject 1, in the optimal time window (see below for the determination of this time window). The dashed line represents the significance threshold for $\alpha = 0.01$.

Adjacent significant frequencies (with probability of type I error below $\alpha = 0.01$) were gathered into a single frequency band. Then, for every frequency band, a *shrinking* step was performed which consisted in reducing the frequency band (making it 1 Hz shorter) and computing a new statistic for this band. If the new statistic was higher than the previous one, the shrunk frequency band was selected. The shrinking process was repeated until the statistics could not be increased any further.

To identify the optimal time window in which to extract the BP features, we performed the statistical analysis mentioned above for several time windows, and selected the one with the highest mean value of significant statistics. The parameters used for BP feature extraction are summed up in Table 1. In this table, the window start value is given in seconds after the start of the feedback presentation.

Table 1: Parameters of band power feature extraction for each subject.

Subject	C3 α band (Hz)	C3 β band (Hz)	C4 α band (Hz)	C4 β band (Hz)	window start (s)	window length (s)
1	11	21-29	11-13	21-27	0.4	2.5
2	8-13	20-24	11-14	20-29	1.4	1.5
3	9-12	21-22	11-12	18-25	1.4	1.5

Thus, this BP feature extraction method represents each trial by a four dimensional feature vector: $[C3_\alpha, C3_\beta, C4_\alpha, C4_\beta]$ in which Cp_y is the BP value for electrode Cp in the y band. These feature vectors will be used as input data for the following classification step.

5 Results

Five different approximation methods for GP classification were applied to the three data sets described in section 3, using the Band Power features presented in section 4. The approximation methods employed are expectation propagation (EP) [11, 5], variational Bayes (VB) [4], Gibbs sampling [4] and the Laplace approximation [20]. For Laplace approximation we consider both the probit and logistic functions in the noise model. For comparison purposes, SVM, KNN, pSVM and KLR were also employed for tackling this problem. For KLR, each weight was assumed to follow a Gaussian distribution with zero mean and variance σ^2. Note that it would be interesting to consider a sparse prior such as a Laplacian or a student-t distribution, which induces sparsity of the weights. After the prior was fixed, the weights were then estimated using the Laplace approximation. For all the methods used here, we employ the kernel function $c(\mathbf{x}_i, \mathbf{x}_j) = \exp\{-\phi \sum_{d=1}^{D}(x_{id} - x_{jd})^2 + 2\}$ where one hyper-parameter ϕ is required to be tuned. In order to obtain the relatively optimal hyper-parameters, ten-fold cross-validation (CV) was employed for tuning them. Note that one more hyper-parameter C, i.e. the box constraint parameter, was also optimized using ten-fold CV for SVM, and also the parameter k of KNN, i.e. the number of neighbors. After the hyper-parameters were selected using the training data sets, the classifier was obtained by learning the training data set and then applied to the test data set. The results of the log-predictive likelihood (PL) and the 0-1 loss class prediction error (PE) of those methods employed are shown in Table 2. Note that the PL and PE are respectively defined as $\frac{1}{M}\sum_{i=1}^{M} \log\{P(t_i^* = t_{true}^i | \mathcal{D}, \boldsymbol{\theta}, \mathbf{x}_i^*)\}$ and $\frac{100}{M}\sum_{i=1}^{M} \mathcal{I}(t_{pred}^i \neq t_{true}^i)$, where M is the number of test samples, t_{true}^i and t_{pred}^i denote the true and predicted labels of the i^{th} test sample respectively, and $\mathcal{I}(\cdot)$ denotes the indicator function. The results show that there are no obvious differences in terms of predictive likelihood between those probabilistic methods employed for the current binary classification problems. However, the results show that the GP outperforms SVM in terms of prediction error across all the data sets. On the other hand, except for the third data set, the GP is superior to KNN in terms of the prediction error. As a by-product we collected the total prediction time of each learned classifier when applied to the test data sets. The results suggest that except for the Gibbs sampler all the classification methods are likely to be efficient enough for some real time applications such as a BCI system. Note that the experiments presented here were done using Matlab-6.5 under Windows XP, running on an

	S1			S2			S3		
	PL	PE	PT	PL	PE	PT	PL	PE	PT
EP	-0.340	**10.691**	0.329	-0.556	28.333	2.187	-0.483	24.814	2.172
VB	-0.340	**10.691**	0.047	-0.542	27.592	0.5	-0.491	24.814	0.437
GIBBS	-0.375	**10.691**	53.125	-0.540	27.037	123.891	-0.482	25.740	125.75
PL	-0.342	**10.691**	0.203	-0.561	27.962	1.938	-0.487	24.814	1.5
LL	-0.341	11.320	0.047	-0.542	27.222	0.47	-0.484	24.814	0.37
PSVM	-0.384	15.094	0.015	-0.542	**25.556**	0.078	-0.540	25.370	0.078
KLR	-0.359	12.578	0.016	-0.558	27.407	0.109	-0.483	25.370	0.094
SVM	-	13.836	0.016	-	29.074	0.078	-	25.555	0.062
KNN	-	14.465	0.078	-	37.777	0.172	-	**24.629**	0.171

Table 2: The prediction error of SVM and KNN, and the log-predictive likelihood (PL) and prediction error (PE) of Gibbs sampler, EP, VB, probit Laplace (PL) and logistic Laplace (LL) approximations, the probabilistic SVM of Platt [15] (pSVM) and kernel logistic regression (KLR), when applied to the data sets obtained from Subject 1 (S1), Subject 2 (S2) and Subject 3 (S3). The total prediction time (PT) in seconds of each learned classifiers when applied to the test data sets are also shown. Best results are highlighted in bold.

Intel Pentium 4 CPU 3.4GHz, with 1GB RAM.

6 Discussions and Conclusions

Binary classification based on GP's has been applied to EEG. Experimental results have shown that all the approximate GP methods employed in this paper give similar performance on the EEG data in terms of predictive likelihood. It has been shown that the GP outperforms SVM and KNN in terms of prediction error on the EEG data sets employed. Experimental results indicate that no evidence was sufficient to suggest that KLR or pSVM are superior to the GP based methods in terms of predictive likelihood. Therefore, following the advantages of GP described in the introduction, we suggest using GP based methods in BCI. Furthermore, when classifying a new test sample to one of the k classes, the classifier which can produce predictive probabilities of the test sample is of great utility in practical recognition circumstances. This posterior probability, which can facilitate the separation of inference and decision, essentially represents the uncertainty in the prediction in a principal manner [3]. The SVM only produces a threshold value which is not a probability for making a decision for a new test sample. We argue this is a potential shortcoming for the SVM in BCI applications. By contrast, as we have seen, the GP-based classification method can naturally produce posterior probabilities. Importantly, the predictive probabilities can be used for further processing for a BCI system. For example, this quantity can isolate the test feature vector which has great uncertainty with similar class posterior probability values of binary classification problems. In this case, the subject might not attend the designed tasks and the data sample is not suitable for further use in a BCI system. In our observation, there is a case that the predictive probabilities of misclassification for some data samples are very high. The reason might be that the subject was actually doing an opposite task with respect to the expected one. Imagine a motor task of imaging left (class -1) and right (class $+1$) hand movements in an experiment, the subject was asked to imagine left hand movement for instance. Unfortunately, the subject actually imagined right hand movement which is opposite to the task. The predictive probability of classifying the sample to class $+1$ in this case should be very high, though the data sample is labeled as class -1. Besides, GP-based

classification could be used for an asynchronous BCI, in which no cue stimulus is used and the subject can intend a specific mental activity as he wishes. The posterior probability can then be used as a quantity for detecting the mental events and discriminating them from noise and nonevents [18]. These have shown that GP provides a suitable quantity for further processing for a BCI.

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