Near Optimal Broadcast with Network Coding in Large Sensor Networks

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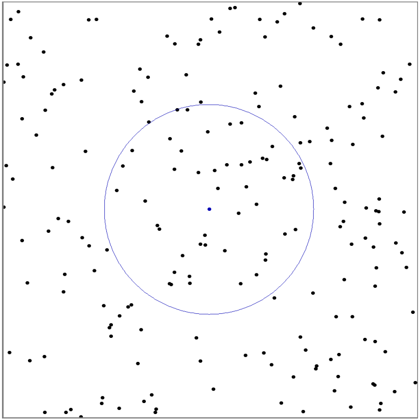
Motivation

Wireless Sensor Networks

- Low cost elements: large networks
- Limited battery power
- Wireless communication



Efficient Broadcast

Problem studied

- Specific form of communication
  - Broadcast: from one source to all nodes in the network
- Energy Efficiency:
  - minimize the total number of transmissions for broadcasting one message for the source

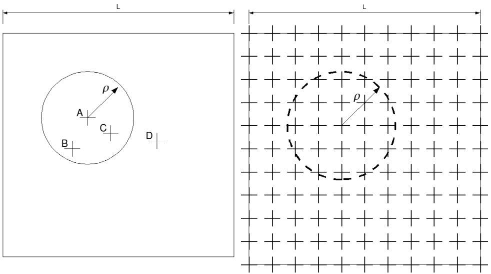
Network Model

Idealized Model

- Unit-disk graph (boolean)
  - range: ρ
- No packet loss
- No interference
- No capacity limit

Topologies

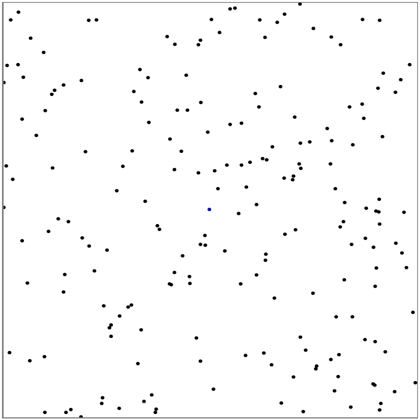
- Lattice
- Random (uniform)



Efficient Broadcast

Without Network Coding

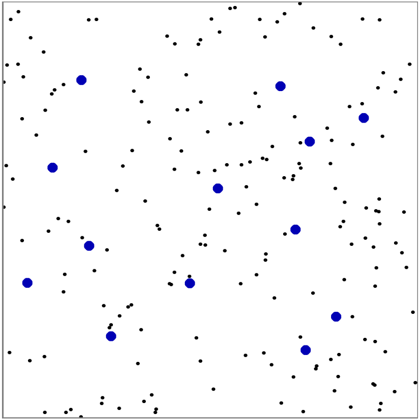
- Wireless case: heuristics exist



Efficient Broadcast

Without Network Coding

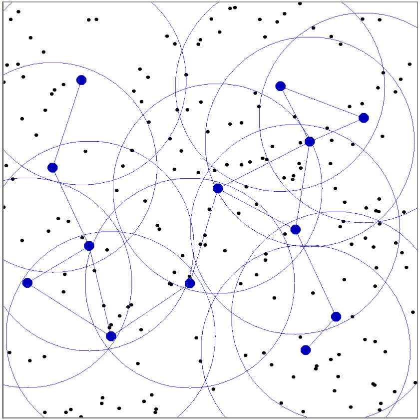
- Wireless case: heuristics exist
- Some subset of nodes retransmits messages



Efficient Broadcast

Without Network Coding

- Wireless case: heuristics exist
- Some subset of nodes retransmits messages
- Connected Dominating Set



Efficient Broadcast with Network Coding

Question:

- What about Energy Efficient Broadcast with Network Coding in Wireless Networks?

Efficient Broadcast with Network Coding

Results:

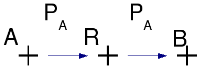
- Propose one way to do network coding ("rate selection")
- Show that it is energy efficient: "optimal at the transmission level"
  – asymptotically for some classes of networks
- Offers advantages over routing

Network Coding Principle 1/2

Transmission without coding

Image nodes A and B wants to communicate, through one relay node R

- A sends a packet P_A to node R
- R forwards the packet P_A to node B



Network Coding Principle 1/2

Transmission without coding

Image nodes A and B wants to communicate, through one relay node R

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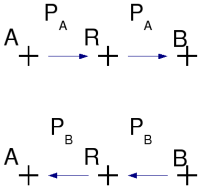
- B sends a packet P_B to node R
- R forwards the packet P_B to node A



Network Coding Principle 2/2

With coding

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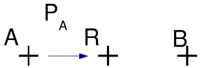

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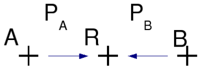

Network Coding Principle 2/2

With coding

- A sends a packet P_A to node R
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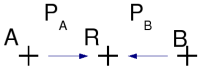
- R computes the packet $P_A \text{ xor } P_B$



Network Coding Principle 2/2

With coding

- A sends a packet P_A to node R
- B sends a packet P_B to node R
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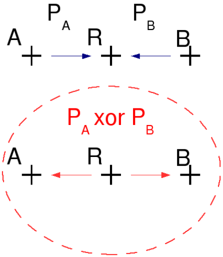
- R sends the coded packet to both nodes A and B
- A and B can decode and get P_b and P_a



Coding performed inside the network

- Ex. of another form of coding: linear coding, packets are vectors of a Galois Field (ex: $GF(2^8)$)
- linear combination: $P_{\text{send}} = \sum_i \alpha_i P_i$

Transmission-Level Efficiency

Transmission-level efficiency

- One transmission reaches several neighbor nodes
- Efficiency (at the trans. level): useful for several nodes
- Innovative $\triangleq$ useful
- Transmission-level optimality $\triangleq$ the transmission is useful for every receiver
- Difference with point-to-point link

Energy-Efficient Broadcast with Network Coding

Problem of Energy-Efficient Broadcast

- Single source broadcast: perf. depends on avg. rate of nodes??
- Select the rate of each node
- For these rates: maximum broadcast rate
- $\rightarrow$ number of transmissions per broadcast message

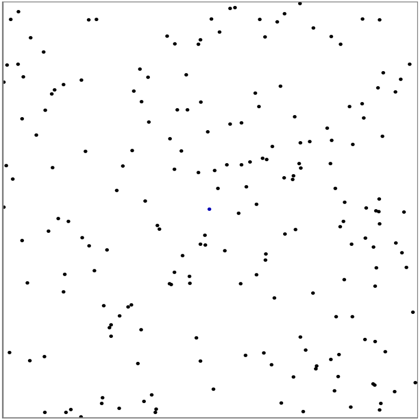
Optimal Solution: Energy-Efficient Broadcast

- Finding rate and cost (at same time):
  - Optimization problem
  - Linear Program [Wu et al. 2004] [Lun et al. 2004]
  - Solved in polynomial time
  - Distributed algorithms exist

Energy-Efficient Broadcast

Sample solution

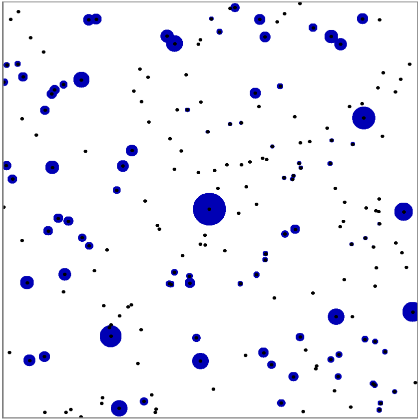
- Sensor Network
- Linear Program:
  - $\approx N^2 M$ variables (here sparse matrix with 3710156 coefs)



Energy-Efficient Broadcast

Sample solution

- Sensor Network
- Linear Program:
  - $\approx N^2 M$ variables (here sparse matrix with 3710156 coefs)
- Solution for the rate
  - Source rate = 1
  - Total rate = 9.0625



Issues

- Simple rate selection?
- How much to expect?

Our Approach: overview

Our Approach

- Start with a simple rate selection

Rate Selection

Rate Selection

- Starting point:
  same rate for all nodes $= 1$

Further Reasoning

- Assume homogeneous network: each node has $\approx M$ neighbors
- Consider: innovative packets
- If every received transmission is innovative,
- Then: node with M neighbors, would receive M useful packets per unit time
- Hence: the source needs to send M packets per second
- Problem: nodes on border may have less than M neighbors

Rate Selection

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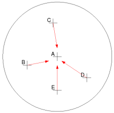

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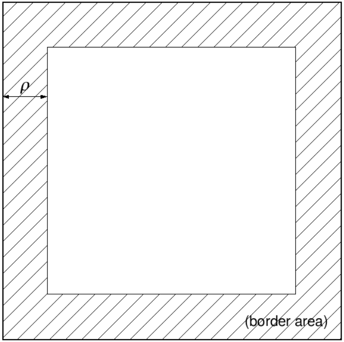
Proposed Rate Selection

Rate Selection: IREN/IRON

- *Increased Rate for Exceptional Nodes* (*IREN*):
  • rate M for the source, and nodes in the border area
- *Identical Rate for Other Nodes* (*IRON*):
  • rate 1 for other nodes

(M: number of neighbors)



Performance of the Rate Selection IREN/IRON

Key result

Maximum broadcast rate with IREN/IRON

Theorem

For a lattice: the maximum broadcast rate is exactly M

Theorem

For a random graphs: $\frac{\text{maximum broadcast rate}}{M} \xrightarrow{p} 1$, *asymptotically, when density M grows (and density M grows fast enough compared to network width L*

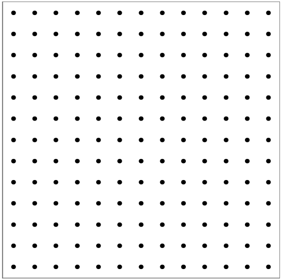
– e.g. $M = L^{\theta}$ *with* $\theta > 0$*)*

Proof.

Discrete geometry, *min-cut* computation and probability □

Overview of the proof for lattice

Proof: maximum broadcast rate $\geq M$



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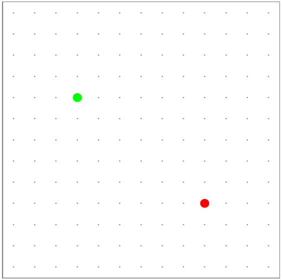

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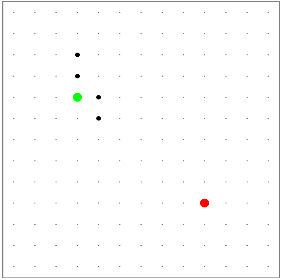

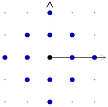

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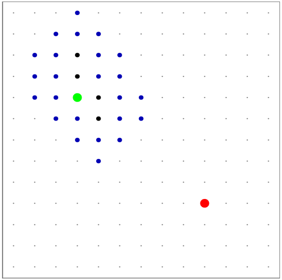

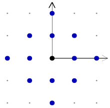

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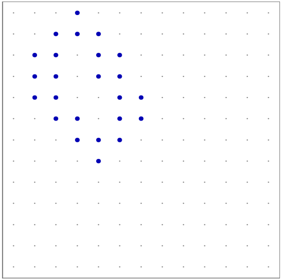
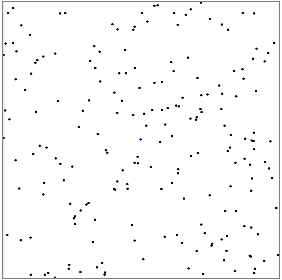
- Neighborhood of U (including U): Minkowski sum: $(U \oplus R) \triangleq \{u + v : u \in U, v \in R\}$
- Neighbors of U (without U), ΔU: $\Delta U = (U \oplus R) \setminus U$
- Brunn-Minkowski inequality for finite sets: $|A \oplus B| \geq |A| + |B| - 1$
- Sum of rates to $U \geq |R| - 1 = M$



Overview of the proof for unit disk graphs

Unit disk graphs

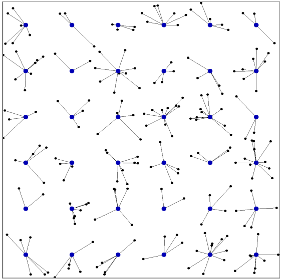
- Seq. of random unit disk graphs



Overview of the proof for unit disk graphs

Unit disk graphs

- Seq. of random unit disk graphs
- Create a virtual (*embedded*) lattice
- Map the nodes to the virtual lattice
- Capacity of an s-t cut, related to capacity of s_L-t_L cut on the virtual lattice: $C_{\min} \geq m_{\min} C_{\min}^{(L)}$
- When $M \to \infty$ fast enough, the results of the lattice are generalized to the seq. of unit disk graphs.



Energy-efficiency

Energy-efficiency and optimality

- Assumption behind IREN/IRON:
  - every transmission is *innovative*, and received rate = M (1 per M neighbors)
- Proven: broadcast rate $\approx M$
- Indeed: $\approx$ every transmission is innovative
- Optimality: comes from reaching information-theoretic bound for each transmission
  - Transmission-level optimality
- Except: issues for the border nodes and random graphs

Asympotic Optimality

Bound for Energy-efficiency

- N nodes; and at most $M_{\max}$ neighbors for any node
- Minimum transmissions per broadcast: $E_{\text{bound}} = \frac{N}{M_{\max}}$
- Actual transmissions per broadcast: $E_{\text{cost}} = \frac{\text{transmission rate}}{\text{broadcast rate}}$

Theorem

For lattice graphs, cost per broadcast of IREN/IRON converges towards cost of the bound, i.e. $\frac{E_{\text{cost}}}{E_{\text{bound}}} \to 1$, *when* L *grows to* ∞ *and range* ρ *is fixed.*

Theorem

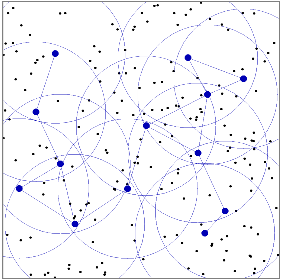
For random graphs, $\frac{E_{\text{cost}}}{E_{\text{bound}}} \xrightarrow{p} 1$ *when* L *grows to* ∞, *density grows as* $M = L^{\theta}$, *with* $0 < \theta < 1$.

Asymptotic Optimality

- Stronger result than optimality of linear program

Comparison with routing

- In wired networks, [Edmonds, 1972], for broadcast routing is sufficient.
- In wireless networks, it is different, logic from [Widmer et al. 2005]:
  - except the source, every retransmitter receives from a node
  - when retransmit, common neighbors receive
- Hence asymptotically, at least $(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}) \approx 0.391\ldots$ of the area receives redundant transmissions.



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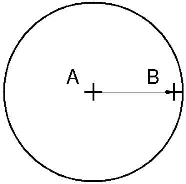

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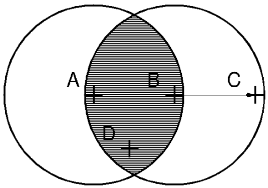

Summary

Summary

- Presented a rate selection: IREN/IRON
- Computed its performance (maximum broadcast rate from the source)
- Shows that it is optimal, asymptotically
- Offers advantages over routing

Future Work

- Simple rate selection for given graph? Non-homogeneous?
- Fully distributed operation?
- Realistic wireless models?

Thank you for your attention