Arterial blood pressure analysis based on scattering transform I

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Abstract— **This article presents a new method for analyzing arterial blood pressure waves. The technique is based on the scattering transform and consists in solving the spectral problem associated to a one-dimensional Schrödinger operator with a potential depending linearly upon the pressure. This potential is then expressed with the discrete spectrum which includes negative eigenvalues and corresponds to the interacting components of an N-soliton. The approach is analogous to the Fourier transform where the solitons play the role of sinus and cosinus components. The proposed method seems to have interesting clinical applications. It can be used for example to separate the fast and slow parts of the blood pressure that correspond to the systolic (pulse transit time) and diastolic phases (low velocity flow) respectively.**

I. INTRODUCTION

The cardiovascular system, composed of the heart and a complex vascular network, provides oxygen and nutrients to all the body. Pressure and flow waves are created by the beating heart and propagate through the aorta and the major arteries to the periphery. The analysis of the Arterial Blood Pressure (ABP) in daily clinical practice is often restricted to the maximal and the minimal values of the pressure wave called systolic and diastolic pressures respectively. However, in this case, none information about the instantaneous variability of the pressure is provided. Many studies were devoted to model and analyse the ABP waveform and many models were proposed from the well-known windkessel model [22] to the more complex three dimensional models [13].

A standard description of the blood pressure and flow waves uses the linear Fourier analysis where the waves are decomposed into sinus and cosinus components. The Fourier decomposition is applicable if two assumptions are satisfied. First, the arterial system is supposed to be linear so that the superposition principle applies. Then, the pressure and flow must be measured in steady state conditions at constant heart rate [21]. About 12 to 15 harmonics are needed for a good reconstruction of the ABP waves. To explain the waves contour and interpret the changes in the pulse pressure when it propagates along the arterial tree like the increase in the amplitude and the decrease in the width called "Peaking" and "Steepening" phenomena respectively, this approach supposes the existence of backward waves. Each harmonic consists then of an incident wave propagating away from the heart and a reflected wave travelling towards the heart. Many studies were carried out in order to separate the ABP into its forward and backward components as in the pioneering work of Westerhof et al [23] followed by many others [16], [17], [18].

Instead of the usual decomposition of the ABP into a linear superposition of sinus and cosinus, in this article, we propose to use a nonlinear superposition of solitary waves or solitons. The concept of soliton refers in fact to a solitary wave emerging unchanged in shape and speed from the collision with other solitary waves [19]. They fascinate the scientists with their very interesting coherent-structure characteristics and are used in many fields and to model natural phenomena. Solitons are solutions of nonlinear dispersive equations like the Korteweg-de Vries (KdV) equation arising in a variety of physical problems, for example to describe wave motion in shallow water canals. This third order nonlinear partial derivative equation (NPDE) includes both nonlinear and dispersive effects and solitons result here from a stable equilibrium between these effects [20], [25].

The use of solitons to describe the ABP was already introduced in [24] and in [15] where a KdV equation and a Boussinesq equation were respectively proposed as a blood flow model. Recently, in [2], [9] an interesting reduced model of the ABP was introduced. The latter consists of a sum of a 2 or 3-solitons, solution of a KdV equation, describing fast phenomena during the systolic phase and a 2-element windkessel model describing slow phenomena during the diastolic phase. We recall that the systolic phase corresponds to the contraction of the heart, driving blood out of the left ventricle while the diastolic phase corresponds to the period of relaxation of the heart. We point out that the introduction of solitons in the ABP model explains the peaking and the steepening phenomena [9].

The decomposition of the ABP into a nonlinear superposition of solitons introduced in this article is based on an elegant mathematical transform : the scattering transform for a one-dimensional Schrödinger equation [1], [5], [6]. This Scattering-Based Signal Analysis (SBSA) method was introduced in [10]. The main idea in the SBSA consists in solving the spectral problem of a one-dimensional Schrödinger operator with a potential depending linearly upon the pressure wave. This potential is then expressed with the discrete spectrum which includes negative eigenvalues and corresponds to the interacting components of an N-soliton.

In the next section, we recall the basis of the SBSA method that is used to reconstruct and analyse the ABP waves. Section III illustrates a good agreement between real and reconstructed pressures using the SBSA. We also present an interesting clinical application which consists in separating the fast and slow components of the ABP which are related

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to the systolic and diastolic phases respectively. Finally a conclusion summarizes the different results.

II. A SCATTERING-BASED SIGNAL ANALYSIS METHOD

In this section, we introduce an original method for reconstructing signals based on the scattering transform. We start by briefly recalling the basis of the Direct and Inverse Scattering Transforms (DST & IST) and then present the main idea in the SBSA technique.

A. Scattering transform for a Schrödinger equation

The stationary one-dimensional Schrödinger equation is given by :

$$-\frac{\partial^2\psi}{\partial x^2}+V\psi=\lambda\psi, \quad -\infty<x<+\infty. \tag{1}$$

where x is the space variable, λ and ψ are respectively the eigenvalues and the associated eigenfunctions. V is the potential of the Schrödinger operator $L(V)=-\frac{\partial^2}{\partial x^2}+V$.

In this study we suppose that the function V belongs to the Schwartz space $\mathscr{S}(\mathbb{R})$ of regular and rapidly decreasing functions on $\mathbb{R}$.

The DST of the potential V is the solution of the spectral problem for $L(V)$. The spectrum of this operator has two components : a continuous spectrum including positive eigenvalues and a discrete spectrum with negative eigenvalues [1], [4], [5], [6], [12]. Denoting the positive eigenvalues by $\lambda=k^2$, the continuous spectrum is characterized by the following asymptotic boundary conditions where $T(k)$ and $R(k)$ are respectively the transmission and the reflection coefficients associated to V :

$$\psi(x,k)\to T(k)\exp(-ikx), \quad x\to-\infty, \tag{2}$$

$$\psi(x,k)\to \exp(-ikx)+R(k)\exp(ikx), \ x\to+\infty. \tag{3}$$

Conservation of energy leads to $|T(k)|^2+|R(k)|^2=1$.

We note the negative eigenvalues $\lambda_n=-\kappa_n^2$, $n=1,\cdots,N$ with N their number. The discrete eigenfunctions ψ_n are L^2-normalized such that :

$$\int_{-\infty}^{+\infty}\psi_n(x)^2dx=1. \tag{4}$$

and behave as $\psi_n(x)\sim c_n\exp(-\kappa_n x)$ in the limit of large x where the coefficients c_n are defined by :

$$c_n=\lim_{x\to+\infty}\exp(\kappa_n x)\psi_n(x), \ n=1,\cdots,N. \tag{5}$$

So, the DST S of V is the collection of data $S(V)$ called scattering data and defined by :

$$S(V)=(R,\ \kappa_n,\ c_n,\ n=1,\cdots,N). \tag{6}$$

The inverse problem or IST consists in reconstructing the Schrödinger operator's potential from its spectral data. The IST is based on the Gel'fand-Levitan-Marchenko (GLM) integral equation [1], [5]. In this study we are only interested in a special situation which corresponds to a reflectionless potential.

A reflectionless potential is a potential for which the reflection coefficient $R(k)$ is zero. This means that there is no contribution from the continuous spectrum. This situation leads to an interesting representation of the potential with the discrete spectrum only [6] as it is given in the following theorem :

Theorem **:** If V is a reflectionless potential of the Schrödinger equation (1) then :

$$V(x)=-4\sum_{n=1}^{N}\kappa_n\psi_n^2(x), \quad x\in\mathbb{R}. \tag{7}$$

An interesting relation between solitons, solutions of a KdV equation and the discret spectrum of the Schrödinger operator was introduced in [6]. Indeed, from a reflectionless potential of (1) that evolves, in time and space, according to a KdV equation, there emerge, for $t\to+\infty$, N solitons, each one characterized by a pair (κ_n, c_n) such that $4\kappa_n^2$ gives the speed of the soliton and c_n its position. Therefore each component $-4\kappa_n\psi_n^2$ of the previous sum refers to a single soliton.

B. SBSA principle

We now present the SBSA technique which is essentially inspired from the results established in the case of a reflectionless potential that have been recalled in the previous subsection.

We note y the signal that we want to reconstruct and analyse and we suppose that the potential V of the Schrödinger operator L depends linearly upon y :

$$V(y)=-\chi(y-y_{min}), \tag{8}$$

where y_{min} is a constant such that $y-y_{min}>0$ and χ a positive parameter to determine.

The main idea is then to find the parameter χ such that the signal y is well approximated by the reflectionless part of the potential which can be written then using equation (7) :

$$\hat{y}=4\chi^{-1}\sum_{n=1}^{N}\kappa_n\psi_n^2+y_{min}, \tag{9}$$

where $-\kappa_n^2$ and ψ_n, $n=1,\cdots,N$ are respectively the N negative eigenvalues and the corresponding L^2-normalized eigenfunctions for the potential V, determined by the DST. This situation corresponds in fact to the decomposition of the signal y into a nonlinear superposition of solitons [10].

It is important to recall the role of the parameter χ and its influence on the number of negative eigenvalues. The potential V is a well of variable depth which is determined by χ. The number of the negative eigenvalues $N(\chi)$ is a nondecreasing function of χ and there is an infinite unbounded sequence of values of χ at which $N(\chi)$ is incremented by one, the new eigenvalue being born from the continuous spectrum [7], [10], [14].

Determining the parameter χ determines the number $N(\chi)$ of negative eigenvalues and hence the number of solitons components required for a satisfying approximation of the signal y. Fig. 1 summarizes the SBSA method.

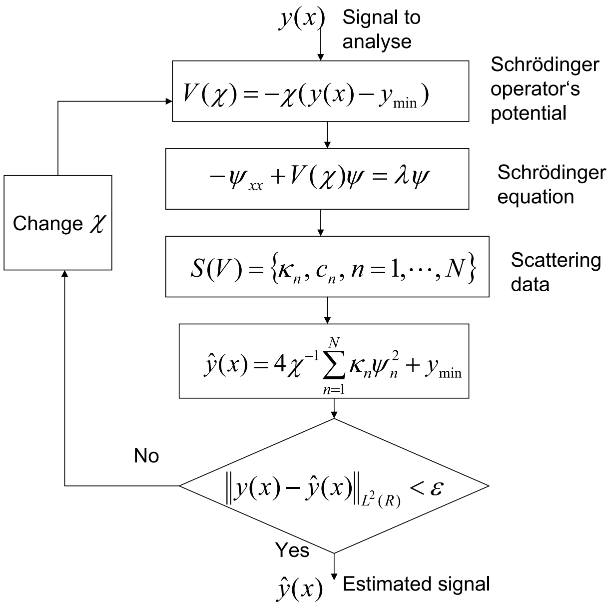


Fig. 1. Signals analysis with the SBSA method

III. A SOLITON-BASED DECOMPOSITION OF THE ABP

In this section, the SBSA method is applied to the ABP signal. An interesting application is also presented which consists in separating systolic and diastolic phases. For a convenient use, we solve the SBSA by replacing the space variable x by the time variable t.

A. Reconstruction of the ABP

Let $P(t)$ be the ABP signal. The Schrödinger operator potential V depends linearly upon the ABP signal $V(t) = -\chi P(t)$.

Fig. 2 and Fig. 3 compare measured and estimated pressures at the aorta and at the finger levels for different values of the number $N(\chi)$ of the solitons' components. We notice that only 5 to 10 components are sufficient for a good approximation of the ABP. In Fig. 4, several beats of measured and reconstructed pressures are considered.

B. Separation of the systolic and diastolic phases

Now we exploit the SBSA to separate the pressure into its fast and slow parts which correspond respectively to the systolic and diastolic phases. We suppose that the $N_f = 2$ or 3 largest κ_n^2 describe fast phenomena and the $N_s = N - N_f$ smallest κ_n^2 describe slow phenomena. Then, the following

$$\hat{P}_f(t) = 4\chi^{-1}\sum_{n=1}^{N_f}\kappa_n\psi_n^2, \tag{10}$$

$$\hat{P}_s(t) = 4\chi^{-1}\sum_{n=N_f+1}^{N}\kappa_n\psi_n^2, \tag{11}$$

are good candidates to represent respectively the fast and slow phenomena in the ABP. This is confirmed by the

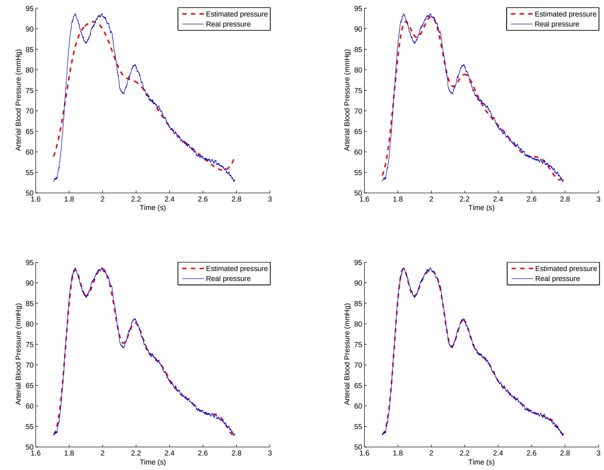


Fig. 2. Measured and reconstructed pressures at the aorta with N solitons. From left to right : $N = 3$, $N = 5$ (Up). $N = 7$, $N = 9$ (Down)

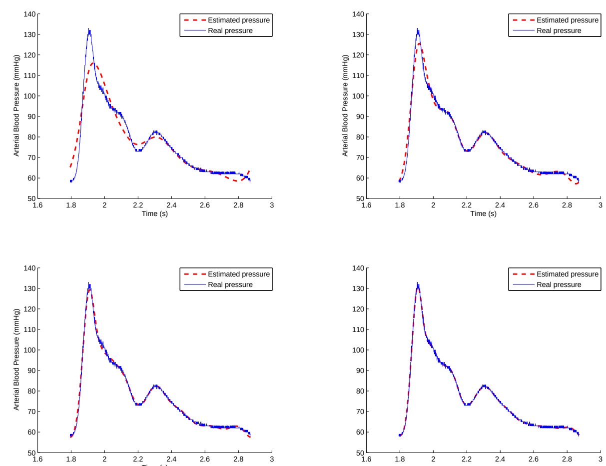


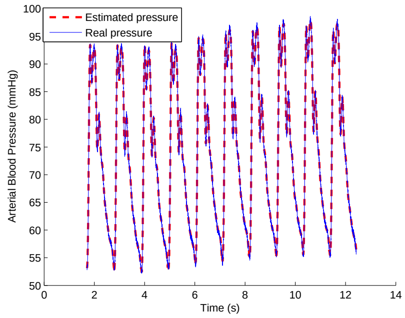
Fig. 3. Measured and reconstructed pressures at the finger with N solitons. From left to right : $N = 3$, $N = 5$ (Up). $N = 7$, $N = 9$ (Down)

experimental results. For $N_f = 2$ and $N = 9$, Fig. 5 and Fig. 6 show that $\hat{P}_f$ and $\hat{P}_s$ are respectively localized during the systole and the diastole, as expected.

This decomposition of the ABP into fast and slow parts completes the results obtained in [2], [3], [8], [9].

IV. CONCLUSIONS

This article presents a new method for analyzing ABP waves. This approach is based on the scattering transform and deals with the solution of the spectral problem of a perturbed Schrödinger operator for a given potential. The latter is then expressed in a new base which components are solitons. It seems through the satisfactory results obtained that this method can lead to interesting clinical applications, for instance the separation of the ABP into its systolic and diastolic phases. Moreover, the SBSA provides interesting ABP indices that can be analyzed in various clinical and physiological conditions. Promising results are presented in



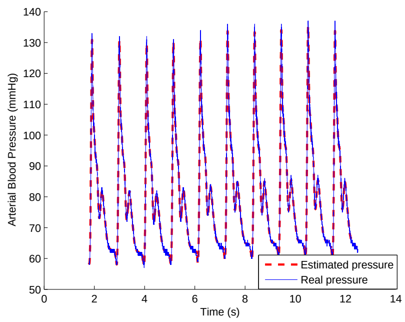


Fig. 4. Multi-beat measures and estimates : Aorta (up) and Finger (down)

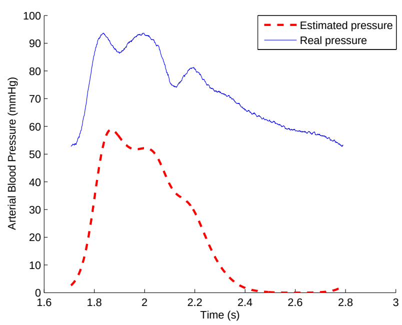


Fig. 5. $\hat{P}_f$ and fast systolic phenomena

our second article [11].

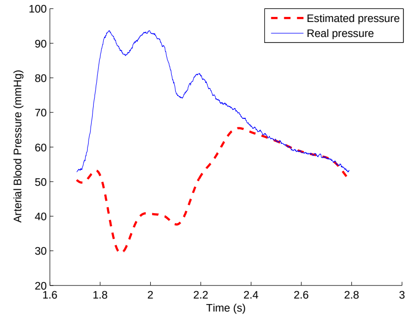


Fig. 6. $\hat{P}_s$ and slow diastolic phenomena

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