Space Time relaying versus Information Causality

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Abstract: We investigate the connection between causality and Information Theory. We show that a simple reading of Information Theory makes that quantum entanglement allows super-luminal information transfer. We show that introducing the concept of space-time information relaying partially solves the paradox of non causal information transfer. Surprisingly information causality is only dependent on the quantum unitarity which is a weaker principle than physical causality.

Key-words: Quantum information, causality, super-luminal, retro-information, entanglement

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Relayages spatio-temporels et principe de causalité dans l'information

Résumé : Nous analysons la connexion entre le principe de causalité et théorie de l'information. Nous montrons qu'une lecture basique de la théorie de l'information peut amener à d'crire le phénomène quantique d'intrication comme un transfers d'information super-luminique. Nous montrons comment l'introduction du concept de routage spatio-temporel résoud partiellement ce paradoxe de transfers d'information non causal. De manière surprenante la causalité del'information dépend du principe d'unitarité quantique, celui-ci étant un principe physique plus faible que celui de la causalité.

Mots-clés : Information quantique, causalité, super-luminique, rétro-information, intrication

1 Introduction

Since the early ages men have sent their messages on pieces of matter or on energy flows, *e.g.* on stones, paper, flags, smokes, messengers, electrical signals, radio waves. In the near future, the quantity of mass or energy needed to carry the same amount of information will certainly continue to decay, closing to quantum limits, but this will still be energy flow. Therefore it is natural to expect that information transfer should endorse the same constraints that apply on energy transfer, for example the light speed limit and the principle of causality. But is that so obvious? For example can we imagine information faster than light? This paper will show that surprisingly energy causality does not imply information causality. But this in fact comes from an incompleteness of Shannon theory that lead to a misinterpretation of quantum entanglement in information theory. But it is known that quantum entanglement cannot carry "useful" information random entropy. Nevertheless we can break this apparent paradox by introducing the concept of space-time relaying which restore a kinf of information causality. Interesting enough the information causality is not directly dependent on energy causality but is rather a consequence of a much weaker axiom in physics, namely the principle of unitarity in quantum mechanics.

To the author best knowledge it is the first time such issue is addressed (following [4]). It outlines the unexpected result that indeed information transfers and energy transfers don't play in the same arena and need definitely to be treated differently. The paper is organized as follows: the next section reviews closely the notion of causality. Section III addresses the concept of causality with information and shows how a basic reading of this concept is broken by the concept of quantum information entanglement. Section IV introduces the concept of space relaying that restore the principle of causality in information theory as a consequence of unitarity principle.

2 The principle of causality

2.1 Philosophical Causality

The popular acception of causality is that "the cause precedes the effects", this means that if an event X has an effect on another event Y, then X precedes Y. But imagining "cause" and "effect" is difficult without an implicit time reference. One could try to express "X has effect on Y" by the fact that the mutual information $I(X,Y)$ is non zero, transposing causality into an information transfer. But this viewside does not distinguish at all between *emitter* and *receiver*. In fact, by virtue of the identity $I(X,Y) = h(X) + h(Y) - h(X,Y)$, $h(\cdot)$ being the entropy, we have the symmetry $I(X,Y) = I(Y,X)$.

Restricting causality to non zero mutual information may lead to absurb situations. For example let imagine the mother of two twin traveler salesmen who travel to two different locations the same day. Before they leave their house their mother leave the same amount of money in their luggage. In the evening in their respective hotel each twin discovers its money, respectively X and Y. Since $X = Y$ then the mutual information is non zero. But none of the twins is the emitter nor the receiver. This example illustrates the paradox known as the Einstein-Podolski-Rosen (EPR) paradox where a correlation occurs between two particles at a speed that can exceed the speed of light but which does not result from any information transfer.

There is no proper definition of emitter and transmitter, excepted that the emitter contains the information to be transmitted just before the emission, and the receiver contains the information just after the reception. Notice that these definitions require a temporal order related to time. Also notice that if time were to be reversed, then the emitter and receiver would switch their status.

2.2 Physical causality

The present acception of causality involves the concept of causality lines (or causal lines). A causality line is an ordered sequence of events such that event i is a cause to event $i+1$. For example the space-time trajectory of a particle from the past to the future is a causality line. In this perspective the physical principle of causality is the following: two causality lines that have in common two events X and Y must contains these events in the same order, *i.e.* if event X precedes event Y in one causality line, then event X precedes event Y in all causality lines that contains both events. Of course by event we mean local event such as collision of particles and not a remote consequence of this event (which will be a separate event, created by the flow of photons generated by the first event for example).

As we can see this physical definition of causality does not bring any insight on how to distinguish between cause events and effect events, but at least it states the rules that makes all causal lines consistent. Two consecutive events in a causality line can be separated by a flow of energy or matter, for example the trajectory of a particle. Therefore causality line slopes must satisfy the light speed limit. In other words, the effect events must always stand in the light cone of a cause event.

If we apply the concept of causality lines to particle trajectory, then it turns out that two particles A and B that meet twice, respectively on event X and Y, must experience these events in the same sequence in their respective history. In other words, if X precedes Y in A's history, then X must precedes Y in B's history. This rule directly derived from causality lines property actually excludes the concept of time warp derived via unclassical solution of general relativity. Indeed if particle B after event X travels backward in time through a black hole, or what so ever, and loops back with particle A on event Y before event X there is a causality loop failure.

In the following we denote $\mathcal{P}(X)$ the set of events that belongs to the past of X, *i.e.* the set of events that precede X in any causality line that contains X. By convention the event X itself is excluded from $\mathcal{P}(X)$. Indeed $\mathcal{P}(X)$ is included in the past light cone of X. But it does not necessarily contain all events in the light cone since some causal lines that intersects the light cone may not contain X, because the particle are refrained interfering (because of an opaque screen for example).

Using this terminology the causality principles becomes

- $X \notin \mathcal{P}(X)$;
- $\forall Y \in \mathcal{P}(X)$: $\mathcal{P}(Y) \subset \mathcal{P}(X)$.

3 Information causality

Admitting the acception of physical causality, the definition of information causality comes as follow:

The receiver must be on a causal line of the emitter.

Or in other words, to take the classic characters of information theory, Bob must stand on a causal line of Alice. When information transfer are made via energy-matter flows, then the information causality is simply implied by physical causality. But information theory foundations are not based on the concept of physical flows, and only deal with the correlation between the event X (the emission) and the event Y (the reception).

More generally we can generalize to the assertion that if none of events X and Y are not in a causal line of the other (*i.e.* $X \notin \mathcal{P}(Y)$ and $Y \notin \mathcal{P}(X)$) then

$$I(X, Y \mid \mathcal{P}(X) \cap \mathcal{P}(Y)) = 0 \; . \tag{1}$$

It means that if we fix the common past history of two events, then the latter become independent because the only events that may make them correlated should be in their common history which is fixed. With this definition we immediately eliminate the *pseudo* information transfer between the twin traveler salesmen. Indeed their mother is in their common history. Fixing the latter means that X and Y are fixed then no mutual information is exchanged.

In this section we closely investigate the concept of information causality in the framework of modern physics. Quantum physics revolutionized physics during the twenties. Having less famous fathers than relativity it less known by the public, but its consequence are far more astounding. It introduced the concept of incertainty, the concept of of vector state. Its first victim was determinism. Its second victim was information causality.

3.1 Quantum non causal information transfers

A source of information is deterministic X when $h(X|\mathcal{P}(X)) = 0$. Conversely a source is non deterministic, or is "purely random", when $h(X|\mathcal{P}(X)) > 0$. Or equivalently $h(\{X\} \cup \mathcal{P}(X)) > h(\mathcal{P}(X))$.

Quantum mechanics enables the existence of non deterministic sources. The result X of the measurement of the spin of a particle is non deterministic. $X \in \{-1, 1\}$ (spin $\frac{1}{2}$) and can take any value independently of its past history.

The key concept of quantum mechanics is that the state of a particle parameter, for example its spin, is given by a complex state vector ψ. The spin can have two pure states $|s_u\rangle$ for spin value $+1$ (up), and $|s_d\rangle$ for spin value -1 (down).

$$\psi = \alpha_u |s_u\rangle + \alpha_d |s_d\rangle$$

The vector is assumed to be unitary, *i.e.* $|\alpha_u|^2 + |\alpha_d|^2 = 1$ and indicates the probability distribution of the spin measurement outcome X: $P(X = +1) = |\alpha_u|^2$ and $P(X = -1) = |\alpha_d|^2$.

A spin is measured via a polarizer and the result $X(\theta)$ depends on the angle θ of the polarizer.

The first known of information causality failure, due to quantum physic is a consequence of this simple measurement on entangled particles. Two particles with a common past are entangled: they have a quantum state in common. For example their spin may be exactly inversed. If one measures the spin $X(\theta)$ of the first particle on a given angle θ and if another operator measures the spin of the other particle $Y(\theta)$ on the same angle then

$$X(\theta) + Y(\theta) = 0 \tag{2}$$

In other words the quantities $(X(\theta), Y(\theta))$ are strongly correlated.

Theorem 1. *The spin measurement on entangled particles are correlated source of information that breaks the information causality.*

Proof If the spins are measured according to different angle θ_1 and θ_2 then we have

$$\left\{ \begin{array}{rcl} P(X(\theta_1) = -Y(\theta_2)) & = & \cos^2 \frac{\theta_1 - \theta_2}{2} \\ P(X(\theta_1) = Y(\theta_2)) & = & \sin^2 \frac{\theta_1 - \theta_2}{2} \end{array} \right.$$

or $E(X(\theta_1)Y(\theta_2)) = -\cos(\theta_1 - \theta_2)$.

The above result does not depend on the relative position of the measures in the space-time. The measures can be done when the particle are very far apart. One measure can be far in the future. it will not change the correlation equation. This is a non local phenomenon. In particular the measure $X(\theta_1)$ and $Y(\theta_2)$ can done such that the events cannot be linked by a causal line, for example such that none is in the light cone of the other. Therefore one should have correlated information sources that are not causal. Since the source are not deterministic $I(X(\theta_1), Y((\theta_2) \mid \mathcal{P}(X) \cap \mathcal{P}(Y)) = I(X(\theta_1), Y(\theta_2)) = 1 + \ell(\cos^2 \frac{\theta_1 - \theta_2}{2})$ with $\ell(x) = -x \log_2 x - (1-x) \log_2(1-x)$. We have $I(X(\theta_1), Y(\theta_2)) > 0$ as long as θ_1 and θ_2 are not orthogonal. Therefore the entangled particles contradicts the information causality principle. ■

Remark In fact the proof is more complicated and is due to Bell that succeeded in eliminating so-called hidden variables, *i.e.* hypothetical variables that were supposed to complete any quatum system to make it determinist.

However this dependence between sources does not allow to transmit any other information than the one created by the source themselves: this cannot be used to move information faster than light. Indeed since $E(X(\theta_1)Y(\theta_2)$ does not vary whatever which measure occurs the first, the entanglement phenomenon is just a non local correlation between sources of information. However this a very clear blow against information causality.

4 Space time relaying and Anti-causal information transfers

We have seen that information can actually break causality if we are not careful with the concept of information transfer. Entangled sources leads to information transfers on random bits created by the measurement themselves which is not "useful" information.

In order to get further we need to define what we would like to exactly mean by useful information. But this is a very difficult question. Instead we will use a side concept we call space-time relaying.

Definition 1. *A space-time relay emitter is an event X that transmits information that can come from any source in the past of X. A space-time relay receiver is an event Y which receives information which can be delivred to any point in its future.*

Internet is based on space time: a web page on a server can contain any kind of information obtained from the past and the internet can transfer this information to another machine that can deliver the information to any receiver in its future. Conversely, in the example of the twin salersmen, none of the twin can play the role of the relay emitter of the information contained in his luggage, since in order to the other twin to receive the information the source of information shall exclusively be his mother in the common past of X and Y. Similarly, in the example of entangled pairs, none is relay emitter, since they are the source of information and cannot relay any information coming from their past.

4.1 Faster than light information

We will show that faster than light information transfer leads to anticausal information. As a direct application of theorem 2 that will be developped in the next section, anticausal information implies unitarity violation. Therefore it would be impossible to develop a faster than light information system in an unitary model.

Lemma 1. *Faster than light information makes anticausal information possible.*

Proof The special relativity theory states the invariance of light speed with respect to moving coordinate systems thanks to the Lorentz transform for space-time coordinate systems. When a coordinate system leaves at time $t = 0$ the origin at a constant speed v (oriented on x axis), then the ccordinate of a point (x, y, z, t) becomes

$$(x', y', z', t') = (x, y, z, t)\frac{1}{\sqrt{1 - v^2/c^2}} \begin{bmatrix} 1 & 0 & 0 & -\frac{v}{c^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -v & 0 & 0 & 1 \end{bmatrix}$$

Consequently if a node moves at light speed c then it also moves at speed c in the new coordinate system: the light speed is invariant. If a node moves at a speed $V < c$ then it will move at a speed still smaller than c in the new coordinate system: the light cones are invariant. The rule of relativistic speed composition makes the mobile trajectoryin space time is oriented with $(V - v, 1 - \frac{vV}{c^2})$ in the new coordinate reference, therefore the new speed is $\frac{V-v}{1-vV/c^2}$. This is an old and well known property of Lorentz transform: when a mobile moves at a speed V, faster than light, then there exists a coordinate system moving at a speed $\frac{c^2}{V} < v < c$ where the mobile moves backward in time. Therefore faster than light moving physical object should not be allowed. Since $1 - \frac{vV}{c^2} < 0$ the mobile moves backward in time. In other words, in hyper-relativistic ping-pong, a ball moving at speed V received on a racket moving at speed v makes the ball back to the past of the sender. Indeed the ball leaves the racket with relative speed

$(-V+v, 1-\frac{vV}{c^2})$ which lead to a trajectory back to the sender in the past of the incoming trajectory.

The only thing we need to show is that we can apply this property to superluminal information transfer. We take the assumption that information delivery at speed $V > c$ is possible. To this end we consider a superluminal trajectory as above with point A on the first end point such the vector AB is colinear to speed V. The vector BC is colinear to the speed $(-V, 1)$ in the referential of the racket and C is in the past of point A. Without lost of generality we can assume a small translation of point B in its future, called point B', and a similar translation of C, called C' such that $B'C'$ remains colinear to BC and C' is still in the past of A.

At point A we assume a relay emitter, Alice, immobile, in other words her space-time trajectory is oriented with vector $(0, 1)$, and a relay receiver on point B, Bob. Information is transmitted from Alice to Bob. Now assume a relay emitter, Bob Prime moving at speed v on B'. Assume that Bob relays its information to Bob prime via a classical channel. Now put a relay receiver, Carol Prime on C' also moving at speed v. Since $B'C'$ is colinear to speed $(-V, 1)$ in the referential moving at speed v, then the same system of information transfer between Alice and Bob, can be applied between Bob Prime and Carol Prime. Therefore the information can be transmitted from Alice to Carol Prime in the Alice's past. ■

4.2 Anti-causal information

We will show that non causal information transfer between a relay emitter and a relay receiver is equivalent into breaking the principle in physics called *unitarity*.

4.2.1 The quantum unitarity

Unitarity is a property in quantum physics that makes the modulous of a quantum state $\psi(t)$ invariant with time t. We have

$$\psi(t) = \mathbf{U}(t)\psi(0) \tag{3}$$

where $\mathbf{U}(t)$ is a linear unitary operator. By implication the trace of the derived density operator $\rho(t)$ is invariant with time t. Unitarity makes universe time lines to diverge from pure state superposition to state mixture.

How strong is the unitarity axiom? Certainly it is weaker than the causality principle. Early in 1976 Hawking [1] was the first to conjecture that black holes evaporation was violating unitary. Although recently Hawking has partially refuted his previous argumentation the question is still open. Wald has noticed [6] that some quantum fields in curved spacetime (as it is the case with general relativity) would necessarily be non unitary. More generally the extension of string theories rise the existence of ghost states for spin 0 and spin 2 fields which could be the first sign of non unitary effects [3].

These considerations have a considerable impact on *grand unification* and the problem is still widely open. We can be sure that there will be many exotic theories before getting the right one. However it must be noted and stressed that if non-unitary physics was a necessary amendment, then its effects would be only noticeable on very curved space-time, in conditions existing only on the edge to a black hole, or approachable in the very dense heart of stars. Of course the effects would be undistinguishable from unitary physics in conditions that physicists can reproduce in laboratories.

4.2.2 Information causality and unitarity

We show the following theorem. It is not a "standard" theorem since it is related to physical considerations, and the proof is not a standard proof but rather a hint.

Theorem 2. *Anti-causal information between a relay emitter and a relay receiver implies unitarity violation*

Proof: Let consider two spacetime points A (Alice) and B (Bob) such that the vector (B, A) is time like and A in the future of B. We assume that Bob makes a physical measurement on B, for example a spin measurement, whose result is Y. We suppose that the measurement depends on the value of some setting X operated by Alice so that retro-information occurs between A and B. In other words, X and Y are not independent random variables.

Let an observer be on a point Q located in both futures of A and B. We assume a classical channel between B and Q. The channel has output Z and is correlated with Y: $Z = Y$ (for example Z is the spin measurement on a particle that gave the same spin on the same angle on A).

Without loosing generality, we also suppose that Y and X both take binary values, 0 and 1. We denote e_i the state corresponding measurement $Z = i$ and $\mathbf{P}_i$ the Hilbertian projection $e_i^\star \otimes e_i$. Since there is information transfer from A to B and to Q, $P(Z = 0 \mid X = 0) \neq P(Z = 0 \mid X = 1)$. Let ρ_j be the density operator of measurement Z when $X = j$. $\rho_j = \rho_j(0)\mathbf{P}_0 + \rho_j(1)\mathbf{P}_1$. We assume *a contrario* unitarity, thus $\mathrm{Tr}(\rho_j)$ does not depend on j: $\mathrm{Tr}(\rho_0) = \mathrm{Tr}(\rho_1) = 1$.

In order to show non-unitarity we imagine the same experimental system but now with a classical channel established from B to A that does not disturb the measurement on B and Q and such that setting X is the output of the channel. The channel depends on an operator Carol which can activate a switch C in the past of B. As usual Carol is in charge of disrupting the communication between Alice and Bob. Switch C has a binary setting σ. The switch operates to make setting X depending on measurement Y as follow: $X = Y + \sigma$ *modulo* 2. For example X is the spin measurement on a particle that gave the same spin and on the same angle on B. When $\sigma = 0$, the angle is the same on A when $\sigma = 1$ the angle is reversed. Now the result Z depends on setting σ. Let $\rho_j^\star$ be the density operator of measurement Z when $\sigma = j$. When $\sigma = 0$, (Z, X) attains only values $(0, 0)$ and $(1, 1)$ with respective weight given by Hilbertian projection (since we apply linear unitary evolution): $\rho_0^\star = \rho_0(0)\mathbf{P}_0 + \rho_1(1)\mathbf{P}_1$. When $\sigma = 1$, only $(1, 0)$ and $(0, 1)$ are attainable: $\rho_1^\star = \rho_1(0)\mathbf{P}_0 + \rho_0(1)\mathbf{P}_1$ Since we have $\mathrm{Tr}(\rho_0^\star) \neq \mathrm{Tr}(\rho_1^\star)$, there is an unitarity violation. ∎

4.2.3 Reverse theorem

From now we handle non-unitary systems: we couple a particle in an non unitary environment (e.g. in a non unitary black hole) with a classical system in an unitary environment (e.g. outside the black hole). Depending on the outcome of the classical system we apply a different operator to the particle in the non unitary environment. For example we throw or we do not throw the particle in the black hole. The density operator of the non-unitary system is not unitary. Hartle [5] has proven the following lemma.

Lemma 2. *Let ρ be the density operator of an non-unitary system. The probability $P(\phi)$ of a time line ϕ is given by*

$$P(\phi) = \frac{\phi^\star \rho \phi}{\mathrm{Tr}(\rho)} \tag{4}$$

Remark: Notice that Hartle renormalization acts like a non-linear effect and justifies a link with non-linear quantum of Weinberg. Polchinsky [2] in his discussion about Weinberg non-linear quantum mechanics, has acknowledged that certain non-linear effects can allow super-luminal transmissions via entanglement, that we have shown is a special case of anti-causal information.

We now turn to specific non-unitarity effects. If non-unitarity effect comes from space curvature as suggested by Wald, then one may expect that the trace of the density operator varies with the space curvature, *i.e.* with the density of matter. In other words, if an operator changes the density of matter it changes the trace of the density operator of the universe. Equivalently if matter or energy is thrown in a black-hole, then the trace of the density operator takes a value different of that it would have taken without thrown matter. In other words the trace value depends on history (past and future).

In the proof of theorem 2 setting σ affects the trace of the density operator.

We will prove that if we have a non-unitary operator $\mathbf{U}$ (e.g.$\mathbf{U} = e^{i\mathbf{A}}$ for a **non-adjunt operator $\mathbf{A}$) then can have retro-information. In fact we should need two operators $\mathbf{U}_{\mathrm{out}}$ and $\mathbf{U}_{\mathrm{in}}$ such that $\mathbf{U}_{\mathrm{out}}\psi \neq \mathbf{U}_{\mathrm{in}}\psi$, but we take $\mathbf{U}_{\mathrm{out}}$ unitary since we assume that we keep the particle outside on non unitary environmenet (the black hole). Imagine that point A is close to a black-hole about to fade. From point A there are particles or group of particles called *probes*. Alice has the choice to either throw a probe in the black-hole or to throw it away. Let ρ_{in} be the trace of the density operator of the system probe plus black hole after black hole fading in the state "probe thrown-in". Let ρ_{out} be the trace when the system is in the state "probe thrown-out". Since we assume directed unitarity violation between state "probe thrown-in" and state "probe thrown-out", there is a unitarity violation and $\rho_{\mathrm{in}} \neq \rho_{\mathrm{out}}$.

Assume that a spin measure on a particle decides on the state "thrown-in" and "thrown-out". If spin is 1, then the probe is thrown in, and if the spin is 0, then the probe is thrown out. The trace of the density operator of system made of the spin and the probe is $\mathrm{Tr}\rho = \frac{1}{2}\rho_{\mathrm{in}} + \frac{1}{2}\rho_{\mathrm{out}}$ and the probability to have the spin value equal to 1 is $\frac{\frac{1}{2}\rho_{\mathrm{in}}}{\mathrm{Tr}\rho}$ and to 0 $\frac{\frac{1}{2}\rho_{\mathrm{out}}}{\mathrm{Tr}\rho}$.

This discrepancy in unitary makes that the time lines containing the state "probe thrown-in" have a different weight than the time lines containing the state "probe thrown-out" and distorts the probability of the events attached to these time lines. This distorsion gives rise to retro-information as described below.

Theorem 3. *Non-unitary operators can lead to retro-information.*

Proof: Let imagine, as in proof of theorem 1, that a quantum measurement with binary output occurs on point A. We assume that the density operator without non-unitary distorsion is $\rho = \frac{1}{2}\mathbf{P}_0 + \frac{1}{2}\mathbf{P}_1$ (for example a 1/2 spin measurement).

We imagine that there is a classical noiseless channel (figure 1) between point B and point A.

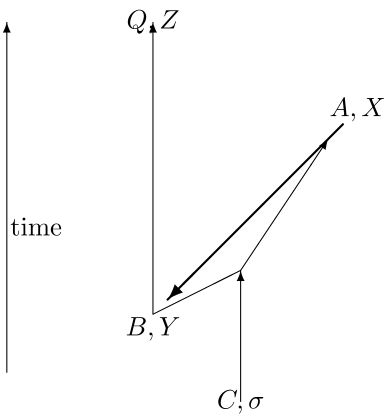

Figure 1: Retro-information from A to B

We imagine the following algorithm: when the output measured by Bob on B is 0 then Alice on point A throws a probe inside the black hole, otherwise she throws the probe out. We also assume that the black hole is the only non-unitarity effect on the causality line of the output measurement (or more generally the other non-unitary effects on the causality line occur independently of the actual output measurement). We assume that the history of the system decoheres between state “throw-in” and state “thrown-out” (e.g. the support of the wave functions are non-overlapping).

In this case the density operator of the measurement on B becomes

$$\rho^\star = \frac{\rho_{\text{in}}}{2}\mathbf{P}_0 + \frac{\rho_{\text{out}}}{2}\mathbf{P}_1. \tag{5}$$

This density operator, even when renormalized by its trace $\frac{1}{2}\rho_{\text{in}} + \frac{1}{2}\rho_{\text{out}}$, is different of ρ. Therefore there is an information transfer from Alice to Bob. This is retro-information since the transmitter is on the causality line of the receiver.■

The determination of the channel capacity is a specific problem. If we tune the number of spins to be measured at B, *i.e.* namely the size of the message received by Bob, with Alice still holding a single probe and without changing the non unitary parameter of the system, then it turns out that the capacity of the channel is at least equal to $|\log \frac{\rho_{\text{out}}}{\rho_{\text{in}}}|$ as proven in [4].

5 Conclusion and perspective

In this short paper we have addressed the unusual concept of causality in information. To the author best knowledge this simple question has not been addressed before. Surprisingly the physical causality does not imply information causality. In other words information transfers could travel backward in time without affecting the principle of causality. But this is merely due to some incompleteness in information theory since the information that can be transfered by this mean is not useful. We suggest to partially fill this gap by introducing the concept of space-time relaying. In this case we show that the principle of information causality is equivalent to the principle of unitarity, a weaker axiom in physics.

Assuming that unitarity holds, we prove that informations cannot travel faster than light.

Is unitarity an inalterable principle of physics is still an open question since unified theories such as strings and super-strings seem to have hard nuts to crack in this domain. But of course this far outside the scope of this paper.

More generally the results contained in this paper establish that energy and information don't necessary obey to the same rules, and finally bring an interesting new perspective on the intricate relations between information and space and time.

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