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Romain Bourdais, Michel Fliess, Cédric Join, Wilfrid Perruquetti

TOWARDS A MODEL-FREE OUTPUT TRACKING OF SWITCHED NONLINEAR SYSTEMS

Romain BOURDAIS * **Michel FLIESS** **
Cédric JOIN *** **Wilfrid PERRUQUETTI** ****,[1]

* *École Centrale de Lille, LAGIS (UMR-CNRS 8146), BP 48, Cité Scientifique, 59 651 Villeneuve-d'Ascq, France. E-mail:* `bourdais.romain@ec-lille.fr`
** *Projet ALIEN, INRIA Futurs & Équipe MAX, LIX (UMR-CNRS 7161), École polytechnique, 91128 Palaiseau, France. E-mail:* `Michel.Fliess@polytechnique.edu`
*** *Projet ALIEN, INRIA Futurs & CRAN (UMR-CNRS 7039), Université Henri Poincaré (Nancy I), BP 239, 54506 Vandœuvre-lès-Nancy, France. E-mail:* `Cedric.Join@cran.uhp-nancy.fr`
**** *Projet ALIEN, INRIA Futurs & École Centrale de Lille, LAGIS (UMR-CNRS 8146), BP 48, Cité Scientifique, 59 651 Villeneuve-d'Ascq, France. E-mail:* `wilfrid.perruquetti@ec-lille.fr.`

Abstract: We extend previous works on model-free control to switched nonlinear SISO systems. Our contribution, which is utilizing new algebraic methods for numerical differentiations, yields PID-like regulators which ensure practical stability. Several academic examples, with convincing computer simulations, are illustrating our approach.

Keywords: Nonlinear SISO systems, switched systems, hybrid systems, model-free control, numerical differentiation.

1. INTRODUCTION

Many systems encountered in practice exhibit switchings between several subsystems, both as a result of controller design, such as in switching supervisory control, and inherently by nature, such as when a physical plant has the capability of undergoing several operational modes, for instance a walking robot during leg impact and leg swing modes, different formations of a group of vehicles, reactions during chemical operations ...

Switched systems may be viewed as higher–level abstractions of hybrid systems, obtained by neglecting the details of the discrete behavior. Informally, a switched system is composed of a family of dynamical subsystems (linear or nonlinear), and a rule, called the switching law, that orchestrates the switching between them. In recent years, there has been increasing interest in the control problems of switched systems due to their significance from both a theoretical and practical point of view and also because of their inherently interdisciplinary nature. So several important results for switched systems have been achieved, including various stability results, controllability results (Sun *et al.*, 2002; Xie *et al.*, 2002), and input-to-state properties, ... For a survey on these results, the reader may refer to (Liberzon *et al.*, 1999; Liberzon, 2003; Sun *et al.*, 2005).

[1] Two authors (RB, WP) were partially supported by Région Nord Pas de Calais et Feder: ARCIR Robocoop `http://syner.ec-lille.fr/robocoop/`

More precisely on the stability and control issues, different operating tools have been proposed, such as multiple Lyapunov functions (Branicky, 1998), dwell-time (Persis *et al.*, 2003) and average dwell time (Hespanha *et al.*, 1999). Among all the problems linked to switched systems, two main questions are:

- The switching stabilizability: given a family of subsystems, how the switching law can be constructed in order to ensure the stability of the switched system (see (Wicks *et al.*, 1994; Wicks *et al.*, 1998; Feron *et al.*, 1996; Wicks *et al.*, 1997; Pettersson *et al.*, 1996; Pettersson, 1999; Wicks *et al.*, 1997; Pettersson *et al.*, 1996; Pettersson, 1999))?
- The uniform stability: given a family of subsystems, which conditions on vector fields is ensuring the stability of the switched system under any switching law (see (Vu *et al.*, 2005; Mancilla-AguilarSun *et al.*, 2000; Bourdais *et al.*, 2006))?

But in all the proposed results, a complete description of the subsystems is necessary in order to obtain explicit stability conditions and control laws. Moreover, even if the description is complete, searching for a stable convex combination is a $NP-$hard problem (Skafidas *et al.*, 1999), such as constructing a common Lyapunov function. **Here the control problem is tackled without a complete description of the subdynamics and even without knowing the switching signal**. The proposed control design methodology is based on a new point of view: the system output on a small time window is approximated by a polynomial (w.r.t time) which leads to some local model on a time varying window. The obtained model relies on fast, i.e., real-time, estimations of derivatives for noisy signals. These technics were first developed for closed-loop parametric identification for linear systems in (Fliess *et al.*, 2003; Fliess *et al.*, 2007) and were shown to be efficient alternative to existing technics mentioned in (Sjöberg *et al.*, 1995; Kerschen *et al.*, 2006; Ljung *et al.*, 1994; Ljung *et al.*, 1994). Further developments of these technics have great impacts on automatic control and related topics:

- nonlinear state reconstructors and feedback control (see (Fliess, Sira-Ramírez, 2004; Fliess *et al.*, 2005)),
- fault tolerant control (see (Fliess *et al.*, 2005)),
- model-free control (see (Fliess, Join, Sira-Ramírez, 2006; Fliess, Join, Mboup, Sira-Ramírez, 2006), and (Join *et al.*, 2006) for a concrete application),
- ciphering using chaotic nonlinear systems (see (Sira-Ramírez *et al.*, 2006)),

See (Fliess, 2006) for a new analysis of the notion of corrupting noises, and the references therein for other applications, especially in signal processing.

The proposed technics, leading to fast derivatives estimations and on-line parametric estimations, give a new framework for control design without accurate modeling of the process (see (Fliess *et al.*, 2006; Fliess *et al.*, 2006)). Here we will develop further this approach to encompass a wide class of stabilizers for switched systems (without state jumps). To this end we consider switched systems without state jumps as a collection of ordinary differential equations (ODEs) which can be seen as differential relations between the input and output variables. During a short time window those ODEs may be given the elementary form $y^{(p)} = a(.) + b(.)u$, where the terms $a(.)$ and $b(.)$ depend on the input, output variables, their derivatives up to some finite order, and on the switching signal. Now using fast online estimations of these two terms (as soon as $b(.)$ is non zero) and eventually the successive derivatives of the output up to order $(p-1)$, one can obtain the desired tracking performances using either a popular PID (see, e.g., (Aström *et al.*, 1995), (O'Dwyer, 2003)), or a GPID (see (Fliess *et al.*, 2002)). For a kind of "state" feedback, one can use, for example, a control of the form ($e_{y,i,\mathrm{estim}} = \left[y^{(i)}\right]_{\mathrm{estim}} - y^{(i)}_{\mathrm{ref}}$):

$$[b(.)]_{\mathrm{estim}}\, u = y^{(p)}_{\mathrm{ref}} - [a(.)]_{\mathrm{estim}} - \sum_{i=-1}^{(p-1)} \alpha_i e_{y,i,\mathrm{estim}} \quad (1)$$

Section 2 sets up the problem formulation then gives an outline of the control procedure which is illustrated through a simple example. Section 3 recalls some background on algebraic fast estimations. Then section 4 gives some details about the assumptions and the numerical implementation of the control for two large classes of switched systems that is illustrated in section 5 with several academic examples. Lastly, section 6 serves as a conclusion.

2. MAIN PRINCIPLES

2.1 Problem formulation

As mentioned in the introduction we consider nonlinear switched systems of the following input/output form:

$$0 = f_{\sigma(t)}(t, y, \dot{y}, \ldots, y^{(p_{\sigma(t)})}, u, \ldots, u^{(m_{\sigma(t)})}, d) \quad (2)$$

where $\sigma(t)$ is the switching signal taking value within the index set $I = \{1, ..., N\}$. We **will assume** that the zero dynamics is asymptotically stable. For a large class of such systems, for any switching signal with a minimal given activation time $T^{\mathrm{active}}_{\min}$ (this means that any of the (2) is active for at least $T_{\min}^{\text{active}}$ units of time) and only using the measured output which is eventually noisy we want the output to track a given signal. This problem up to now, using conventional results, has never been addressed for the following reasons: switching signal is unknown and each subsystem can be complex (nonlinear ones, etc...). But here, we do not need the exact dynamics description (see section 1 for some bibliographical comments).

2.2 Outline of the control design procedure

Assumption 1: Assume now that for all ODE there exist an integer $p \in \{1, \ldots, \min_{i \in I}(p_i)\}$ such that during a short time window we have

$$y^{(p)} = a_{\sigma(t)}(.) + b_{\sigma(t)}(.)u \tag{3}$$

where the functions $a_{\sigma(t)}(.)$ and $b_{\sigma(t)}(.)$ depend on $(t, y, \ldots, y^{(p_{\sigma(t)})}, u, \ldots, u^{(m_{\sigma(t)})}, d)$. If fast estimations of $a_{\sigma(t)}$ and $b_{\sigma(t)}$ are available then the tracking problem is solved using control (1). It remains to show that:

(1) there exists a "large" class of switched systems that meet assumption 1 (see Section 4),
(2) there exists "good" realtime estimation of time derivative of noisy signal (see subsection 3.1),
(3) there exists "good" realtime estimation of $a_{\sigma(t)}$ and $b_{\sigma(t)}$ using only the noisy output and the input (see subsection 3.2),
(4) the closed-loop system is uniformly asymptotically stable (see Section 4).

2.3 Example

Let us consider the speed regulation problem of the following simplified model of a manual transmission (Brockett, 1993). The car dynamics can be represented by the equation:

$$\dot v = -\frac{\beta v^2}{M}\operatorname{sgn}(v) - g\sin(\alpha(t)) + \frac{T(i)}{M}u, \tag{4}$$

where the output v is the speed, M the mass of the vehicle, $\alpha(t)$ the road incline and $T(i)$ is the motor torque, which depends on the selected gear $i \in \{1,2,3,4,5\}$. Let us mention that $a(.) = -\frac{\beta v^2}{M}\operatorname{sgn}(v) - g\sin(\alpha(t))$ is usually not well known and time varying. Now let us consider that we can have an estimation of $a(.)$ and $b(.)$, the problem here is to construct a control law such that v follows a given trajectory v_{ref}. Since the system is flat (cf. (Sira-Ramírez *et al.*, 2004)) with flat output $y = v$ one can define a planned trajectory to be tracked v_{ref}. Suppose that, on some small time window, we have $\dot y = a_0 + b_0 u$ (this assumption is valid for a sufficiently small time window and if $y = v$ is sufficiently smooth). From now, using only the measured output y, if we can get some fast estimations of $\dot y = \dot v$ and b_0, then applying the following sampled control[2] $u((k+1)T_s) = -\frac{\left(k_p e_y + k_i \int e_y\right) - \left(\left([\dot v]_{\text{estim}} - \dot v_{\text{ref}}\right) - b_{0_{\text{estim}}} u(kT_s)\right)}{b_{0_{\text{estim}}}}$, where $e_y = v - v_{\text{ref}}$ is the relative error for y, ones get that $\ddot e_y + k_p \dot e_y + k_i e_y \approxeq 0$, in fact $o(T_s)$, which implies the desired stabilization (see figures below). The gear is changing incrementally every 0.5 s. Figure 1 presents the speed regulation. We can notice on this figure that some oscillatory phenomena can appear (see 1 in Fig. 1) whereas the regulation seems to be correct, this can be explained by the fact that in order to obtain a good estimation, the system has to be excited. This is why the regulation is still correct when some disturbance appears on the output (see 2 in Fig. 1).

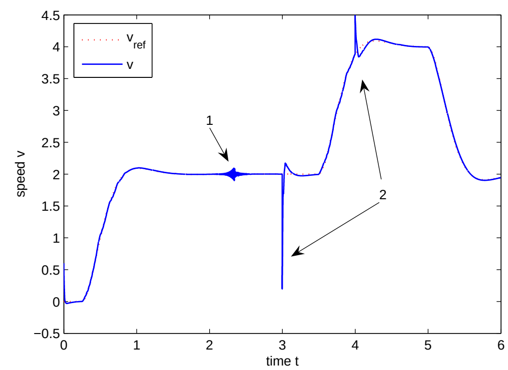

Fig. 1. Trajectory of the switched system 2

3. BACKGROUND ON ALGEBRAIC ESTIMATION

3.1 Numerical differentiation

This algebraic setting for numerical differentiation started in (Fliess, Sira-Ramírez, 2004; Fliess, Join, Mboup, Sira-Ramírez, 2004). See (Mboup *et al.*, 2002) for further developments, and (Nöthen, 2007) for interesting discussions and comparisons. Consider a signal $y(t) = \sum_{i=0}^{\infty} y^{(i)}(0)\frac{t^i}{i!}$ which is assumed to be analytic around $t = 0$ and its truncated Taylor expansion $y_N(t) = \sum_{i=0}^{N} y^{(i)}(0)\frac{t^i}{i!}$ at order N. The usual rules of symbolic calculus in Schwartz's distribution theory (Schwartz, 1966) yield $y_N^{(N+1)}(t) = y(0)\delta^{(N)} + \ldots + y^{(N)}(0)\delta$, where δ is the Dirac measure at zero. Multiply both sides by $(-t)^i$ and apply the rules $t\delta = 0$, $t\delta^{(i)} = -i\delta^{(i-1)}$, $i \geq 1$. We obtain a triangular system of linear equations from which the derivatives $y^{(i)}(0)$ can be obtained $(1 \leq i \leq N)$

$$(-t)^i y_N^{(N+1)}(t) = \frac{N!}{(N-i)!}\delta^{(N-i)}y(0) + \ldots + \delta y^{(N-i)}(0) \tag{5}$$

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[2] This is a sampled version of (1) since $\left([\dot v]_{\text{estim}} - b_{0_{\text{estim}}} u(kT_s)\right) \approx a_{0_{\text{estim}}}$ as soon as T_s is small enough with respect to the variation of $a(.)$ and $b(.)$ which here are approximated by constants $(a_{0_{\text{estim}}}, b_{0_{\text{estim}}})$ on a sliding window of length T_{window}: $T_{\text{window}} >> T_s$.

It means that the coefficients $y(0), \ldots, y^{(N)}(0)$ are *linearly identifiable* (Fliess *et al.*, 2003; Fliess *et al.*, 2007). The time derivatives of $y_N(t)$, the Dirac measures and its derivatives are removed by integrating with respect to time both sides of Eq. (5) at least ν times ($\nu > N$):

$$\int_0^t \int_0^{t_{\nu-1}} \cdots \int_0^{t_1} (-\tau)^i y_N^{(N+1)} dt_{\nu-1} \cdots dt_1 d\tau =$$
$$\frac{N!}{(N-i)!} \frac{t^{\nu-N-i-1}}{(\nu-N-i-1)!} y(0) + \ldots + \frac{t^{\nu-1}}{(\nu-1)!} y^{(N-i)}(0)$$

It is clear that the numerical estimation rely on $\lim_{N\to+\infty} [y_N^{(i)}(0)]_{\text{estim}}(t) = y^{(i)}(0)$.

Remark 1. These iterated integrals are low pass filters which attenuate the noises, which are viewed as highly fluctuating phenomena (see (Fliess, 2006) for more details).

Remark 2. The above formulae may easily be extended to sliding time windows in order to obtain real time estimates (see (Mboup *et al.*, 2002) for further details).

3.2 Parametric estimation

Consider (see (Fliess *et al.*, 2003; Fliess *et al.*, 2007) for more details) the first order ODE $\dot{y} = a(.) + b(.)u$, where $a(.)$ and $b(.)$ are functions which on some small time intervals may be approximated by polynomial time functions, and, for instance, by constants, i.e., $a(.) = a_0$, $b(.) = b_0$, and $\dot{y} = a_0 + b_0 u$. The classic rules of operational calculus yield $sY(s) - y_0 = \frac{a_0}{s} + b_0 U(s)$. The initial condition is annihilated by taking derivatives of both sides w.r.t. s. The unknown parameters are linearly identifiable, i.e., $b_0 = \frac{\int_0^t P(t,\tau)y(\tau)d\tau}{\int_0^t Q(t,\tau)u(\tau)d\tau}$, $P(t,\tau) = -(t-\tau)^2 + 4\tau(t-\tau) - \tau^2$, $Q(t,\tau) = -\tau(t-\tau)^2 + \tau^2(t-\tau)$, $a_0 = \frac{b_0\int_0^t R(t,\tau)u(\tau)d\tau + \int_0^t S(t,\tau)y(\tau)d\tau}{t^3}$, $R(t,\tau) = -6\tau(t-\tau)$, $S(t,\tau) = -6(t-2\tau)$. A singularity may arise in in the above formulae when the stabilization is achieved since numerators and denominators will be both zero. This will be of importance later on for our purpose when dealing with switched systems.

Remark 3. Similar results may be obtained for more general ODEs, such as $y^{(p)} = a(.) + b(.)u$.

4. SOME DETAILS ABOUT CONTROL DESIGN

Comments about Assumption 1: If we look at Eq. (3) it appears that the exact knowledge of $b_{\sigma(t)}$ is not required since $a_{\sigma(t)}(.)$ may contain some control effects: what is needed is only an order of magnitude for this term. When, among the collection of dynamics f_i involved in (2), we can find an integer p leading to (3) with b_i:

(1) of the same order of magnitude: then we **will not** estimate this parameter,
(2) varying with the active dynamics then we **will** estimate this parameter online.

The reason for making a distinction between those two cases is for efficient numerical implementations. It is much simpler in the first case.

First case, the required conditions are:

(1) $\forall i \in I$: there exists a set of integers $p_{i,j} \in \{1, \ldots, p_i\}$ denoted by P_i such that $\frac{\partial f_i}{\partial y^{(p_{i,j})}} \neq 0$ then we set $p = \min(\cap_i P_i)$. This is the smallest integer such that $\frac{\partial f_i}{\partial y^{(p)}} \neq 0, \forall i \in I$. Thus, from the implicit theorem, we have (at least locally) ($\hat{y}^{(n_{\sigma(t)})} \triangleq y, \ldots, y^{(p-1)}, y^{(p+1)}, \ldots, y^{(n_{\sigma(t)})}$):
$$y^{(p)} = F_{\sigma(t)}(t, \hat{y}^{(n_{\sigma(t)})}, u, d). \qquad (6)$$
(2) $\forall i \in I$: $\left.\frac{\partial f_i}{\partial u}\right|_{u=0} \neq 0$. Then we may obtain from experiments or physical laws a rough estimate of $\alpha_i = \left.\frac{\partial F_i}{\partial u}\right|_{u=0}$: $\alpha_{\sigma(t)} \in [m, M]$: $\alpha_{\sigma(t)}$ is of order $10^o, o \in \mathbb{N}$. This will be of importance later on for numerical implementation. Lastly, rewrite (6) as
$$y^{(p)} = a_{\sigma(t)}(.) + 10^o u. \qquad (7)$$

Second case, the required conditions are:

(1) $\forall i \in I$: there exists an integer $p \in \{1, \ldots, p_i\}$ such that $\frac{\partial f_i}{\partial y^{(p)}} \neq 0$. Then Eq. (6) holds at least locally from the implicit function theorem.
(2) $\forall i \in I$: $\left.\frac{\partial f_i}{\partial u}\right|_{u=0} \neq 0$. Rewrite then Eq. (6) as (3).

Taking in practice $p = 1, 2$ is most of the time sufficient. It **does not imply** that the system is of order 1 or 2.

Digital implementation of the control law: Let T_s be the sampling period. The discrete version of Eq. (1) reads

$$[b(.)]_{\text{estim}}(kT_s)u(kT_s) = -[a(.)]_{\text{estim}}(kT_s) + y_{\text{ref}}^{(p)}(kT_s)$$
$$-\sum_{i=-1}^{(p-1)} \alpha_i \left(\left[y^{(i)}\right]_{\text{estim}}(kT_s) - y_{\text{ref}}^{(i)}(kT_s) \right) \qquad (8)$$

Using (7) $\left[y^{(p)}(kT_s)\right]_{\text{estim}} + o(T_s) \approxeq y^{(p)}(kT_s)$ $= a_{\sigma(t)}(.)\big|_{(kT_s)} + 10^o u(kT_s)$. Thus $[a(.)]_{\text{estim}}(kT_s)$ is obtained thanks to $\left[y^{(p)}(kT_s)\right]_{\text{estim}} - 10^o u(kT_s)$. If we plug this estimate into Eq. (8) an algebraic loop appears: thus $[a(.)]_{\text{estim}}(kT_s)$ is replaced by $[a(.)]_{\text{estim}}((k-1)T_s)$. Some analysis (assuming that there is no switching time in this time interval) leads to $e_y^{(p)} + \sum_{i=-1}^{p-1} \alpha_i e_y^{(i)} = o(T_s)$, where $e_y = y(t) - y_{\text{ref}}(t)$. Let us mention that, for (3), similar arguments lead to a similar estimations.

Closed-loop practical stability: Adjust the time response t_r thanks to well-chosen coefficients α_i: after a total time of $T_{\text{window}} + t_r$ the response error is close to zero. From well known results (Persis *et al.*, 2003; Hespanha *et al.*, 1999) about asymptotic stability of switched asymptotically stable systems for any switching law having a given dwell time, we need

$$T_{\text{window}} + t_r < T_{\min}^{\text{active}} \qquad (9)$$

to guarantee uniform asymptotic stability of the closed-loop system for any switching signal having a minimum activation time $T_{\min}^{\text{active}}$. Note that t_r can be tuned using the α_i and $T_{\text{window}} = \kappa T_s$. Eq. (9) gives thus a rule in order to select the sampling period knowing only the minimal time of activation of a subsystem:

$$T_s < \frac{T_{\min}^{\text{active}} - t_r}{\kappa}$$

The error will then enter some ball of radius $o(T_s)$ centered at the origin.

5. EXAMPLES

Consider, in order to demonstrate the effectiveness of our results, the following switched system where the subsystems are defined as follows:

$$\begin{aligned} \dot{x} &= f_i(x) + g_i(x)u + d, \quad x \in \mathbb{R}^{n_i} \\ y &= h_i(x,u) + n \end{aligned}$$

where n is a noise, d a disturbance. The other functions are given by the following table

i	$f_i(x)$	$g_i(x)$	$h_i(x,u)$	n_i
1	$-x$	1	x	1
2	$2x$	1	x	1
3	$\begin{pmatrix} 0 & -3 & -3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$x_1 + 3x_2 + x_3$	3
4	$\begin{pmatrix} 2 & -3 \\ 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \end{pmatrix} x$	2
5	$\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} x$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \end{pmatrix} x$	2
6	$5x + 10\sin(x)$	1	x	1
7	$-2x + 10\exp(x)$	1	x	1

The subsystems are either linear and stable ($i = 1$), unstable ($i = 2, 3, 4$), non-minimum phase ($i = 5$), or even nonlinear ($i = 6, 7$).

The switching signal is randomly selecting any subsystem every 0.5 s (thus here $T_{\min}^{\text{active}} = 0.5$ s). The desired time response is chosen to be $t_r = 0.4$ s. Moreover, $\kappa = 100$, $T_s = 0.001$ s, $T_{\text{window}} = \kappa T_s = 0.1$ s. The sliding window has to be large enough to estimate the signal and not only the disturbance.

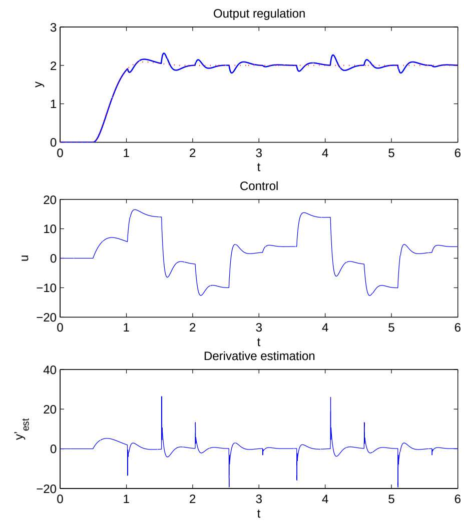

Fig. 2. Simulation results in the linear case

For the two first Figures, since the b_i are of the same magnitudes, we use, according to case 1, Eq. (7) with $p = 1$, the control law given by Eq. (8).

Figure 3 concerns a disturbed case with some additive output noise. It includes nonlinear subsystems. If some noise effects appears in the derivative estimation, the control law ensures the regulation, even when a large disturbance appears at $t = 3$ s.

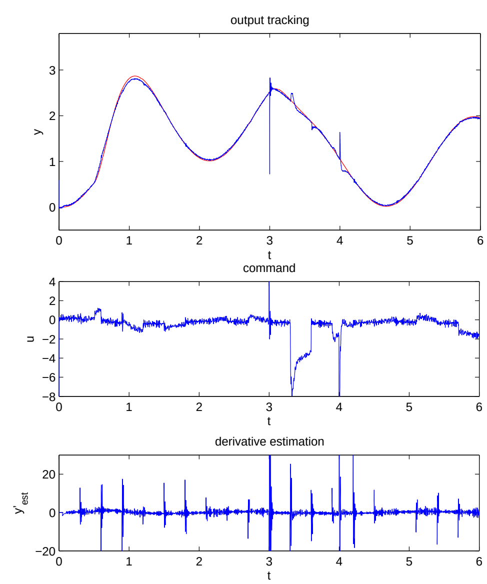

Fig. 3. Simulation results in the nonlinear case

As it is shown in the previous Figures, the proposed technics provides encouraging results: the obtained control laws ensure the output tracking. But this control is only valid for subsystems where the b_is are of the same magnitude in Eq. (3). Indeed, in the introductive example, the coefficient $\frac{T(i)}{M}$ takes its values among $\{1, 5, 10, 30, 50\}$ (the behavior is completely different between the first gear and the fifth one). Then, the control law (8), without an estimation of this parameter, is not enough for obtaining a correct tracking. This is illustrated by Figure 4: the oscillations which appear are direct consequences of the non-estimation of the parameter b. The fact that the oscillations disappear is the result of the switching and not because the derivative estimation is correct. This is the reason why in the introduction the applied control is of the form (8) with an online estimation of the parameter b.

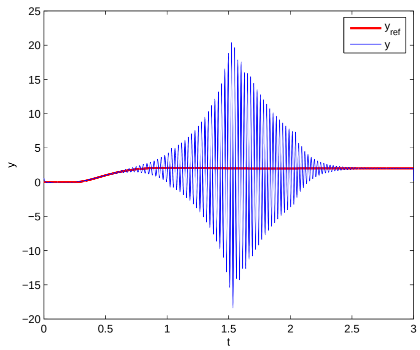

Fig. 4. Simulation results without an estimation of b

6. CONCLUSION

We proposed in this communication a new approach for the control of hybrid systems, where no description of the sub-dynamics is needed. Our key idea is to get fast derivative estimations in order to give an explicit formula of the control law which ensures the output tracking. When a switching occurs, the current controller is not adapted during the estimation windows: we will therefore try to improve our algorithm in order to limit its influence during this period.

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