On variational data assimilation for 1D and 2D fluvial hydraulics

I. Gejadze[2], M. Honnorat[1], FX Le Dimet[1], and J. Monnier[1]

[1] LJK - MOISE project-team, Grenoble, France. `Contact: Jerome.Monnier@imag.fr`

[2] Civil Engineering Dept, Univ. Strathclyde, UK.

1 Introduction

We address two problems related to variational data assimilation (VDA) as applied to river hydraulics (1D and 2D shallow water models). In real cases, available observations are very sparse (especially during flood events). Generally, they are very few measures of elevation at gauging stations. The first goal of the present study is to estimate accurately some parameters such as the inflow discharge, manning coefficients, the topography and/or the initial state. Since the elevations measures (eulerian observations) are very sparse, we develop a method which allow to assimilate extra lagrangian data (trajectory particles at the surface e.g. extracted from video images). The second goal aims to develop a joint data assimilation - coupling method. We seek to couple accurately a 1D global net-model (rivers net) and a local 2D shallow water model (zoom into a flooded area), while we assimilate data. This "weak" coupling procedure is based on the optimal control process used for the VDA. Numerical twin experiments demonstrate that the present two methods makes it possible to improve on one hand the identification of river model parameters (e.g. topography and inflow discharge), on the other hand an accurate 1D-2D coupling combined with the identification of inflow boundary conditions.

The 2D forward model

The 2D forward model considered rely on the shallow water equations (SWE) (h is the water elevation, $\mathbf{q} = h\mathbf{u}$ the discharge, $\mathbf{u}$ the depth-averaged velocity):

$$\begin{cases} \partial_t h + \mathrm{div}(\mathbf{q}) \;=\; 0 & \text{in } \Omega \times]0,T] \\ \partial_t \mathbf{q} + \mathrm{div}(\frac{1}{h}\mathbf{q} \otimes \mathbf{q}) + \frac{1}{2} g \nabla h^2 + gh\nabla z_b + g\frac{n^2\|\mathbf{q}\|}{h^{7/3}}\mathbf{q} \;=\; 0 & \text{in } \Omega \times]0,T] \end{cases} \tag{1}$$

with initial conditions $(h_0, \mathbf{q}_0)$ given, g the magnitude of the gravity, z_b the bed elevation, n the Manning roughness coefficient. Boundary conditions are:

at inflow, the discharge $\bar{q}$ is prescribed; at outflow, either the water elevation $\bar{z_s}$ is prescribed or incoming characteristics are prescribed; and walls conditions. Given the control vector $\mathbf{c} = (h_0, \mathbf{q}_0, n, z_b, \bar{q}, \bar{z_s})$, the state variable $(h, \mathbf{q})$ is determined by solving the forward model.

2 Assimilation of lagrangian data

Lagrangian DA consists in using observations described by lagrangian coordinates in the DA process. Here, we consider observations of particles transported by the flow (e.g. extracted from video images). The link between the lagrangian data made of N particle trajectories denoted by $X_i(t)$ and the classical eulerian variables of the shallow water model is made by the following equations, see [4]:

$$\begin{cases} \frac{d}{dt} X_i(t) = \gamma\, \mathbf{u}\Big(X_i(t), t\Big) & \forall\, t \in]t_i^0, t_i^f[\\ X_i(t_i^0) \;\; = x_i^0 \;, & \end{cases} \qquad \text{for} \;\; i = 1, \ldots, N \tag{2}$$

where t_i^0 and t_i^f are the time when the particle enter and leave the observation domain, γ is a multiplicative constant. We consider two kinds of observations (classical eulerian observations $h^{obs}(t)$ and trajectories of particles transported by the flow $X_i^{obs}(t)$). Then, we build the following composite cost function:

$$j(\mathbf{c}) = \frac{1}{2} \int_0^T \left\| \, Ch(t) - h^{obs}(t) \, \right\|^2 dt + \frac{\alpha_t}{2} \sum_{i=1}^{N} \int_{t_i^0}^{t_i^f} \left| \, X_i(t) - X_i^{obs}(t) \, \right|^2 dt \tag{3}$$

where α_t is a scaling parameter, C the observation operator.

Numerical results.

Particle trajectories associated with local water depth measurements are used for the joint identification of local bed elevation z_b and initial conditions $(h^0, \mathbf{u}^0)$. A constant discharge $\bar{q}$ is prescribed at inflow, Fig. 1(a). A vertical cut of the fluid domain in the longitudinal plane in Fig. 1 (b) shows the bed and the free surface elevation for this configuration.

Twin DA experiments are carried out: observations are created by the model from the reference steady flow described above. Water depth is recorded continuously in time at the abscissae $x_1 = 15\,m$ and $x_2 = 70\,m$, for the whole width of the domain. These measurements are used as observations denoted by $h_i^{obs}(y;t)$ for $i = 1$, 2. With regards to the creation of trajectories observations, virtual particles are dropped in the reference steady flow and transported by a turbulent surface velocity $\mathbf{u}^t = \gamma \mathbf{u} + \mathbf{u}^p$, where $\gamma = 1$ and $\mathbf{u}^p$ is a Gauss-Markov process. A total of $N_{obs} = 640$ particles is released in the flow.
We seek to identify jointly the reference topography and the reference initial conditions (water depth h^0 and velocity $\mathbf{u}^0$) used to create the observations,

from the *a priori* hypothesis that the bed is made of a longitudinal slope of without bump and the initial conditions correspond to the steady state obtained with the modified topography. To that purpose, we carry out DA using cost function (3). As shown in Fig. 2 (a), the identified topography is close to the reference, with a good recovery of the bump. As for the initial conditions, we can see in Fig. 2 (c) and (d) that it reproduces the same main features as the reference.

3 A joint assimilation-coupling procedure

Operational models used in hydrology are generally net-models based on the 1D Saint-Venant equations with storage areas. Here we shortly describe a method which superpose locally the previous 2D SWE along the 1D channel, see [3]. The first issue is to nest properly the local 2D model into the 1D global model. To this end, we specify incoming characteristics at lateral boundary conditions (BC), Fig. 3:

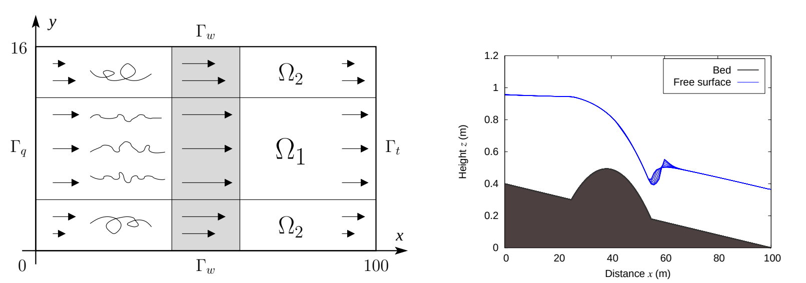
$$(x,y) \in \Gamma_3 : \begin{array}{l} q + (c-u)h = w_1(x,y,t) \\ p - vh = w_3(x,y,t), \; \forall u > 0 \end{array} \tag{4}$$

$$(x,y) \in \Gamma_4 : \begin{array}{l} q - (c+u)h = w_2(x,y,t) \\ p - vh = w_3(x,y,t), \; \forall u < 0 \end{array} \tag{5}$$

where the coefficients in (4)-(5) are: $c = (gh|_{t-\tau})^{1/2}$, $u = (q/h)|_{t-\tau}$, $v = (p/h)|_{t-\tau}$ and τ is a time shift, which is taken equal to the time integration step in the numerical implementation; w_1, w_2 and w_3 (depending on the sign of u) are the incoming characteristic variables that must be specified based on their counterparts W_1, W_2 from the global model defined as follows

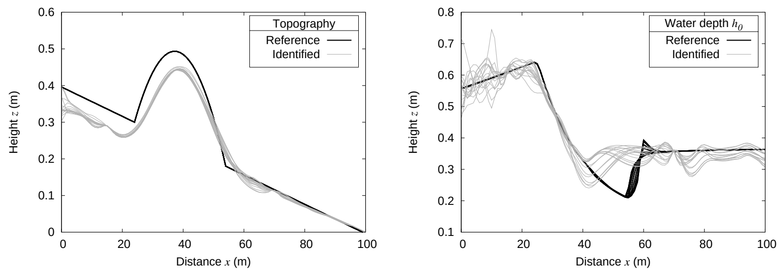
$$Q + (c-u)H = W_1(x,t), \; Q - (c+u)H = W_2(x,t) \tag{6}$$

where H, Q are variables of the 'dimensional' 1D SWE problem (i.e. scaled by the main channel width assuming the rectangular cross-section) and



(a) Domain description (b) Vertical cut of the fluid domain

Fig. 1. Flow configuration



(a) Identified topography (c) Identified h^0

Fig. 2. Joint identification of the topography and the initial conditions using water depth measurements and particle trajectories with $\alpha_t = 1 \times 10^{-4}$.

$c = (gH|_{t-\tilde{\tau}}/b)^{1/2}, u = (Q/H)|_{t-\tilde{\tau}}$.
The feedback from local to global model is achieved by computing a generalized defect correction term, which will be a source term to the global model equations, see [3] for more details. Another issue is that we couple two different models (1D and 2D). The problem is formulated as a DA problem, while the local model boundary conditions are considered as unknown controls. The coupling conditions in this formulation become penalty terms of the extended objective function $J = (\gamma J^* + J_1 + J_2)$ with $J^* = \sum_i \int_0^T (U_i - \hat{U}_i)^2 \; dt$ and $J_k = \int_0^T (\int \Gamma_{k+1} w_k d\Gamma - W_k|_{\Gamma_{k+1}})^2$, while as constraints we consider the one-way relaxed model described by the following steps: a) given current approximation w_k solve the 2D SWE local problem; b) compute 'defect correction'; c) given current approximation (or known values) $W_1(0)$, $W_2(L)$ and the 'defect correction' computed at previous stage solve the 1D SWE problem; d) compute extended objective function J. We refer to this method as to a 'joint assimilation-coupling method'(JAC).
In the numerical examples given below we solve DA problem for the 1D section (main channel) looking for the unknown inflow BC (characteristic $W_1(t, 0)$), while data is measured in the area covered by the 2D local model and is assimilated into this model correspondingly. The control problem for J is solved using the adjoint of the one-way relaxed model described by steps a)-c). Data is collected in two points as shown in Fig.3(a). The reference value is chosen to cause a 'flooding event' i.e. massive overflowing of the main channel in the area where the 2D local model is superposed. (Rem. Under these conditions assimilation of measurements from sensor A into the 1D model alone may fail to produce meaningful results because this model is not adequate. Data from sensor B cannot be assimilated in principle).

In the following assimilation examples, Fig.3(b), we can see the reference BC (in dashed line) and the retrieved value after k iterations of the JAC algorithm (in sharp solid lines). A line that corresponds to $k = 0$ is the initial guess. This example shows that the JAC method converges and allows retrieving the unknown BC of the 1D model, while data is assimilated into the weakly connected local 2D model.

References

1. F.-X. Le Dimet and O. Talagrand. Variational algorithms for analysis and assimilation of meteorological observations. *Tellus*, 38A:97–110, 1986.
2. I. Gejadze, M. Honnorat, X. Lai, J. Marin, and J. Monnier. DASSFLOW V2.0 : a variational data assimilation software for 2d shallow water river flows. To appear, INRIA reasearch report, 2007.
3. I. Gejadze and J. Monnier. On a 2d zoom for 1d shallow-water model: coupling and data assimilation. *Submitted*, 2007.
4. M. Honnorat, J. Monnier, and FX Le Dimet. Lagrangian data assimilation for river hydraulics simulations. *Submitted*, 2007.
5. J.-L. Lions. *Optimal Control of Systems Governed by Partial Differential Equations*. Springer-Verlag, 1971.

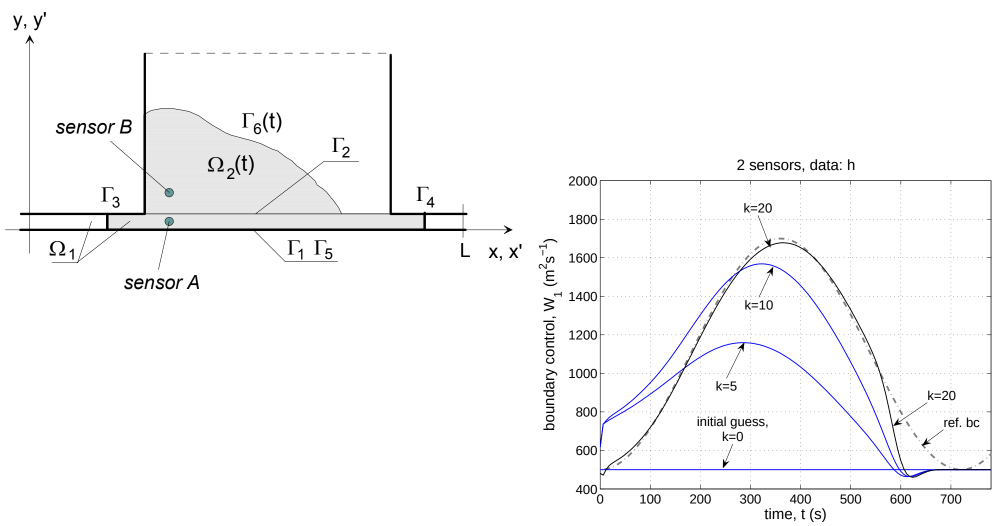


Fig. 3. (a) Problem layout (b) Assimilation of data (h) by the JAC algorithm: $W_1(t)$ after k iterations