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Abstract: Multipoint relays were introduced to optimize flooding in ad hoc networks. They are also used to determine a sub-topology containing shortest paths in the OLSR routing protocol. We show that a generalized version of multipoint relays can be used to construct a sub-topology preserving bi-connectivity and more generally k-connectivity. Moreover, we show that the multipoint relay structure is intrinsic to any sub-topology with similar properties.

Key-words: ad hoc, bi-connectivity, k-connectivity, multipoint relays, OLSR

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Bi-connexité, k-connexité et multipoints relais

Résumé : Les multipoints relais ont été introduits pour optimiser l'inondation dans un réseau ad hoc. Ils servent aussi, dans le protocole OLSR, à déterminer une sous-topologie qui conserve les plus courts chemins. Nous montrons, comment une généralisation des multipoints relais permet d'obtenir une sous-topologie conservant des propriétés de bi-connexité et plus généralement de k-connexité. Nous montrons de plus, que cette structure de multipoints relais est intrinsèque à toute sous-topologie montrant les mêmes propriétés.

Mots-clés : ad hoc, bi-connexité, k-connexité, multipoints relais, OLSR

1 Introduction

To allow all pairs of nodes of a network to communicate, the underlying graph has to be connected. If it is k-connected, this property is preserved even under the failure of $k-1$ nodes. However, optimized link state protocols use only a subset of links, and the resulting sub-topology used for routing may loose this k-connectivity property met by the full network. For better resilience, it is thus interesting to design some sub-topology selection mechanism that preserves k-connectivity if possible. Similarly, we also focus on resilience to link failures.

We show that OLSR routing protocol [1] when it is parameterized with multipoint relay coverage k preserves k-connectivity of the topology included in the link state advertisements. Moreover, if $k' \leq k$ disjoint paths exist between two nodes, the OLSR topology will include k' disjoint routes between them with smallest length sum. In other words, any node is able to compute disjoint routes from the topology it learns from OLSR and these routes are short.

Related work study the influence of power-control on k-connectivity of the network [6, 4, 3]. Our approach is complementary, and acts at routing level instead of link level. Given some power control policy, a link state routing protocol such as OLSR detects neighborhoods and optimizes the quantity of topology information broadcasted to enable efficient pro-active route construction. The results given here apply to any link state protocol where a subset of the topology is broadcasted.

2 Connectivity graph and k-connectivity

The set of all possible connections in a network can be modeled through a graph $G = (V, E)$ where the vertices $u \in V$ are the nodes of the network and where an edge $uv \in E$ in the graph indicates that u and v can communicate together. In the case of OLSR, E is made of the links uv with symmetric state, *i.e.* u and v receive HELLO messages from each other. We then say that u and v are *neighbors*. We also say that u *covers* v (or that v covers u) by analogy with radio communications. Let $N(u)$ denote the neighbors of u (excluding u itself). By extension, we will abusively write $N(A)$ to denote the set of neighbors of some node in A excluding nodes in A ($N(A) = \{v | v \notin A$ and there exists $u \in A$ such that $uv \in E\}$). $N(N(u))$ is thus the set of *two hop neighbors* of u.

A *path* is a sequence $u_0, \ldots, u_n$ of nodes such that $u_{i-1}u_i \in E$ for all $1 \leq i \leq n$. n is the *length* of the path. A graph is *connected* when all pair of nodes can be connected by a path. Two paths $u_0, \ldots, u_n$ and $v_0, \ldots, v_m$ are *internal node disjoint* when the sets of nodes $\{u_1, \ldots, u_{n-1}\}$ and $\{v_1, \ldots, v_{m-1}\}$ are disjoint. They are *edge disjoint* when the sets of edges $\{u_0u_1, \ldots, u_{n-1}u_n\}$ and $\{v_0v_1, \ldots, v_{m-1}v_m\}$ are disjoint.

Definition 1 (k-connectivity) *A graph is* k-connected *when it is connected and remains connected after removal of any subset of $k-1$ vertices. Equivalently, Menger's theorem states that a graph is k-connected iff every pair of nodes is connected by at least k internal node disjoint paths.*

Similarly, connectivity can be viewed with respect to edge deletion.

Definition 2 (k-edge-connectivity) *A graph is* k-edge-connected *when it is connected and remains connected after removal of any subset of $k-1$ edges. Equivalently, a graph is k-edge-connected iff every pair of nodes is connected by at least $k-1$ edge disjoint paths.*

3 Multipoint relays sub-topology

When the density of a wireless network increases, the number of links may become quadratic. To maintain a sub-quadratic control traffic, OLSR protocol disseminates only a subset of links: MPR links (MPR states for multipoint relay). A node computes its routing table based on these links and links to neighbors.

Definition 3 (MPR) *Every OLSR node selects a subset of neighbors called* multipoint relays *(or* MPRs *for short) such that every two hop neighbor is covered by some MPR.*

This covering property is fundamental to the proper functioning of OLSR. It is further parameterized by the MPR coverage parameter [1] described bellow. A simple greedy heuristic [5] allows to compute multipoint relay sets with small size. Finding a minimal size set is NP-hard and the greedy heuristic is within a factor $1 + \log d$ from optimal [5] (d denotes the maximal degree of a node).

When u selects m as MPR, the directed link mu is said to be an *MPR link*. We also say that u is an MPR selector of m. OLSR protocol disseminates links existence by letting nodes flood topology control messages (TC) containing their list of MPR selectors.

Every node computes its routing table from shortest paths in the directed graph containing all MPR links and links to its neighbors. (Recall that neighbors are discovered through HELLO messages.) More precisely, OLSR protocol constructs routes of the form $u_0, \ldots, u_n$ such that u_1 is a neighbor of u_0 (neighbor link) and $u_1u_2, \ldots, u_{n-1}u_n$ are MPR links. Such a path is called an OLSR route.

Every node uses a different topology to compute its routes. Indeed all MPR links are shared but each node uses its own neighbor links.

To enable more redundancy, the definition of MPRs is parameterized through the MPR coverage parameter [1, 2]:

Definition 4 (MPR Coverage) *An MPR set has* coverage k *if every two hop neighbor covered by d neighbors is covered by* $\min(d, k)$ *MPRs.*

In other words, a two hop neighbor must be covered by k MPRs if possible or all its common neighbors if not. The greedy heuristic proposed in [5] can easily be adapted to compute MPRs with coverage k. Higher coverage enables more redundancy in MPR flooding [2]. We are going to see that it also enables some k-connectivity property of the MPR link sub-topology.

4 k-connectivity and MPR coverage k

As OLSR uses a sub-topology, the existence of k disjoint routes in OLSR topology is not guaranteed by the existence of k disjoint paths in the connection graph. The following theorem makes the link between MPR coverage k and k-connectivity.

Theorem 1 *Consider an OLSR network where MPRs are selected with coverage k. If two non neighbor nodes are linked by k internal node disjoint paths in the connection graph, then there exists k internal node disjoint routes from one to the other.*

Proof: Consider a connection graph such that two nodes u and v are connected by k internal node disjoint paths. Suppose that each node has elected an MPR set with coverage k. We show that there must exist k disjoint OLSR routes from u to v.

Let $C_1, \ldots, C_k$ be k internal node disjoint paths from u to v such that the sum of their length is minimal. Let $u_0 = u, \ldots, u_n = v$ be the sequence of nodes in C_1. Let us show by recurrence that there exist a path $m_i, \ldots, m_n = v$ such that m_i covers u_{i-1}, $m_i m_{i+1}, \ldots, m_{n-1} m_n$ are MPR links, and none of the paths $C_2, \ldots, C_k$ include $m_i, \ldots, m_{n-1}$.

It is obviously verified for $i = n$. Suppose that the property is verified for $i > 1$ and let $m_i, \ldots, m_n = v$ be such a path. As m_i covers u_{i-1}, u_{i-2} is a two hop neighbor. (It cannot be a direct neighbor as $C_1, \ldots, C_k$ have been chosen with minimal length sum.) Let d denote the number of neighbors of m_i linked to u_{i-2}. In the case $d < k$, m_i must select all these d neighbors (including u_{i-1}) as MPRs. The property is then verified by the path $m_{i-1} = u_{i-1}, \ldots, m_n$.

Now consider the case $d \geq k$. m_i has selected k MPRs that cover u_{i-2}. Suppose by contradiction that they all lie in some path $C_2, \ldots, C_k$. There must thus exist a path C_j (with $2 \leq j \leq k$) containing at least two MPRs of m_i. Let $v_0 = u, \ldots, v_p = v$ be the sequence of nodes of C_j and let v_a and v_b ($a < b$) be two MPRs of m_i in C_j. Now consider the two paths $C'_1 = v_0, \ldots, v_a, m_i, \ldots, m_n$ and $C'_j = u_0, \ldots, u_{i-2}, v_b, \ldots, v_p$. They form with $C_2, \ldots, C_{j-1}, C_{j+1}, \ldots, C_k$ k internal node disjoint paths from u to v with smaller length sum since $|C'_1| + |C'_j| = (a+n-i+1) + (i-2+p-b+1) = n+p-(b-a) < n+p = |C_1| + |C_j|$. To avoid this contradiction, some MPR of m_i covering u_{i-2} must be outside $C_2, \ldots, C_k$. Let m_{i-1} be this node. The property is then verified by $m_{i-1}, \ldots, m_n$.

We can thus conclude by recurrence that the property must be verified for $i = 1$. This implies the existence of a path $M_1 = u, m_1, \ldots, m_n$ from u to v with same length as C_1, disjoint from $C_2, \ldots, C_k$ and such that links $m_{i-1} m_i$, $1 < i \leq n$ are MPR links. This path is thus an OLSR route.

Similarly, $M_1, C_2, \ldots, C_k$ are k internal node disjoint paths from u to v with minimal length sum. There must thus exist an OLSR route M_2 disjoint from $M_1, C_3, \ldots, C_k$. Iterating the argument, we can conclude to the existence of k internal disjoint OLSR routes $M_1, \ldots, M_k$ included in the OLSR sub-topology known by u.

Remarks:

- The proof still holds if MPR selection is made with MPR coverage $k' \geq k$.
- The proof implies the existence of k disjoint OLSR routes with minimal length sum. In particular, this gives a proof of the existence of shortest path routes in OLSR topology (case $k = 1$).
- A slight modification of the MPR coverage notion allows to extend the theorem to the case where the two nodes are neighbors:

Definition 5 (MPR Compplete Coverage) *An MPR set has* complete coverage k *if every two hop neighbor covered by* d *neighbors is covered by* $\min(d, k)$ *MPRs and every neighbor covered by* d *other neighbors is covered by* $\min(d, k-1)$ *MPRs.*

Consider two neighbors u and v sharing d other neighbors. There thus exist at least $d+1$ internal node disjoint paths from u to v. With complete coverage $k \geq d+1$, u will find these $d+1$ paths as OLSR routes. If there exists $k' > d+1$ disjoint paths, the construction of the previous proof allows to find $k-(d+1)$ more disjoint routes in the OLSR topology known by u. We can thus state the more general theorem:

Theorem 2 *Consider an OLSR network where MPRs are selected with complete coverage* k. *If two nodes are linked by* $k' \leq k$ *internal node disjoint paths in the connection graph, then there exists* k' *internal node disjoint OLSR routes from one to the other. Moreover, these routes can be chosen with minimal length sum.*

5 k-edge-connectivity and MPR coverage k

Consider the similar problem of selection a sub-topology containing k edge disjoint routes if possible. Consider first the special case where two nodes u and v are two hops away from each other. If they have k common neighbors, we would like to find k edge disjoint routes from u to v. If we additionally require that these routes have minimal length sum, this implies that k advertised links join v to $N(u)$ (all routes must have length 2). In OLSR, this means that v has k MPRs that cover u. Requiring some k-edge-connectivity property with smallest length sum again drives to MPR selection with coverage k.

Conversely, it is not immediate that the existence of k edge disjoint OLSR routes is guaranteed by MPR selection with coverage k. However, the construction of the previous proof is still valid considering edge disjoint paths. We can thus state the following theorem.

Theorem 3 *Consider an OLSR network where MPRs are selected with coverage* $k' \geq k$. *If two non neighbor nodes are linked by* k *edge disjoint paths in the connection graph, then there exists* k *edge disjoint routes from one to the other. Moreover, such routes can be chosen with minimal length sum.*

A similar theorem holds for neighboring nodes when MPRs are selected with complete coverage k.

6 Sub-topology preserving k-connectivity

Any link state protocol disseminates in the network the existence of links. If the network may become dense, such as in wireless networks, the protocol has to disseminate only a subset of links to maintain control traffic low. For that purpose, let us formally define the notion of sub-topology.

Definition 6 *A sub-topology $H = (V, E_H)$ is a subgraph of the connection graph G ($E_H \subseteq E$). As every node knows its neighborhood $N(u)$, it knows the graph $H_u = (V, E_H \cup \{uv | v \in N(u)\})$. H is* k-sub-connected *if for every pair u, v of nodes, the existence of $k' \leq k$ internal node disjoint paths from u to v in G implies the existence of k' internal node disjoint paths from u to v in H_u. Additionally, H is* optimally k-sub-connected *if such disjoint paths have minimal length sum.*

This definition simply states that a node u knowing a common broadcasted topology H and its neighborhood can compute k' disjoint routes to any destination v when there exists $k' \leq k$ disjoint paths in the full topology. Considering paths from a two hop neighbor of u to u, we can obviously state the following theorem.

Theorem 4 *If H is an optimally k-sub-connected sub-topology of G, then the neighbors of a node in H form an MPR set with coverage k in G.*

With $k = 1$, this theorem states that MPRs are an intrinsic structure to obtain routes which are shortest paths. As intuitively mentioned before, a similar theorem can be stated concerning k-edge-connectivity.

7 Conclusion

Multipoint relays are a necessary and sufficient structure allowing k-connectivity with minimal length sum of the routes provided. They thus appear as an intrinsic structure when optimizing the sub-topology broadcasted in a link state routing protocol.

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