Reactive planning under uncertainty among moving obstacles

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This paper addresses the problem of navigation of complex systems in dynamic environments under uncertainty. Such environments impose a hard real time constraint. However, computing a complete motion to the goal within a limited time is impossible to achieve in most real situations. The Partial Motion Planning (PMP) approach presented in [1] is used to address the planning problem. However, for real applications, it is important to take into account uncertainty. In this paper, we present an extension to the PMP that accounts for uncertainty in order to plan trajectories robust to the robot's errors. We show that PMP framework is highly suitable to account for these constraints and present original simulation results of robust trajectories for a car-like robot evolving among moving obstacles.

1. INTRODUCTION

The motion planning problem has been widely addressed during last decade for various systems as a mean to define a priori feasible paths (or trajectories) that can safely guide a system from an initial state to a goal state while avoiding obstacles. Basic motion planning problems assume that the current state of the robot is known at execution such that the plan can perfectly be executed. However, in real world application, uncertainty exists. There is at first uncertainty in the model of the environment (geometry, current and predicted position) as well as the model of the robot (model of the physics, geometry, position) that will affect the plan. This type of uncertainty will not be addressed in this paper. Secondly, it is important to consider the case where the state cannot be known. In this case, information regarding the state is obtained from sensors during the execution of the plan. Sensor errors as well as control errors will further affect the execution of a motion.

It is not possible to eliminate these errors and in case these imperfections are not small relative to the tolerance of the task being performed, it is important to generate plans robust to these errors. A solution to this problem is to design a scheme that explicitly takes this uncertainty into account and guarantee that the goal will be reached.

In this paper, we address the problem of motion planning under uncertainty for a car-like robot evolving in a dynamic environment. Trajectory planning under these constraints (non-holonomic and dynamic system, dynamic environment) is already a highly complex task for which an efficient approach was proposed in [1]. The Partial Motion Planning approach is a reactive planning scheme used to iteratively calculate safe partial plans by incrementally exploring the state-time space of the system, until the robot has reached its goal. In this work, we present an extension of this scheme accounting for uncertainty in the real robot's motion, using a probabilistic representation of the errors that appear at execution in the controls. As a model based control method, the PMP is highly suitable for such an extension from which we built an original planning method that accounts for uncertainty as well a dynamic constraints stemming from the system and the environment.

After a presentation in the second part, of the related work, we recall in the third part the principle of the Partial Motion Planner. In the fourth part, we present the error model and the modified planning scheme accounting for uncertainty. Finally, we present original simulation results of robust trajectories within dynamic environment in the fifth part and draw our conclusions in the last part.

2. RELATED WORK

First work on the subject ([2], [3]) considered bounds or worst case on uncertainty within motion planning. A first plan was generated with no uncertainty and then the plan was analyzed and modified in order to produce a robust plan. In these approaches, uncertainty is represented as a set of equiprobable possible values. The preimage backchaining approach is pioneered in [4]. It was extended later [5] and used in simple cases for mobile robots [6]. A comprehensive state of the art of preimage backchaining is presented in [7]. A preimage for a given motion command and a given goal region in configuration space is a set of free configurations from which the command can be started with the guarantee that the robot will reach the goal. In [8] the concept of sensory uncertainty field (SUF) is introduced. A SUF represents for each configuration its estimation error computed by a sensor based localization. A planner using SUF can generate a path that minimizes expected errors by traversing workspace areas where visible environment features yield low sensory uncertainty. More specific approaches are presented in [9,10,11] for which non-holonomic constraints are added to the problem.

3. PARTIAL MOTION PLANNING

3.1 Partial Motion Planning (PMP) Cycle

Planning in a dynamic environment implies that the plan is anchored in time, ie. starts at a specific time and reaches the goal at a precise time. This timing constraint is a real time constraint that the planner has to fulfill when planning. In a real world indeed, a robotic system cannot safely remain passive in a dynamic environment during an unlimited time as it might be collided by a moving obstacle (*decision constraint*). This decision time is function of the environment's dynamicity. Besides, in a real environment, the model of the future will usually to be predicted. Such predictions have limited validity duration. The planning time (or calculation time) is hence strictly limited to the minimum of these two bounds. After completion of a planning cycle of limited calculation duration, it is most likely the planned trajectory of an arbitrary time horizon will not reach the goal and be partial. Thus, the PMP algorithm [1] iterates over a cycle of limited calculation duration. We consider in this paper a constant planning cycle in order to be able to regularly get an update of the model.

Let us focus on the a planning cycle as depicted in Fig.1:

(1) An updated model of the future is acquired.

(2) The state-time space of the robot is searched using an incremental exploration method that builds a tree.

(3) The computation time for the current iteration has expired; the best partial trajectory in the tree is selected according to given criteria (safety, metric) and is fed to the robot that will execute it from now on.

The algorithm operates until the last state of the planned trajectory reaches a neighborhood of the goal state.

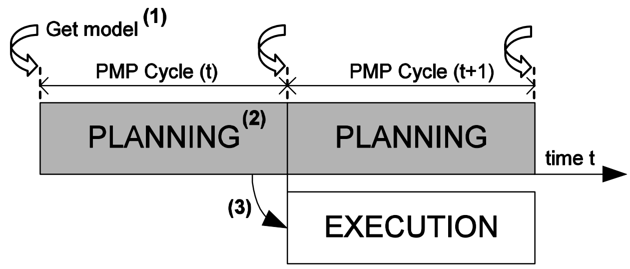

Figure 1: Partial Motion Planning Cycle

3.2 Exploration Tree

The exploration of the state-time space consists in building incrementally a tree as follows (Fig.2): a milestone s_r is generated within the workspace. The closest state s_c to s_r is selected. A control is selected from a set of controls (usually bang-bang controls) and applied to the system during a fixed time (integration step). In case the new state s_n of the system is safe, this control is valid, otherwise it is rejected.

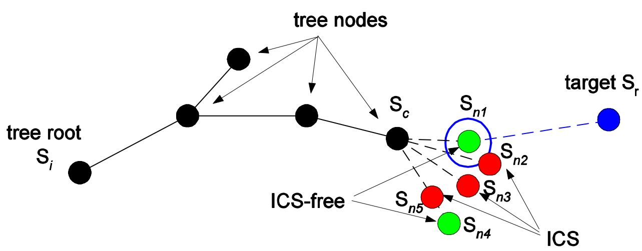

Figure 2 : Exploration Tree

The safety issue is addressed from the perspective of Inevitable Collision States (ICS) a concept that defines the safety of a system according to its own dynamics [13]. The operation is repeated over all control inputs of the set and finally the new state, safe and closest to s_r, is finally selected and added to the tree.

In case the state s_r is selected at random, this scheme reduces to the famous Rapidly-Exploring Random Tree method [14]. Otherwise s_r can be selected as the goal, which provides high quality collision avoidance trajectories. Finally, another interesting variant is to keep in the tree all safe nodes instead of the closest only. By this mean, the generated trajectory can retrieve its way by back tracking the tree in case a local minimum is reached (Fig.3).

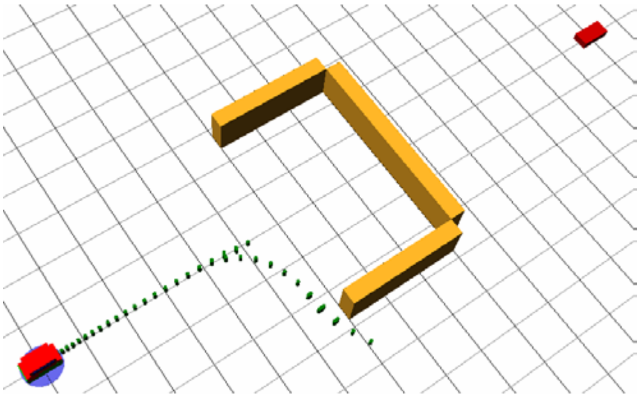

Figure 3 : Exploration tree avoiding local minimum

4. PLANNING UNDER UNCERTAINTY

4.1 Car-like vehicle model

The dynamic model of the car-like robot is defined by the following set of differential equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} v.\cos\theta \\ v.\sin\theta \\ v.\tan\phi / L \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2 \qquad (1)$$

Let the considered mobile robot placed in flat a workspace This equation is of the form $\dot{X} = f(X, U)$ where X is the state of the system, $\dot{X}$ its time derivative and U a control. A state X of the robot is defined by the 5-tuple (x, y, θ, ϕ, v) where (x, y) are the coordinates of the rear wheel, θ is the main orientation of the robot, v is the linear velocity of the rear wheel, and ϕ is the orientation of the front wheels. A control U of the robot is defined by the couple $(u_1, u_2) = (\alpha, \gamma)$ where α is the rear wheel linear acceleration. and γ the steering velocity. The actuator constraints are $|\dot{v}| \leq |\dot{v}_{\max}|$ (acceleration constraints), $|v| \leq |v_{\max}|$ (velocity constraints) $|\dot{\phi}| \leq |\dot{\phi}_{\max}|$ (steering speed constraint) and mechanical constraints $|\phi| \le |\phi_{\max}|$ and $|k| \le k_{\max} = {\tan\phi_{\max}}/{L}$ (maximum turning radius, resp. curvature, constraints) are considered as well.

4.2 Error propagation

In this paper the uncertainty stemming from the actuation imperfections is considered at the planning stage. Indeed, since the model for which motion planning is performed is known in advance, it is possible to establish an error propagation model for this error. The model of error propagation is analyzed using the predictive step of the Extended Kalman Filter (EKF) using the linearized form of the bicycle model of the car. The linearized model is of the form $\dot{X} = A(t)X + B(t)U$ with $A(t) = {\partial f}/{\partial X}(t, X(t), U(t))$ and $B(t) = {\partial f}/{\partial U}(t, X(t), U(t))$.

The error prediction is given by the covariance matrix $P(t_{k+1}|t_k) = A_k P(t_k|t_k) A_k{}^t + R_{sys} + B_k R_{com} B_k{}^t$ with R_{sys} the noise on the model and R_{com} the noise on the command represented using gaussian probabilistic density functions.

4.3 Partial Motion Planning under uncertainty

The covariance matrix informs about the propagation errors of the system. These errors appear as uncertainty of the robot configuration during exploration. Since the exploration tree method is a sample-based method, it relies on a geometric collision checker. The collision detection is performed over a circular bounding box of our system. Therefore, the maximum calculated component of the estimated position error is added to the radius of the circular bounding box of our system in order to provide safe planning with respect to the system's actuator errors.

4.4 Towards Information Feedback plans

The planning problem under uncertainty lies on the assumption that limited information only on the state of the system can be sensed. Thus, instead of estimating the state and pretend that there is no longer any uncertainty, the uncertainty is modeled within the planning scheme. In fact, such a planning problem is expressed in terms of an information space whose elements represent accumulated information about a system [12]. In our case, we suppose that there are no sensors and therefore no observations. In this case the future states are predictable. In the real world, observations from various sensors can be provided and incorporated in the presented method within the EKF framework in the update phase. Furthermore, this method can be seen as a step toward information feedback planning in the probabilistic information space. This space is derived from the information space, where each history information state is converted into a probability distribution over the state space and a Markov probabilistic model is assumed.

4. RESULTS

The results show the trajectories of a car after a few iterations where the model of the future of the environment is known a priori. Figure 4 shows two trajectories, without considering the system's uncertainty (Fig.4a) and a trajectory considering the uncertainty of the system (Fig.4b). We can see that the trajectory between the two obstacles is not safe in case uncertainty is considered and therefore not preferred.

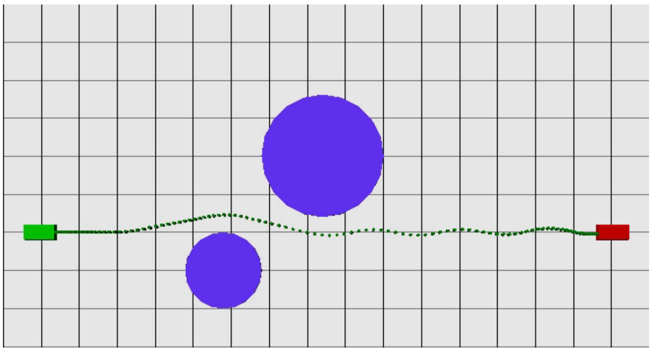
Figure 4a: Planning without uncertainty

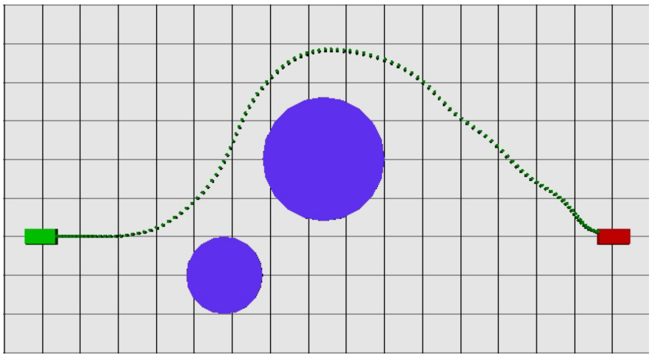
Figure 4b: Planning with uncertainty

Figure 5 and 6 significantly illustrate the result of a planned motion for a car evolving within a dynamic environment (the arrows represent the velocity vector of the moving obstacles). The trajectory planned with uncertainty (Fig.6) bypasses the obstacles and is safer than the plan with no uncertainty (Fig.5) which travels through the obstacles.

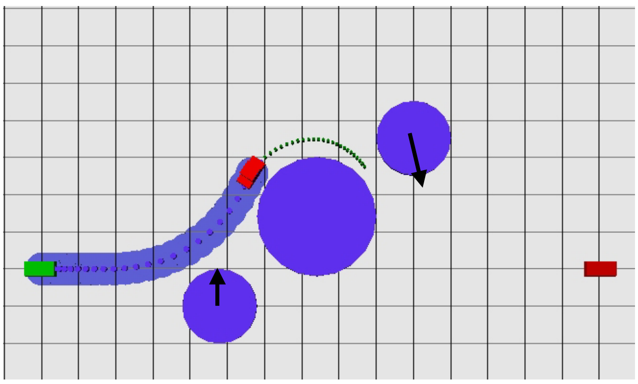
Figure 5a: planning without uncertainty among moving obstacles

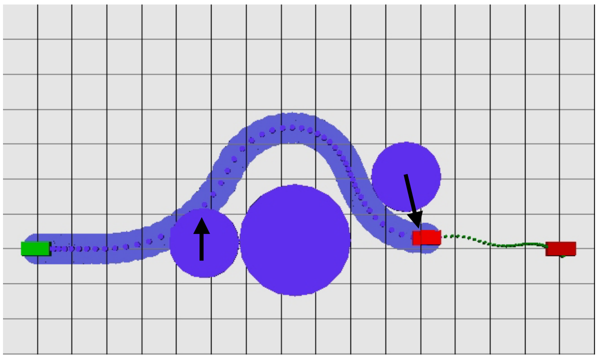
Figure 5b: planning without uncertainty among moving obstacles

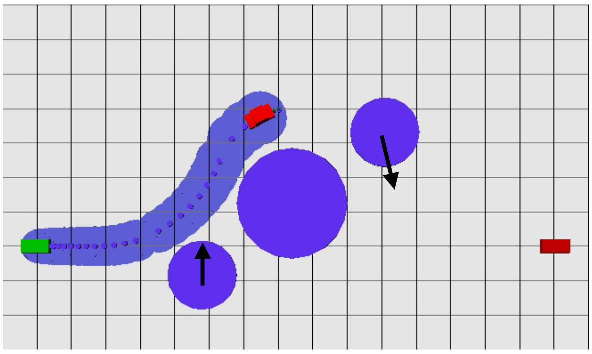
Figure 6b: planning with uncertainty among moving obstacles

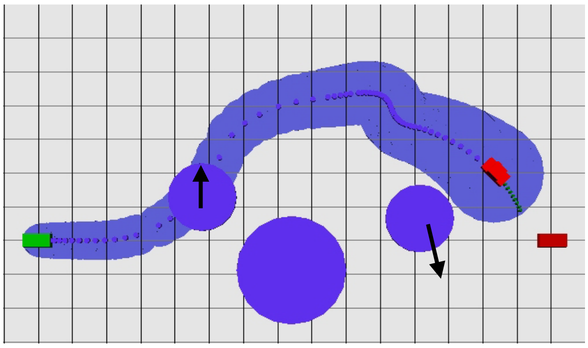
Figure 6b: planning with uncertainty among moving obstacles

4. CONCLUSION AND FUTURE WORK

In this paper, an extension to the Partial Motion Planning scheme presented in [1] is presented to take into account the uncertainty in the robot's controls at the planning stage. The PMP is a real time trajectory planning method for complex systems evolving within dynamic environments. Since PMP is a model based scheme it is extremely well suited to interleave with the Extended Kalman Filter (EKF). This approach can be seen as a way to express the problem in the probabilistic information space computed using the EKF. In the presented work, it was assumed that no observations on the system's state could be performed. The predictive phase of the EKF was used in order to calculate the error propagation in the robot's (speed and steering) controls. The largest error in position is added to the radius of the bounded circle of the robot used for collision detection. Thus, the PMP generates robust trajectories accounting for the drift in the controls. The safety of the trip is therefore increased as illustrated by simulation results.

Depending on the task to be performed, it might not be necessary to perform observation during the trip and still reach the goal. A future work would consist in gathering observation during execution and update the state of the system in the update phase of the EKF.

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