From Path to Trajectory Deformation

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Abstract: Path deformation is a technique that was introduced to generate robot motion wherein a path, that has been computed beforehand, is continuously deformed on-line in response to unforeseen obstacles. This paper introduces the first trajectory deformation scheme as an effort to improve path deformation. The main idea is that by incorporating the time dimension and hence information on the obstacles' future behaviour, quite a number of situations where path deformation would fail can be handled. The trajectory deformation scheme presented operates in two steps: a collision avoidance step and a connectivity maintenance step, hence its name *2-Step-Trajectory-Deformer* (2-STD). In the collision avoidance step, repulsive forces generated by the obstacles deform the trajectory so that it remains collision-free. The purpose of the connectivity maintenance step is to ensure that the deformed trajectory remains feasible, *ie*, that it satisfies the robot's kinematic and/or dynamic constraints. Moreover, unlike path deformation wherein spatial deformation only takes place, 2-STD features both *spatial and temporal* deformation. It has been tested successfully on a planar robot with double integrator dynamics moving in dynamic environments.

Key-words: motion planning, trajectory deformation, collision avoidance.

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De la déformation de chemin à la déformation de trajectoire

Résumé : La déformation de chemin est une technique qui a été introduite pour calculer le mouvement d'un robot. Elle consiste à déformer de façon réactive un chemin nominal en réaction à la présence d'obstacles imprévus. Dans le but d'améliorer la déformation de chemin, ce rapport présente la première technique de déformation de trajectoire. Son principe consiste à prendre en compte la dimension temporelle et donc des informations sur le mouvement futur des obstacles mobiles afin de résoudre des problèmes que la déformation de chemin seule ne peut pas traiter de façon satisfaisante. Le schéma de déformation de trajectoire comporte deux étapes: une étape d'évitement de collision et une étape de maintien de la connectivité de la trajectoire, d'où son nom *2-Step-Trajectory-Deformer* (2-STD). Lors de l'étape d'évitement de collision, des forces répulsives engendrées par les obstacles de l'environnement déforment la trajectoire de sorte qu'elle demeure sans collision. Le rôle de l'étape de maintien de la connectivité est d'assurer que la trajectoire ainsi déformée demeure faisable, *ie* qu'elle vérifie les contraintes cinématiques et dynamiques du robot. A la différence des techniques de déformation de chemin, 2-STD comporte des déformations spatiales mais aussi temporelles. Elle a été testé avec succès dans le cas d'un robot planaire contrôlé en accélération placé en environnement dynamique.

Mots-clés : planification de mouvement, déformation de trajectoire, évitement de collision.

Contents

1 Introduction

1.1 Background and Motivation

Designing an autonomous robotic system requires to solve a number of challenging problems in domains as different as perception, localisation, environment modelling, reasoning and decision-making, control, *etc*. However, whatever the robotic system and whatever the kind of tasks it is expected to carry out, at some point, it has to move. Motion determination is therefore a fundamental issue that has been widely addressed by the Robotics community. In the late sixties, Shakey [1] was one of the first robotic system able to move and perform simple tasks autonomously. Since then a large number of autonomous navigation architectures have been proposed (*cf* the recent review established in [2]).

From the motion determination perspective, these navigation architectures can be broadly classified into *deliberative* (*aka motion planning-based*) versus *reactive* approaches: deliberative approaches aim at computing a complete motion all the way to the goal using motion planning techniques, whereas reactive approaches determine the motion to execute during the next time-step only[1].

Deliberative approaches have to solve a motion planning problem [7]: they require a model of the environment as complete as possible and their intrinsic complexity is such that it may preclude their application in dynamic environments[2]. Reactive approaches on the other hand can operate on-line using local sensor information: they can be used in any kind of environment whether unknown, changing or dynamic. This accounts for the large number of reactive approaches that have been developed over the years, *eg* [8, 9, 10, 11, 12, 13, 14], *etc*. Reactive approaches however face a challenge: the convergence towards the goal is not guaranteed (local minima problem).

In an attempt to bridge the gap between deliberative and reactive approaches, a complementary approach has been proposed based upon *motion deformation*. The principle is simple: a complete motion to the goal is computed first using a priori information. It is then passed on to the robotic system for execution. During the course of the execution, the still-to-be-executed part of the motion is continuously deformed in response to sensor information acquired on-line, thus accounting for the incompleteness and inaccuracies of the a priori world model. Deformation usually results from the application of constraints both external (imposed by the obstacles) and internal (to maintain motion feasibility and connectivity). Provided that the motion connectivity can be maintained, convergence towards the goal is achieved (Fig. 1).

It appears that the different motion deformation techniques that have been proposed [16, 17, 18, 19, 20] all performs *path deformation*. In other words, what is deformed is a geometric curve, *ie* the sequence of positions that the robotic system is to take in order to reach its goal. Path deformation is not entirely satisfactory when applied to environments featuring moving obstacles: the problem is illustrated in Fig. 2. When a moving obstacle is to cross the path of the robotic system, path deformation takes place. Deformation increases in

[1] In a few approaches, the motion is computed for a number, fixed or arbitrary, of time-steps [3, 4, 5, 6].
[2] Arguments about this issue can be found in [4] and [6].

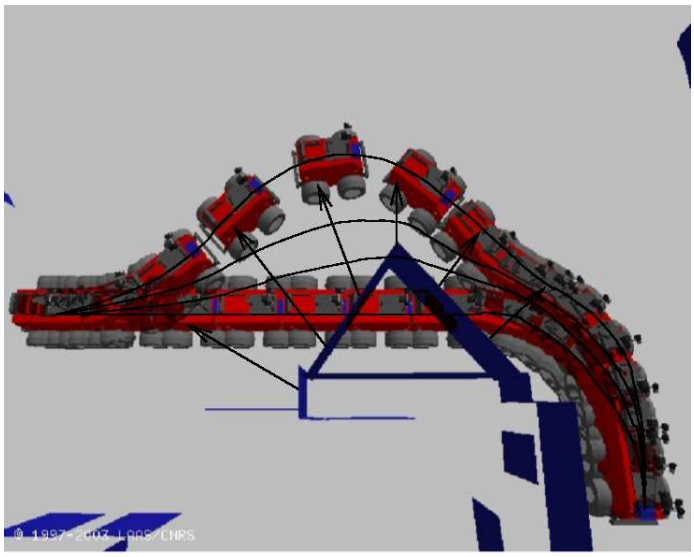


Figure 1: Path deformation principle: the initial path is deformed so as to account for the triangular obstacle that was unknown at planning time [15].

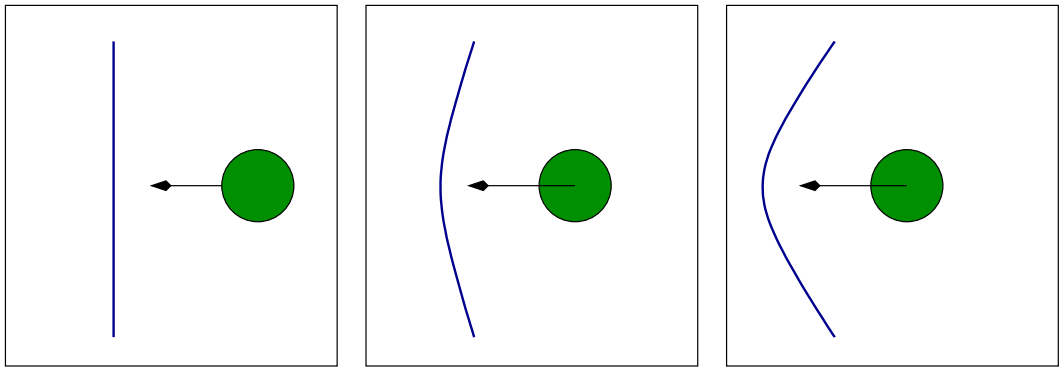


Figure 2: Path deformation problem: in response to the approach of the moving disk, the path is increasingly deformed until it snaps (like an elastic band).

response to the approach of the moving obstacles. At some point, the path connectivity can no longer be maintained (when the internal constraints are violated). Like an elastic band, the path snaps and it is necessary to resort to path planning in order to determine a new path to the goal.

1.2 Contribution and Paper Outline

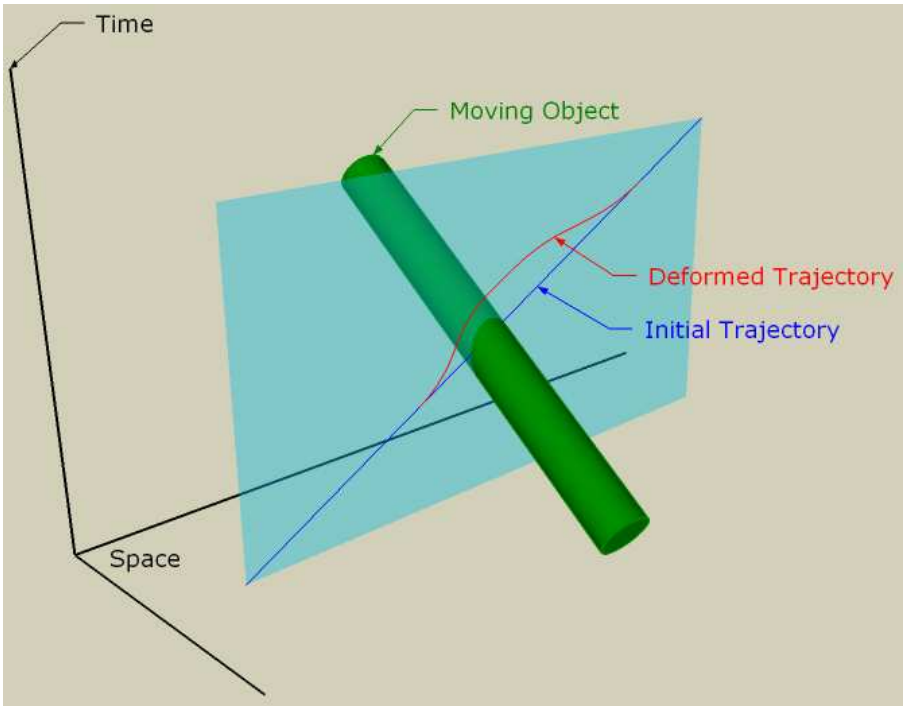


Figure 3: Temporal deformation of a trajectory.

The problem with path deformation techniques is that, by design, they cannot take into account the time dimension of a dynamic environment. For instance in a scenario such as the one depicted in Fig. 2, it may be more appropriate to leave the path as is and simply adjust the velocity of the robotic system along the path so as to avoid collision with the moving obstacle (by slowing down or accelerating). This is illustrated in Fig. 3 that depicts the scenario of Fig. 2 with the time dimension included. The cylinder represents the volume swept by the moving disk as times passes by (the disk is moving at constant linear velocity). Let us assume that the robotic system were to follow its nominal path with a constant velocity. If such a motion eventually yields a collision (when the trajectory intersects the

cylinder), a local deformation of the velocity profile near the collision point suffices to obtain a new collision-free trajectory.

In general, depending on the obstacle's future behaviour, it may be necessary to deform *both* the path *and* the velocity of the robotic system simultaneously. To achieve this, it is necessary to depart from the path deformation paradigm and resort to *trajectory deformation* instead. Broadly speaking, a trajectory is a continuous time-sequence of states. It tells us not only where the robotic system will move but also *how* and *when* it will do it. Let us picture a trajectory as a geometric curve in the state×time space of the robotic system [21]. Let us assume as well that knowledge about the future behaviour of the moving obstacles is available, each obstacle, moving or not, yields a forbidden subset of the state×time space (the equivalent of the well-known configuration-obstacle). A trajectory in the state×time space must avoid these "state×time-obstacles" otherwise it means a collision will take place at some time. Now, when a moving obstacle changes its behaviour, the corresponding state×time-obstacle changes accordingly. Should the trajectory be no longer collision-free, it should be deformed accordingly.

This is precisely what trajectory deformation is about. Unlike path deformation wherein spatial deformation only takes place, trajectory deformation features both *spatial and temporal* deformation meaning that the planned velocity of the robotic system can be altered thus permitting to handle gracefully situations such as the one depicted in Fig. 2.

The contribution of this paper is the first trajectory deformation scheme. The robotic system starts with an initial trajectory to its goal. Then, periodically, an updated model of the environment is obtained (using on-board sensors for instance). It should include information about the dynamics of the moving obstacles (*eg* linear and/or angular velocity, *etc*). Using this information and long-term motion prediction techniques such as [22, 23] or [24], the model of the future evolution of the environment can be updated. At each update, the model of the future is used to determine whether the current trajectory is collision-free. If not, it is deformed accordingly. Note that we do not need information on the whole environment. We can just update the environment information around the current position of the robot, and assume the other part of the environment is still the same. Later on, when the robot receive information about the other part of the environment and realize that the trajectory is in-collision, it can again deform the trajectory as needed. By doing so, of course at any moment, some parts of the trajectory may be in-collision. However, due to fast trajectory deformation procedure, the robot traverses a collision-free trajectory.

The trajectory deformation scheme operates in the state×time space of the robotic system considered. It involves two steps: a collision avoidance step and a connectivity maintenance step, hence its name 2-Step-Trajectory-Deformer (2-STD). In the collision avoidance step, repulsive forces generated by the obstacles deform the trajectory so that it remains collision-free. This step applies repulsive forces in the workspace×time space to several points on the robot, and then transform these forces to generate a repulsive force in the robot's state×time space. By doing so, this step can be applied to high dofs robot. The purpose of the connectivity maintenance step is to ensure that the deformed trajectory remains feasible, *ie* that it satisfies the robotic system's kinematic and/or dynamic constraints.

Depending on the dynamic equation of the robot, the connectivity maintenance step is performed either independently per dimension or using a method similar to the bang bang approach. Hence, this step can be applied to high dofs robot efficiently, too. And therefore, 2-STD is applicable to robots with any number of dofs.

The paper is organised as follows: 2-STD is overviewed in 2 whereas 3 details the overall algorithm, the collision avoidance and the connectivity maintenance steps. Experiments carried out with a planar robot with double integrator dynamics (subject to velocity and acceleration bounds) are presented and discussed in 4. Limitations and possible improvements of 2-STD are discussed in 5.

2 Overview of the approach

Throughout the paper, we consider that the robot's motion is governed by an equation of the form:

$$\dot{s} = f(s, u) \quad ; \quad u_{\min} \leq u \leq u_{\max} \tag{1}$$

where $s \in \mathcal{S}$ is the robot's state. In a first order system, a state refers to the robot's configuration, while in a second order system, a state refers to a tuple of the robot's configuration and velocity. The variable $\dot{s} \in \mathcal{V}$ is the state's first derivative with respect to time or velocity for short, and $u \in \mathcal{U}$ is a control input bounded by minimum control u_{min} and maximum control u_{max}. The set $\mathcal{S}$, $\mathcal{V}$, and $\mathcal{U}$ are the state space, velocity space, and control space, respectively.

2-STD assumes that a collision-free trajectory γ taking the robot to its goal has been computed prior to execution. γ is discretized into a sequence of nodes. Each node is denoted by (s, t), where $t \in \mathcal{T}$ is the time. It is assumed that γ is discretized in such a way that two subsequent nodes (s_i, t_i) and (s_{i+1}, t_{i+1}) can be connected by a trajectory of *constant* control input. In general, a node (s_{i+1}, t_{i+1}) is said to be in the *reachable space* of a node (s_i, t_i) iff there is a valid constant control input that will move the robot from s_i at t_i to s_{i+1} at t_{i+1}. Similarly, (s_i, t_i) is said to be in the *back-reachable space* of (s_{i+1}, t_{i+1}).

Periodically, the robot receives an updated model of its workspace $\mathcal{W}$. This world model includes the prediction of the future motion of the moving obstacles. Upon receiving the new world model, say at time t_{curr}, 2-STD checks whether the still-to-be-executed nodes of γ are still collision-free or not. It does this one by one starting from the node (s, t) where $t > t_{\text{curr}}$. If a node is no longer collision-free in the updated world model, 2-STD deforms γ.

2-STD deforms γ by moving the nodes in $\mathcal{S} \times \mathcal{T}$ in two steps: obstacle avoidance step and connectivity maintenance step. In the obstacle avoidance step, 2-STD adopts the external force mechanism used in the Elastic Strip approach [18]. A set of points (called control points) are defined over the robot body. Based on a particular distance function, repulsive forces generated by the different obstacles is defined for each control point. However, unlike elastic strip, the repulsive forces in 2-STD are defined in $\mathcal{W} \times \mathcal{T}$, instead of $\mathcal{W}$. Applying a repulsive force to a given control point pushes the control point in $\mathcal{W} \times \mathcal{T}$ away from the

obstacle. Each of the repulsive forces are mapped into a displacement vector in $\mathcal{S} \times \mathcal{T}$ and the sum of these displacement vectors will be applied to the obstructed node.

Once the obstacle avoidance step is completed, γ may no longer be connected. In the connectivity maintenance step, 2-STD tries to restore the connectivity. It operates locally on triples of subsequents nodes. Suppose $(s_j{}', t_j{}')$ is the result of applying the obstacle avoidance step to (s_j, t_j) and $(s_j{}'', t_j{}'')$ is the result of applying the connectivity maintenance step to $(s_j{}', t_j{}')$. To ensure the connectivity of a trajectory, 2-STD tries to ensure the connectivity of each triplet $(s_{i-1}{}', t_{i-1}{}')$, $(s_i{}', t_i{}')$, $(s_{i+1}{}', t_{i+1}{}')$ in the trajectory, sequentially. It tries to move these nodes such that $(s_i{}'', t_i{}'')$ is in the reachability space of $(s_{i-1}{}'', t_{i-1}{}'')$ and the back-reachability space of $(s_{i+1}{}'', t_{i+1}{}'')$. For computational efficiency, the nodes are moved in $\mathcal{S}$ only. Now, to respect the displacements generated by the obstacle avoidance step, 2-STD tries to minimize the total displacements,

$$d(s'_{i-1}, s'_i, s'_{i+1}) = \sum_{j=i-1}^{i+1} |s'_j - s''_j| \tag{2}$$

The overall strategy of 2-STD is shown in Algorithm 1. And the details are described in the next section.

Algorithm 1 2-Step-Trajectory-Deformer (2-STD)

Let γ be the current trajectory followed by the robot.
Upon receiving new world model, check whether γ is still collision-free or not.
if γ is obstructed **then**
  Let (s_j, t_j) be the first node of γ that becomes obstructed with respect to the new world model.
  repeat
    if (s_i, t_i) has not been deformed by the obstacle avoidance step **then**
      Apply the obstacle avoidance step to (s_i, t_i) and keep the result in $(s_i{}', t_i{}')$.
    if node (s_i, t_i) is the last node and $(s_i{}', t_i{}')$ is not the goal state **then**
      Add extra node (s_{i+1}, t_{i+1}).
    if node (s_i, t_i) is not the last node **then**
      if (s_{i-1}, t_{i-1}) has not been deformed by the obstacle avoidance step **then**
        Apply the obstacle avoidance step to (s_{i-1}, t_{i-1}) and keep the result in $(s_{i-1}{}', t_{i-1}{}')$.
      if (s_{i+1}, t_{i+1}) has not been deformed by the obstacle avoidance step **then**
        Apply the obstacle avoidance step to (s_{i+1}, t_{i+1}) and keep the result in $(s_{i+1}{}', t_{i+1}{}')$.
      Apply the connectivity maintenance step to triplet $(s_{i-1}{}', t_{i-1}{}')$, $(s_i{}', t_i{}')$, and $(s_{i+1}{}', t_{i+1}{}')$ and keep the results in $(s_{i-1}{}'', t_{i-1}{}'')$, $(s_i{}'', t_i{}'')$, and $(s_{i+1}{}'', t_{i+1}{}'')$, respectively.
    $i = i + 1$.
  until all nodes from (s_i, t_i) until the last node of γ are collision-free, (s_{i+1}, t_{i+1}) is reachable from $(s_i{}'', t_i{}'')$, and the last node is at the goal state.
  if the deformed trajectory is collision free **then**
    Let the robot follow the deformed trajectory. So, γ is now : $<(s_1, t_1)$, ..., (s_{j-2}, t_{j-2}), $(s_{j-1}{}'', t_{j-1}{}'')$, ..., $(s_i{}'', t_i{}'')$, (s_{i+1}, t_{i+1}), ... $>$.
  else
    Find a new trajectory starting from the current node.

3 2-Step-Trajectory-Deformer

The details of 2-STD are described below, starting with the obstacle avoidance step (step-6 and step-10 of Algorithm 1) in Section 3.1, the connectivity maintenance step (step-11 of Algorithm 1) in Section 3.2, and ending with how we add the nodes (step-8 of Algorithm 1) in Section 3.3.

3.1 Obstacle Avoidance Step

2-STD avoids collisions by applying repulsive forces to the obstructed nodes. Suppose γ is the trajectory that is currently followed by the robot. And suppose at time t, the robot receives a new world model of the future where node (s_i, t_i) of γ becomes obstructed. To avoid collision, 2-STD generates repulsive forces in $\mathcal{W} \times \mathcal{T}$ to move the robot's control points away from obstacles. It then transforms each of these forces to a displacement vector in $\mathcal{S} \times \mathcal{T}$. And the sum of these vectors will then be applied to (s_i, t_i).

Since the repulsive force is defined based on distance function in $\mathcal{W} \times \mathcal{T}$, before describing the repulsive force, we need to first define a metric in $\mathcal{W} \times \mathcal{T}$. The main issue here is in deciding the scaling between a unit distance in the spatial dimension $\mathcal{W}$ and a unit distance in the temporal dimension $\mathcal{T}$. In this paper we simply assume that 1 m in $\mathcal{W}$ is the same as 1 sec in $\mathcal{T}$. Other scalings are of course possible and we discuss them in Section 5. Once the scaling is resolved, we consider $\mathcal{W} \times \mathcal{T}$ as a Euclidean space and use Euclidean metric as the metric in $\mathcal{W} \times \mathcal{T}$.

Now, we can define the repulsive force applied to a control point c. Let's denote the position of c when the robot is at (s_i, t_i) as c_i. The goal of the repulsive force is to move c_i in $\mathcal{W} \times \mathcal{T}$ away from obstacles. Since the robot's control points are supposed to represent the whole robot, 2-STD "enlarges" the obstacles by a constant factor r_{inf}. We call the enlarged obstacle as the obstacle's influence region and a point in $\mathcal{W} \times \mathcal{T}$ will be repulsed whenever it lies inside an obstacle's influence region. To compute the force, 2-STD will first find a point on the boundary of the obstacles' influence region that is nearest to c_i. For computational efficiency, 2-STD also tries to preserve the temporal ordering of the nodes in γ. Thus, the deformation in the time dimension is limited to within the time dimension of the previous node (s_{i-1}, t_{i-1}) and the subsequent node (s_{i+1}, t_{i+1}). To be precise, 2-STD sets the time deformation to be within $[t_{i-1,i}, t_{i,i+1}]$ where $t_{j,k} = (t_j + t_k)/2$. So, 2-STD finds a point on the boundary of the obstacles' influence region within $[t_{i-1,i}, t_{i,i+1}]$ that is nearest to c_i. Suppose this point is (w_n, t_n). The repulsive force can then be defined as follows (see Fig. 4 for an illustration),

$$F_{\mathcal{W}}(s_i, c) = k_{ext}(w_n - c_i) \quad ; \quad F_{\mathcal{T}}(s_i, c) = t_n - t_i \tag{3}$$

where k_{ext} is the repulsion gain. The constant k_{ext} is not used in $F_{\mathcal{T}}$ to ensure that the deformation in $\mathcal{T}$ will not change the temporal ordering of the nodes in γ.

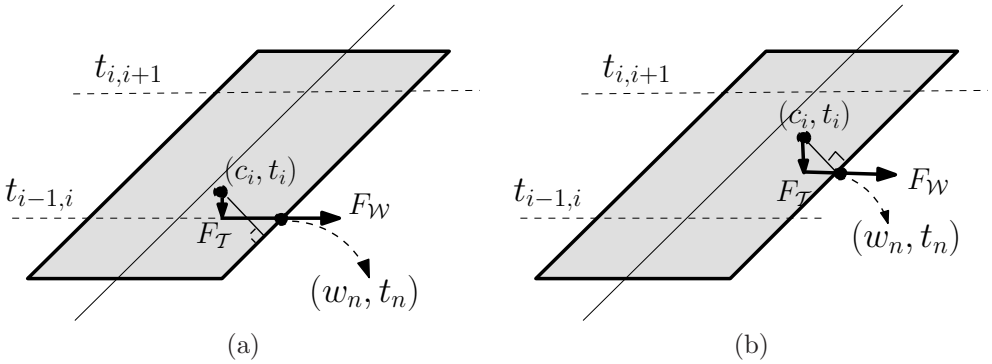


Figure 4: The repulsive force in $\mathcal{W} \times \mathcal{T}$. The grey parallelogram is the influence region of an obstacle in $\mathcal{W} \times \mathcal{T}$. (a) When $t_n = t_{i-1,i}$. (b) When $t_{i-1,i} < t_n < t_{i,i+1}$.

The repulsive forces will then be transformed and summed to generate a displacement vector in $\mathcal{S} \times \mathcal{T}$ as follows,

$$\begin{aligned} D_{\mathcal{S}}(s_i) &= \sum_{\forall c \in C} J_c^T(s_i) F_{\mathcal{W}}(s_i, c) \\ D_{\mathcal{T}}(s_i) &= \frac{1}{|C|} \sum_{\forall c \in C} F_{\mathcal{T}}(s_i, c) \end{aligned} \tag{4}$$

where C is the set of the control points of the robot, $|C|$ is the cardinality of C, and $J_c(s_i)$ is the Jacobian at point c of the robot while the robot is at s_i. This displacement vector will then be applied to (s_i, t_i) to move it away from the obstacle.

3.2 Connectivity Maintenance Step

After the trajectory γ has been deformed to avoid obstacles, it may no longer be connected. The connectivity maintenance step tries to ensure the connectivity of the deformed trajectory while respecting the results of the obstacle avoidance step, as much as possible.

To ensure the connectivity of the trajectory, 2-STD ensures the connectivity of each triplet of nodes, sequentially. To keep the notation simple, we will use triplet (s_0, t_0), (s_1, t_1), (s_2, t_2) and assume that these nodes are the results of the obstacle avoidance step. They are connected whenever there is a pair of velocities $(\dot{s_0}, \dot{s_1})$ such that:

$$s_1 = s_0 + \int_{t=t_0}^{t_1} \dot{s_0}\, dt \;\; ; \;\; s_2 = s_1 + \int_{t=t_1}^{t_2} \dot{s_1}\, dt \tag{5}$$

We also denote the result of the connectivity maintenance step as $({s_0}', {t_0}')$, $({s_1}', {t_1}')$, $({s_2}', {t_2}')$. Maintaining connectivity of triplet means moving the nodes such that the resulting

node $({s_1}', {t_1}')$ is in the reachability space of $({s_0}', {t_0}')$ and in the back-reachability space of $({s_2}', {t_2}')$. Thus, to ensure the connectivity of the triplet, we need to find a valid pair of velocities $\dot{s}_0$ and $\dot{s}_1$ that move the robot from $({s_0}', {t_0}')$ to $({s_1}', {t_1}')$ and from $({s_1}', {t_1}')$ to $({s_2}', {t_2}')$, respectively.

Furthermore, to respect the result of obstacle avoidance step, we also need to minimize the displacement $d(s_0, s_1, s_2)$, as defined in (2). Hence, the main issues here is to find a desired valid pair of velocities $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$, *ie*, a pair of velocities that satisfy the motion equation of the robot and minimizes the displacement $d(s_0, s_1, s_2)$.

Let's now describe the overall method for finding the desired valid pair of velocities $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$, *ie*, both velocities are valid and the pair generates a minimum displacement. First, 2-STD approximates the integration in (5) with summation to get

$$s_1 = s_0 + (t_1 - t_0)\dot{s}_0 \quad ; \quad s_2 = s_1 + (t_2 - t_1)\dot{s}_1 \tag{6}$$

When $(t_1 - t_0)$ and $(t_2 - t_1)$ are small, any motion equation can be approximated quite accurately with the above summation. 2-STD will then find the pair of velocities $(\dot{s_0}^{inv}, \dot{s_1}^{inv})$ that satisfies (6). This pair of velocities minimizes the displacement since it will not move the states at all, but it may not satisfy the robot's motion equation. The idea is to use this pair as an "initial guess" to get $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$. Different motion equations may have different ways for finding this desired valid pair of velocities, efficiently. In the subsequent paragraphs, we give the details when the motion equation is governed by $f(s, u) = u$ and the details for arbitrary motion equation. Once the desired valid pair of velocities $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$ is found, 2-STD applies these velocities to (5) to get the new s'_1 and s'_2. Notice that we do not change s_0. Although changing s_0 is possible, we prefer *not* to move s_0 to ensure that we do not need to go back and ensure the connectivity of the previous triplets again. This enables the robot to use a partially deformed trajectory, instead of having to wait until the whole deformation process is finished. This is important when part of the trajectory that is obstructed by obstacles is quite long and the robot needs to move fast.

When the robot's motion is governed by $f(s, u) = u$, a valid pair of velocities can be found independently for each dimension. Let's now focus on dimension-k of $\mathcal{S}$ and hence of $\mathcal{V}$. When we consider the space of pair-of-velocities $\mathcal{V} \times \mathcal{V}$, the pair of velocities at dimension-k forms a 2-dimensional subspace $\mathcal{V}^k \times \mathcal{V}^k \subset \mathcal{V} \times \mathcal{V}$. In this subspace, the boundaries of the control input at dimension-k, *ie*, $u_{min}(k)$ and $u_{max}(k)$, form a rectangle r_k. Moreover, combining the two equations in (6) gives us the following line l_k equation in $\mathcal{V}^k \times \mathcal{V}^k$.

$$s_2(k) - s_0(k) = (t_1 - t_0)\dot{s}_0(k) + (t_2 - t_1)\dot{s}_1(k) \tag{7}$$

where $s(k)$ and $\dot{s}(k)$ are dimension-k of s and $\dot{s}$, respectively. See Fig. 5 for an illustration. Each point that lies inside r_k and on l_k satisfies the robot's motion equation at dimension-k, and if $s'_1(k) = s_0(k) + (t_1 - t_0)\dot{s}_0(k)$ then $s'_1(k)$ is in the reachability space of $(s_0(k), t_0)$ and in the back-reachability space of $(s_2(k), t_2)$.

The point $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$ always lie on l_k. The issue here is to bring $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$ to be inside r_k. Three cases are possible (Fig. 6), *ie*,

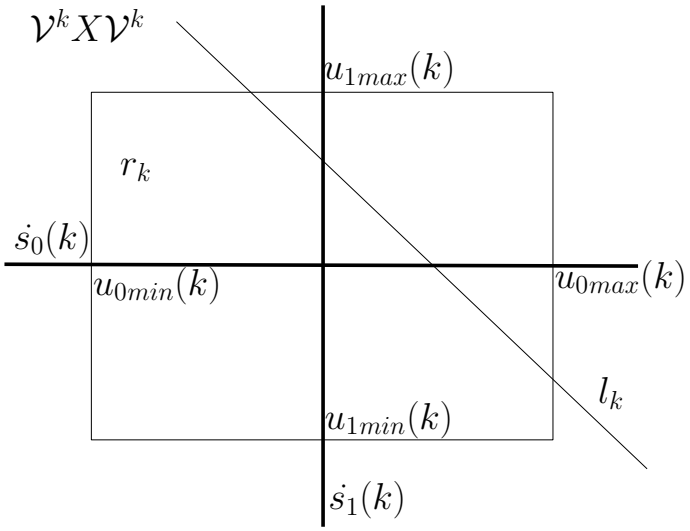


Figure 5: All pairs of $\dot{s}(k)$ inside r_k satisfy the robot's motion equation for dimension-k. All pairs of $\dot{s}(k)$ on l_k satisfy (7).

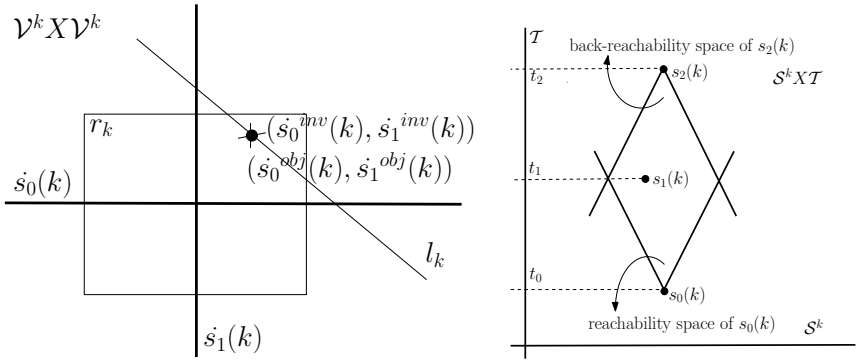
Case-1 (Fig. 6*a*), $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$ is valid. In this case, $(\dot{s_0}^{obj}(k), \dot{s_1}^{obj}(k)) = (\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$. This automatically minimizes the displacement as none of the nodes are moved.

Case-2 (Fig. 6*b*), $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$ is invalid but $r_k \cap l_k \neq \oslash$. In this case, $(\dot{s_0}^{obj}(k), \dot{s_1}^{obj}(k))$ is the point on $r_k \cap l_k$ that is closest to $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$. Simple algebraic manipulation is enough to prove that this pair of velocities minimizes the displacement (assuming the approximation in (6) is accurate enough).

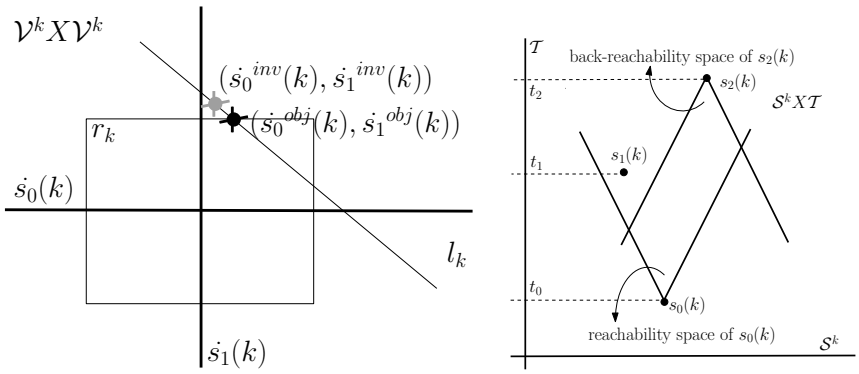
Case-3 (Fig. 6*c*), $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$ is invalid and $r_k \cap l_k = \oslash$. In this case, 2-STD sets $(\dot{s_0}^{obj}(k), \dot{s_1}^{obj}(k))$ to be the point on r_k that is nearest to $(\dot{s_0}^{inv}(k), \dot{s_1}^{inv}(k))$. Note that in this case both $s_1(k)$ and $s_2(k)$, instead of just $s_1(k)$, are moved. Here, the pair of velocities is not guaranteed to minimize the displacement.

The above steps are performed to each dimension independently to get the desired valid pair of velocities $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$.

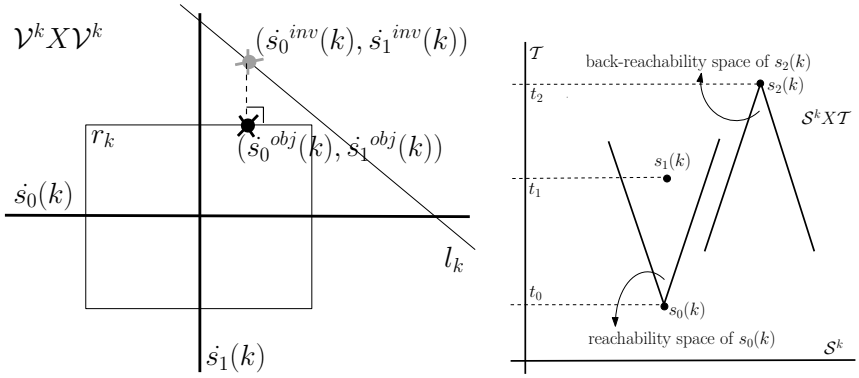
Now, in general $f(s, u) \neq u$ and a valid pair of velocities can not be found independently for each dimension. Nevertheless, the pairs of valid velocities that satisfy $f(s, u)$ form a subspace in $\mathcal{V} \times \mathcal{V}$. When efficient methods are known for projecting a point in $\mathcal{V} \times \mathcal{V}$ to the subspace, 2-STD finds valid pair of velocities that minimizes the displacement, by first projecting $(\dot{s_0}^{inv}, \dot{s_1}^{inv})$ to the subspace. When the projected point satisfies the bounds on the control input, this projected point becomes $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$. When the projected point



(a) Case-1



(b) Case-2



(c) Case-3

Figure 6: Three possible cases for $(\dot{s}_0{}^{inv}(k), \dot{s}_1{}^{inv}(k))$ when $f(s,u) = u$.

does not satisfy the bounds on the control input, 2-STD generates pairs of velocities using the combination of minimum and maximum control input and chooses the pair of velocities that minimizes the displacement $d(s_0, s_1, s_2)$ to be $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$. When this happens, the deformed nodes will be achieved by applying combinations of minimum and maximum control input, similar to the bang-bang approach [7]. In this case too we can not guarantee that $(\dot{s_0}^{obj}, \dot{s_1}^{obj})$ minimizes the displacement. When the motion equation is too complicated for projection to take place, 2-STD bypasses the projection step to directly use the combination of minimum and maximum control.

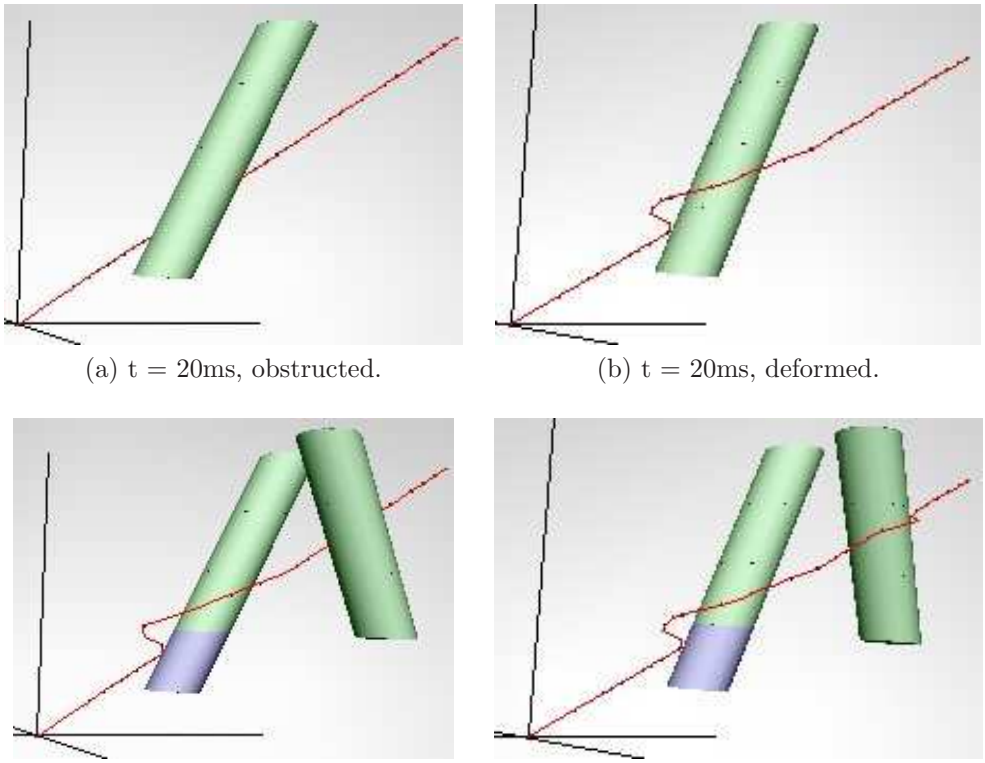
3.3 Adding nodes

When the node (s_{goal}, t_{end}) at the end of the trajectory is deformed, the trajectory may not end at the goal state anymore. When the trajectory does not end at the goal state anymore, 2-STD adds an additional node (s_{add}, t_{add}), $s_{add} = s_{goal}$ and $t_{add} = t_{end} + t_d + t_\epsilon$ at the end of the deformed trajectory. Of course this additional node is not guaranteed to be collision-free nor reachable from its previous node. Therefore, 2-STD will again apply both obstacle avoidance and connectivity maintenance step. This process of adding node and applying the two steps of 2-STD will be repeated until the goal state is collision-free and reachable.

4 Experimental Results

We implemented and tested 2-STD on a simulator written in C++ and ran on an Intel Pentium 4 3GHz 1GHz RAM with Linux operating system. In the test, we assume that a new world model is given every 20ms. Throughout the test, we use a planar square robot with double integrator subject to velocity and acceleration bounds. We ran this robot on two different dynamic environment scenarios.

The first scenario is used to assess the performance of 2-STD in handling scenarios as in Fig. 2. Initially, there is no obstacle and the original trajectory is a straight line. At t = 20ms, the robot receives a new world model (Fig. 7*a*) where there is a new obstacle that crosses the original trajectory. Upon receiving this world model, 2-STD deforms the trajectory to avoid obstacles (Fig. 7*b*). By incorporating the information on the future behaviour of the obstacle, 2-STD is able to deform the trajectory from behind the obstacle. This avoids 2-STD from deforming the trajectory too much such that it breaks the connectivity. Next, 20ms later, the robot receives a new world model where again there is another obstacle moving and crossing the original trajectory (Fig. 7*c*). As before, 2-STD deforms the trajectory to avoid obstacles (Fig. 7*d*). Each of the deformation process took less than 2ms. This scenario shows that 2-STD is able to avoid obstacles where path deformation methods tend to fail. This is expected because by taking the time dimension into account, 2-STD uses information on the future behaviour of the robot in order to deform the trajectory. This enables 2-STD to anticipate the obstacle's motion and deform the trajectory more appropriately.



(a) t = 20ms, obstructed. (b) t = 20ms, deformed.

(c) t = 40ms, obstructed. (d) t = 40ms, deformed.

Figure 7: Scenario-1. The environment and trajectory in configuration×time space. The axis are the black lines, the time axis is the vertical line. The skewed tubes are the obstacles in the configuration×time space, the blue part is the past while the green part is the future. The trajectory is the red line.

The second scenario is to assess the usefulness of 2-STD when the model of the future changes frequently. Initially, there is no obstacle and the original trajectory is shown in Fig. 8*a*. At t = 20ms, new obstacle A is detected to obstruct the trajectory. As a result, 2-STD deforms the initial trajectory (Fig. 8*b*). Then, 40ms later the robot detects new obstacle B that obstructs the trajectory. To avoid collision, 2-STD deforms the trajectory (Fig. 8*c*). Next, 20ms later at t = 80ms, the robot realizes that B has changed its motion and obstructs a different part of the trajectory and hence triggers 2-STD to deform the trajectory again (Fig. 8*d*). Then, 20ms later, the robot realizes that the motion of B, predicted at t = 80ms, is not entirely correct and the new predicted obstacle's motion obstructs again

a different part of the trajectory. So 2-STD deforms the trajectory again (Fig. 8*e*). Note that the trajectory that has been deformed to avoid obstacles according to the world model received at t = 80ms will not be deformed back (to the original trajectory) unless it becomes obstructed. Last, at t = 160ms, new obstacle C is detected and this new obstacle passes through the goal position at the same time the trajectory reaches its goal position. 2-STD deforms the trajectory by adding nodes such that it will reach the goal position sometimes later (Fig. 8*f*). Each of these deformations took less than 8ms. This scenario shows that 2-STD is efficient enough to adapt to the frequent changes of the world model.

Note that although in our experiments, all the obstacles are dynamic, 2-STD respects static obstacles, too. Static obstacles are handled in the same way as dynamic obstacles are handled. Moreover, although we only experimented with low dofs robot, 2-STD is applicable for high dofs robot. Because the two steps of 2-STD, *ie*, obstacle avoidance and connectivity maintenance, are applicable to both low and high dofs robot.

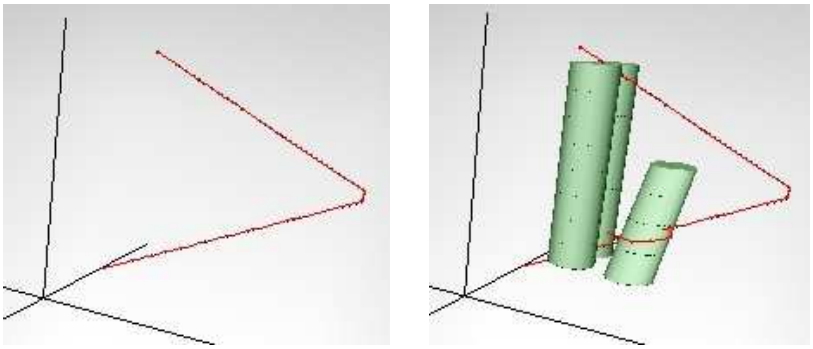
5 Discussion

Several issues still need further investigation, *ie*,

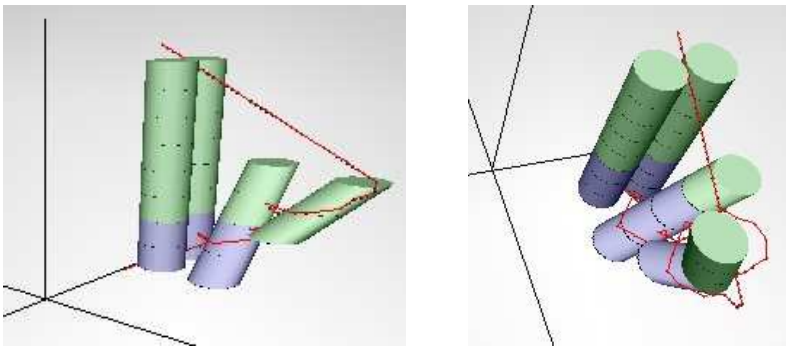
Just as the external force in elastic strip [18], 2-STD uses distance function to avoid obstacles. However, 2-STD considers not just the distance in the spatial dimension $\mathcal{W}$, but also the distance in the temporal dimension $\mathcal{T}$. This means, the distance metric will be a combination of the distance in $\mathcal{W}$ and $\mathcal{T}$. Therefore, we need a good scale between a distance of one unit in $\mathcal{W}$ and a distance of one unit in $\mathcal{T}$. Giving too much weight to $\mathcal{W}$ will cause the deformation to be dominated by spatial deformation, while giving too much weight to $\mathcal{T}$ will cause the deformation to be dominated by temporal deformation. Currently, it is still not clear what the "right" scale should be. One possibility is to relate it to our objective, *eg*, to reach the goal as fast as possible, to minimize energy usage, *etc*. In the current implementation, we simply assume that 1m distance in $\mathcal{W}$ is the same as 1s distance in $\mathcal{T}$. A better scale may be to consider the average speed of the robot.

Currently, to keep the temporal ordering of the trajectory unchanged, the deformation of each node in $\mathcal{T}$ is limited. It would be interesting to see the result when the deformation in $\mathcal{T}$ is unlimited just as the deformation in $\mathcal{S}$. The key issue is of course how to efficiently reorder the trajectory such that the robot can still use the partially deformed trajectory.

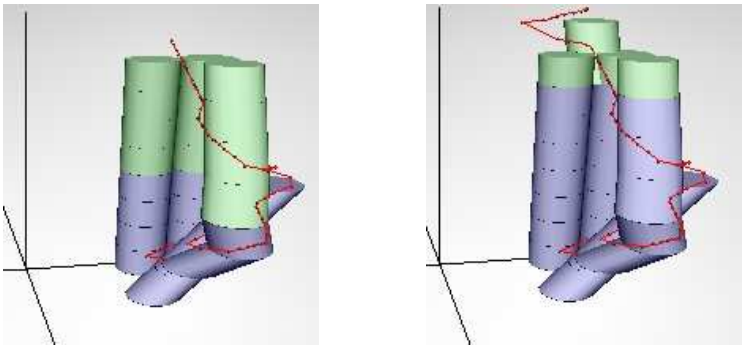
In the connectivity maintenance step, 2-STD fixes the time and assumes that only one control input can be applied for moving the robot from a node to its subsequent node. This is often too restricted, especially when the motion equation is complex. One improvement would be to allow changes in time and allow several changes of control between nodes.



(a) t = 0ms. (b) t = 20ms.



(c) t = 60ms. (d) t = 80ms.



(e) t = 100ms. (f) t = 160ms.

Figure 8: Scenario-2. The environment and trajectory in configuration×time space. The axis are the black lines, the time axis is the vertical line. The skewed tubes are the obstacles in the configuration×time space, the blue part is the past while the green part is the future. The trajectory is the red line.

Although in the connectivity maintenance step, 2-STD tries to respect the deformation generated by the obstacle avoidance step, in general there is no guarantee that the connectivity maintenance step will not nullify the results of obstacle avoidance step. One way to improve on this maybe to "blend" the obstacle avoidance and connectivity maintenance step more smoothly.

6 Conclusion

This paper introduces the first trajectory deformation strategy as an effort to improve path deformation technique for motion planning in dynamic environment. The main idea is that by incorporating the time dimension and hence information on the obstacles' future behaviour, motion deformation approach can be used for robotic system moving in complicated dynamic environment. Based on this idea, we propose a trajectory deformation method, 2-STD, that consists of two steps. First, the obstacle avoidance step to deform the trajectory to avoid obstacles. Second, the connectivity maintenance step to ensure that the deformed trajectory respects the robot's kinematic and/or dynamic constraints.

Our preliminary results on a planar robot with double integrator dynamics show that 2-STD is able to efficiently deform the robot's motion in cases where path deformation methods fail, *ie*, cases such as Fig. 2. Moreover, 2-STD is efficient enough to deform the robot's motion, avoiding collision with dynamic obstacles, even when the world model changes frequently.

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