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Abstract: We investigate the performance of flooding in mobile ad hoc networks when the relay set is a set of nodes uniformly dispatched with density D. We give some bounds on the conditions under which a flooding will actually spread (or percolates) when the relay set density is above a certain threshold that depends on traffic density.

Key-words: wireless mesh networks, flooding, access points, performance and models

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Optimisation de synchronisation point à point par ensemble de liens overlay RNG dans les réseaux mobiles ad hoc

Résumé : Nous nous intéressons aux performances de l'inondation dans les réseaux mobiles *ad hoc* quand l'ensemble des postes relais est uniformément distribué avec densité D. Nous donnons des bornes théoriques sur les conditions pour que l'inondation se propage (ou percole) quand la densité des relais est au dessus d'un seuil qui dépend dela densité de trafic.

Mots-clés : réseaux sans fil maillés, inondation, points d'accès, performances et modèles

1 Introduction

The problem of the management of distributed databases is critical in mobile ad hoc networks where mobility is high and communication ressource is scarse. Example: LSA database synchronization in an hybrid OSPF network. We introduce the Synchronized Link Overlay (SLO) concept where only a subset of the links are used for database synchronization.

Principle of synchronization: Assume that the link between node A and node B belongs to the synchronized overlay, we say that the link is synchronized. Upon a database event or when the link is newly included in the SLO, or more generally any event that triggers a suspicion of synchronization loss, nodes A and Bcompare their databases. The comparisons are done pairwise on the links of the synchronized link overlay. The database can be large and the comparison may lead to heavy overhead, although the comparison could be limited on data header such as it is done in OSPF LSA database, or on an hierarchy of signatures.

Since in a wireless network the number of links can be high, when the node density is heavy, it may be critical to reduce the size of the SLO. However the SLO must connect all nodes in the same connected component. The number of links is lower bounded by the number of nodes (minus one). If the number of links per node is close to one, then the main benefit of a link synchronization overlay is in the fact that the selected links will have the highest link quality, therefore drastically reducing the cost of synchronization.

The most striking example where SLO algorithm shows breakthrough performance is with wireless networks with multiple radio interfaces. Consider a network made of nodes with two interfaces a shot range interface (e.g. wifi at 100 Mbps over 10 m) and a longer range interface (e.g. Wifi at 1 Mbps over 100 m). In this case it is most conevenient to use chain of 100 Mbps instead of using a single hop at 1 Mbps which will use more time and energy for sychronization exchanges.

Future radio networks will operate on on-line programmable radio-interface which makes the throughput to naturally increases when the nodes are closer. In this case the cost metric can be an increasing function of the distance.

In this paper we present and analyse SLO algorithms inspired from Relative Neighbor Graph (RNG [2]). This work is inspired from the fundamental work in [4] which applies the RNG technique to mobile ad hoc networking. This paper is divided into three parts: an introduction to link synchronization overlay based on RNG; an analytic evaluation of the performance of the synchronization overlay based on RNG in the unit disk graph model on an infinite plane; a simulation of these performances on finite networks.

2 Description of SLO algorithms based on RNG

Since the synchronization is done symmetrically on synchronized links, the links must be symmetric. Let $m(A, B)$ the metric on the symmetric link (A, B). We have $m(A, B) = m(B, A)$. For example we take $m(A, B) = \min\{\text{Cost}(A, B), \text{Cost}(B, A)$, where Cost(.) is a specific metric (for example bit delivery delay, remaining bandwidth, financial cost).

The following algorithms are inspired from RNG graph [2, 4]. The RNG graph is inspired from the Gabriel graph [1] but the former does not need positioning device, only link cost estimate.

2.1 SLO-1HOP algorithm

2.1.1 General case

Link (A, B) does not belong to the SLO when there exist a chain of link $(A, C_1), \ldots,$ $(C_i, C_{i+1}), \ldots,$ (C_k, B) such that:

- The C_i are all neighbors of both A and B
- The cost of the links, $m(A, C_1), \ldots, m(C_i, C_{i+1}), \ldots, m(C_k, B)$, are all smaller to $m(A, B)$.

Tie breaking. In case of link cost equality the following tie breaking is proposed: the link with the lower cost will be the link with the minimum node ID. In case this does tie the break (because the two links have their minimum ID node in common), then the tie is not broken and none of the links are removed from the SLO.

To simplify we can use of SLO-T which looks only at triangular relations: link (A, B) is not selected if and only if there exist a common neighbor node C such that $m(A, B) \geq \max\{m(A, C), m(C, B)\}$. This can be used when the metric satisfies the triangular inequality: $m(A, B) \leq m(A, C) + m(C, B)$.

2.1.2 Uniform link cost

When the link have all the same cost, in this case a link (A, B) is removed from the SLO when:

- there exist a node C neighbor to both A and C;
- node C ID is smaller to both node A and node B ID's.

In this case the elimination is equivalent to a triangle elimination: the link holding the two largest node ID is eliminated.

2.2 SLO two-hops

There can be a variant with 2-hop neighbor: SLO-2HOP. In this case it is required that the chain of links $(A, C_1), \ldots,$ $(C_i, C_{i+1}), \ldots,$ (C_k, B) are all both neighbors to A and B, *i.e.* all links have at least one end neighbor to A and one end neighbor to B, possibly the same end.

In case of equal link case, this is equivalent to a square elimination. On all square configuration A, B, C, D we eliminate the link which connect the two largest node ID. Notice that this link may not exist.

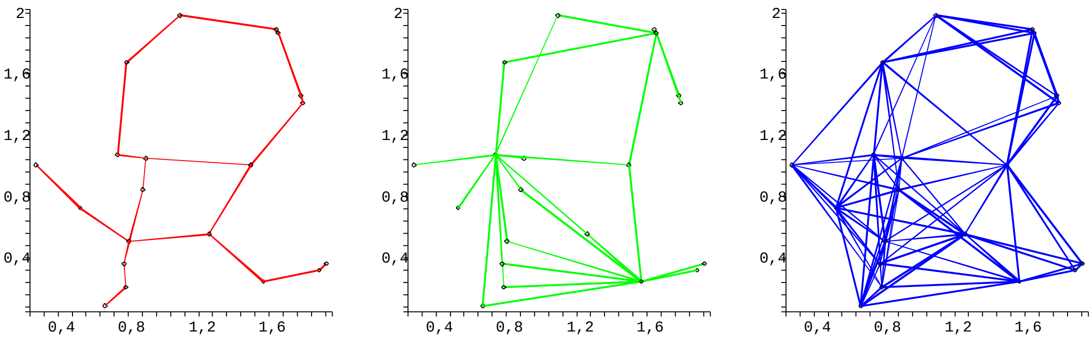

Figure 1: Overlay links with SLO based on distance cost like RNG (left), uniform cost (middle), all links (right), with 20 nodes on a 2×2 map

The 2-hop version is not expected to perform significantly better than the 1-hop version and is mentioned for completeness.

The 2-hop equivalent of the SLO-T is the SLO-Q algorithm when link (A, B) is not advertized if and only if there exists no common neighbor C such that

$$m(A, B) \geq \max\{m(A, C), m(C, B)\}$$

or if there exists a neighbor C of A and a neighbor D of B such that

$$m(A, B) \geq \max\{m(A, C), m(C, D), m(D, B)\}.$$

Figure 1 shows two examples of SLO, one with cost based on distance and the second with uniform cost. Both algorithm are simplified SLO-1HOP restricted to triangular elimination (option T).

3 Theoretical performance of SLO algorithm

In the following we consider the unit-disk graph model of uniform node density ν and we consider an infinite plan. Therefore the average node neighborhood size is $M_f = \pi\nu$. We will consider asymptotic case where $\nu \to \infty$. We will only analyse the performance of triangle version of SLO algorithm.

We assume that each node follows an independent random walk equivalent to an average speed of s. We assume that the random walk is isotropic, so that the stationary probability that a node is in a portion of the map of area σ and has speed in a cone of aperture θ is exactly $\frac{\theta\sigma}{2\pi\mathcal{A}}$.

Let us consider a portion of the map B whose border length is ℓ. We know from [5] that the rate $f(B)$ at which nodes enter the portion B is proportional to the node density, the length of the border and the average speed of the mobile, that is $f(B) = \frac{\ell s}{\pi}\nu$. The average sojourn time in the portion B is such that $T(B) = \frac{b\pi}{\ell s}$ where b is the area of B.

If we consider that the portion of the map B also moves like a mobile node, then the rate at which mobile nodes enter the mobile area becomes $\frac{\ell\Delta(s)}{\pi}\nu$ where $\Delta(s)$ is average relative speed beween the mobile area and the mobile node. If the map and the mobile node move at constant speed we get $\Delta(s) = \frac{4}{\pi}s$, otherwise the formula is more complicated. From now we assume that nodes moves at constant speed (with arbitrary changes of direction).

The rate at which nodes enter a node neighborhood is $V_f = \frac{8}{\pi}s\nu$.

Therefore we have proven the following theorem.

Theorem 1 *The average number of links departing from a node is equal to $M_f = \pi\nu$ and the average rate of new links per second is equal to $V_f = \frac{8}{\pi}s\nu$.*

3.1 link cost based on distance

First we have the easy theorem.

Theorem 2 *The graph of the SLO based on distance is planar.*

Proof This is a well known property of RNG [2], if two links cross over, then it is easy to see that one at least will be eliminated in the SLO algorithm. ■

Theorem 3 *The average number of synchronized links departing from a node in the SLO algorithm with cost based on distance is equal to [3]*

$$M_d = \frac{\pi}{2\frac{\pi}{3} - \frac{\sqrt{3}}{2}}$$

and the average rate of new synchronized links from a node is equal to

$$V_d = \sqrt{\pi}\frac{8}{3}\left(2\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)^{-\frac{3}{2}} s\sqrt{\nu}$$

Proof Let a link between two nodes A and B at distance r of each other. The condition that the link belong to the overlay is that no node C is in the intersection of the two disks of radius r respectively centered on A and B. The area of this intersection is $r^2A(\frac{\pi}{3})$ with $A(\theta) = 2\theta - \sin(2\theta)$. Therefore the probability that link (A, B) is included in the overlay is $\exp(-\nu r^2 A(\frac{\pi}{3}))$.

The average number of links from a random node A, M_d, that belong to the overlay will be

$$\begin{aligned} M_d &= \int_0^\infty 2\pi\nu r dr e^{-r^2A(\frac{\pi}{3})\nu} \\ &= \frac{\pi}{2\frac{\pi}{3} - \frac{\sqrt{3}}{2}} \approx 2.557530242 \end{aligned} \tag{1}$$

The constant M_d, Devroye's constant, is known from [3].

For the rate change the computation are relatively straightforward. Let consider a link (A, B)which belongs to the overlay. The intersection of the disk of radius r is empty. Therefore the rate at which the link will disappear from the overlay is equal to the rate at which the mobile nodes will enter the disk intersection. Since the border of the disk intersection is of length $\frac{4}{3}\pi r$, the number of nodes entering the area between time t and time $t + dt$ is equal to $\frac{4}{3}r\Delta(s)\nu dt$. Therefore the rate V_d at which overlay links vanishes from a random node A is

$$\begin{array}{rcl} V_d & = & \frac{4}{3}\Delta(s)\int_0^\infty 2\pi\nu^2 r^2 dr e^{-r^2 A(\frac{\pi}{3})\nu} \\ & = & \sqrt{\pi}\frac{8}{3}(A(\frac{\pi}{3}))^{-\frac{3}{2}} s\sqrt{\nu} \approx 3.471762654 \times s\sqrt{\nu} \end{array} \quad (2)$$

The rate at which links appears is also V_d. ∎

3.1.1 Uniform link cost

Theorem 4 *The average number of synchronized links departing from a node in the SLO algorithm with uniform link cost, when $\nu \to \infty$ tends to*

$$M_u = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{8\pi \sin 2\theta}{2\theta - \sin 2\theta} d\theta + O(\frac{1}{\nu})$$

with $A(\theta) = 2\theta - \sin 2\theta$ and the average rate of new synchronized links from a node tends to

$$V_u = \frac{128s}{\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\theta \sin 2\theta}{(2\theta - \sin 2\theta)^2} d\theta + O(\frac{1}{\nu})$$

Proof Let assume that node A has I.D qual to x, and B has I.D y. The link (A, B) belongs to the overlay if there is no common neighbor of A and B with I.D greater than $\max\{x, y\}$. Since we consider only I.D comparison, there is no loss of generality to assume that nodes I.D are scalar number uniformly distributed on the unit interval.

Let assume $x > y$ that will address of the cases. The probability that the link (A, B) belongs to the overlay for an arbitrary node B at distance r to node A is $x\exp(-(1-x)A(\theta)\nu)$ with $r = 2\cos\theta$. Counting the case where $y > x$ we get

$$\begin{array}{rcl} M_u(\nu) & = & 2\int_0^1 dx \int_0^1 2\pi\nu r dr e^{-(1-x)A(\theta)\nu} \\ & = & \int_0^1 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 8\pi\nu x \sin 2\theta d\theta e^{-(1-x)A(\theta)\nu} \\ & = & \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{8\pi}{\nu(A(\theta))^2} \sin 2\theta d\theta(\nu A(\theta) - 1 + e^{-\nu A(\theta)}) \end{array} \quad (3)$$

Therefore

$$\begin{array}{rcl} M_u(\nu) & = & \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{8\pi}{A(\theta)} \sin 2\theta d\theta + O(\frac{1}{\nu}) \\ & = & M_u + O(\frac{1}{\nu}) \end{array} \quad (4)$$

with

$$M_u = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{8\pi \sin 2\theta}{2\theta - \sin 2\theta} d\theta \approx 3.603973720$$

To get the rate at which overlay links vanish in the uniform cost algorithm we just have to consider the same link (A, B) such that nodes are at distance r and the I.D are respectively x and y. We assume $x > y$. Therefore the border length of the disk intersection is 4θ with $r = 2\cos\theta$ and the rate of entrance of nodes with I.D larger than x is $4\frac{\theta}{\pi}\Delta(s)$. Therefore the rate at which links vanish (including the case $y > x$) is:

$$\begin{aligned} V_u(\nu) &= \Delta(s) \int_0^1 dx \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 32\nu^2 \theta \sin 2\theta d\theta (1-x) x e^{-\nu A(\theta)(1-x)} \\ &= \Delta(s) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{32\theta \sin 2\theta}{\nu (A(\theta))^3} \left(A(\theta)\nu - 2 + (2 + \nu A(\theta)) e^{-\nu A(\theta)} \right) d\theta \\ &= V_u + O(\tfrac{s}{\nu}) \end{aligned} \tag{5}$$

with

$$V_u = \frac{128 s}{\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\theta \sin 2\theta}{(2\theta - \sin 2\theta)^2} d\theta \approx 4.146111863 \times s$$

■

3.2 Simulations

The simulations have been run in Maple on the unit disk graph model. The discrepancies between simulation and theory mainly come from the fact that the simulations have been run on a finite size map, while the theoretical results hold on infinite map. Therefore the theoretical results can be seen as a theoretical upper bound on the finite size simulated networks.

3.2.1 Number of links

unit disk graph on a 2×2 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
40	44.97	2.24	2.55	51.08	2.55	3.60	373.98	18.69	31.41
60	69.25	2.30	2.55	80.88	2.69	3.60	838.35	27.94	47.12
100	117.65	2.35	2.55	125.30	2.50	3.60	2410.95	48.21	78.53
140	168.62	2.40	2.55	184.47	2.63	3.60	4737.05	67.67	109.95

unit disk graph on a 4×4 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
60	68.42	2.28	2.55	78.52	2.61	3.60	278.95	9.29	11.78
100	118.77	2.37	2.55	140.57	2.81	3.60	754.95	15.09	19.63
140	169.60	2.42	2.55	207.80	2.96	3.60	1495.35	21.36	27.48
180	220.32	2.44	2.55	267.70	2.97	3.60	2512.22	27.91	35.34

unit disk graph on a 6×6 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
100	114.50	2.29	2.55	127.25	2.54	3.60	363.52	7.27	8.72
180	216.67	2.40	2.55	259.90	2.88	3.60	1204.30	13.38	15.70
260	317.25	2.44	2.55	393.85	3.02	3.60	2524.70	19.42	22.68

3.3 Link rate changes

In this section we assume that each node move independently of other node according to a random walk of constant speed of one unit per second. The variation of distances occurs every 0.01 second.

unit disk graph on a 2×2 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
40	385.00	9.62	10.97	176.25	4.40	4.14	451.25	11.28	25.46
60	710.00	11.83	13.44	263.75	4.39	4.14	988.75	16.47	38.19
80	1052.50	13.15	15.52	333.75	4.17	4.14	1770.00	22.12	50.92
100	1576.25	15.76	17.35	366.25	3.66	4.14	2796.25	27.96	63.66
120	2110.00	17.58	19.01	645.00	5.37	4.14	3920.00	32.66	76.39
140	2665.00	19.03	20.53	602.50	4.30	4.14	5550.00	39.64	89.12

unit disk graph on a 4×4 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
60	295.00	4.91	6.72	260.00	4.33	4.14	395.00	6.58	9.54
100	820.00	8.20	8.67	445.00	4.45	4.14	1060.00	10.60	15.91
120	1022.50	8.52	9.50	592.50	4.93	4.14	1525.00	12.70	19.09
180	1940.00	10.77	11.64	937.50	5.20	4.14	3537.50	19.65	28.64
240	3030.00	12.62	13.44	1430.00	5.95	4.14	6510.00	27.12	38.19

unit disk graph on a 6×6 square

Number of nodes	SLO distance	per node	limit theory	SLO uniform	per node	limit theory	All links	per node	limit theory
100	545.00	5.45	5.78	475.00	4.75	4.14	545.00	5.45	7.07
180	1410.00	7.83	7.76	930.00	5.16	4.14	1770.00	9.83	12.73
260	2315.00	8.90	9.33	1660.00	6.38	4.14	3885.00	14.94	18.39
340	3535.00	10.39	10.66	1680.00	4.94	4.14	6315.00	18.57	24.05
420	4790.00	11.40	11.85	2440.00	5.80	4.14	9740.00	23.19	29.70

4 Perspective and conclusion

We have presented and analyzed the performance of the synchronized links overlay protocol inspired from RNG. The selection of links is based on link quality so that the overhead of synchronization is reduced to minimal. This is particularily important when the performance of radio link drastically change with range.

We see that the number of links per node is independent of the network density and topology (with the unit disk graph model), while the total number of non synchronized link is proportional to node density: is $O(\nu)$.

When the nodes are mobile, there is new synchronization process every time a new link comes into the SLO. The uniform link cost shows a per node synchronization rate which is independent of node density and is proportional to average node speed s. When the cost is based on distance the synchronization rate increases to $O(s\sqrt{\nu})$ synchronization, but which is much lower than the total link change rate which is $O(\nu s)$.

However link costs based on real metric is more interesting than uniform link costs, because it shows better performance in synchronization process since the link qualities are better. In order to avoid the $\sqrt{\nu}$ term which can be heavy when the node density increases very much, we can use a *quantized* link cost. For example the cost of a link could be $m(A,B) = \lfloor \log \min\{\mathrm{Cost}(A,B), \mathrm{Cost}(B,A)\}\rfloor$, and when the metric are identical the tie is broken as with uniform link cost. In this case we may expect a per node synchronization rate much smaller than $O(s\sqrt{\nu})$ closer to constant synchronization rate. We conjecture that the per node synchronization rate would be $O(s\log\nu)$.

The future work will be to prove rigorously this conjecture and to better tune the link metric in order to properly balance the synchronization rate and the per synchronization overhead. For example one can also put an upperbound such as $m(A,B) = \min\{C, \lfloor \log \min\{\mathrm{Cost}(A,B), \mathrm{Cost}(B,A)\}\rfloor\}$ for some constant C.

This opens interesting perspectives in the concept of reduced synchronized link overlay.

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