Output control with Internet-in-the-loop : An event-driven realization

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Abstract

This work is devoted to the remote feedback control of a linear process with "Internet in the loop". In such a networked control situation, variable and unpredictable delays arise, which may decrease the global performance or destabilize the system. Our aim is to obtain the best performance despite the variation of the network QoS (quality of service). The considered application is based on a Master-Slave structure. The Slave is a light mobile robot, that receives the control data and sends its sampled position via a UDP protocol. A Master computer realizes the remote control, based on a remote observer and a state feedback. The global strategy is without buffers. The packets are time-stamped so the Master detects the variable time delays (the network QoS). This information is used to adapt its observer/controller gains and guarantee the best possible performances. The design of this gain scheduling strategy relies on Lyapunov-Krasovskii functionals with an LMI optimization which guarantees the stability even with packet losses. Experimental results are provided.

1 Introduction

The web technology of Internet is now widely applied to Networked Control Systems (NCS) thanks to its low costs. The goal is to separate the operating part called the slave and the computing part (the controller) called the master. Both communicate trough a network like Internet. However, Internet unavoidably introduces perturbations (delays, sampling, packet losses) into the system. These perturbations can

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be modeled as some time-varying communication delays ((Richard J.-P. and Divoux T., 2007),(Georges J.-P. *et al.*, 2005) and the references herein) which has to be taken into account in the control design. A variety of stability and control techniques have been developed for general time delay systems (Niculescu S.-I., 2001; Richard J.-P., 2003; Richard J.-P. and Divoux T., 2007). To apply these results, some critical assumptions have to be made.

The first important point is the model of the delay. In fact, to consider the time delay as constant (Fattouh A. and Sename O., 2003; Niemeyer G. and Slotine J.-J., 1998) is actually unrealistic due to the dynamic character of the network. A delay maximizing strategy (Lelevé A. *et al.*, 2001) ("virtual delay", "buffer", or "waiting" strategy) can be carried out so to make the delay become constant and known. This requires the knowledge of the maximum delay values. Whereas the Internet induced delay can be theoretically unbounded, it is assumed that the delay is bounded and any data packets with a longer delay are considered lost. This latter case is treated as a robustness problem.

The second important choice is the control strategy. Some works consider the introduction of a buffer on the slave part (Seuret A. *et al.*, 2006; Jiang W.-J. *et al.*, 2008) which makes the application time of the control known. This allows the use of the separation principle between the control and the observation problems. This technique has one major flaw: the resulting delay is much greater that the real one and decreases the performances.

In this work, the control strategy is event-driven i.e. there is no slave buffer. The control is applied to the slave as soon as this latter receives it. The communication delay is modeled as a bounded one with a possible loss of packets. In the case where the delay is greater than the chosen bound, the packet is treated as lost. The packet loss problem study gives the maximum number of lost packet admissible. The theoretical result is adapted from (Seuret A. and Richard J.-P., 2008). If this number is bigger than the maximum admissible value, the control is stopped. The system is considered as a linear time-invariant one.

2 Problem statement

2.1 Features of the networked control system

The networked control system (NCS) is based on Master-Slave structure. The main features of the system refer to Fig.1.

The transmission protocol UDP is applied to communicate the data between Master and Slave. UDP is preferred to TCP since the retransmission of old data is not very useful. In order to know the instant when the positions of the robot have been measured, the Master and the Slave are synchronized before the system starts by the way of NTP (Network Time Protocol) (Mills D.L., 1995) and time stamps are added to the data packets.

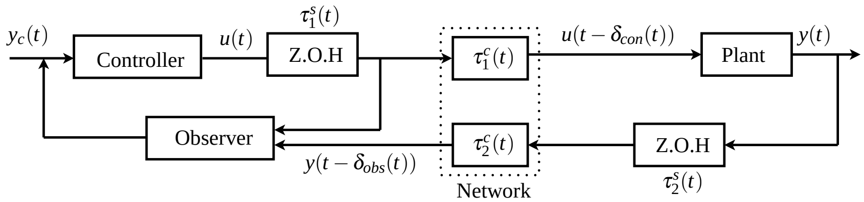

Figure 1: Feedback NCS with observer-based controller

2.2 The three delay sources

In such a situation, the variable delays come from the communication through the Internet, the data-sampling and the packet losses.
In the sequel, $\tau_1^c(t)$ and $\tau_2^c(t)$ denote the communication delays and $\tau_1^s(t)$ and $\tau_2^s(t)$ denote the sampling delays. The total Master-to-Slave delay $\delta_{con}(t)$ results from the addition of these three delays. The same phenomenon stands for the Slave-to-Master delay which is denoted $\delta_{obs}(t)$. The total delay can be represent as $\delta_{con}(t) = \tau_1^c(t) + \tau_1^s(t) + NT$ and $\delta_{obs}(t) = \tau_2^c(t) + \tau_2^s(t) + NT$, where N is the possible number of successive packets loss and T is the maximum sampling period. So the time-delay can be treated as a variable but bounded one.

2.3 Effects of time-delay on the performance and stability

The time-delay of the Internet varies a lot especially between the rush hour and idle time periods. In order to guarantee the exponential stabilization, we have to choose the maximum admissible time-delay, whereas most of the time, the time-delay is much smaller. In other words, the performance is unnecessarily decreased.
In order to enhance the performance and make the system adaptable to the varying time-delay of the Internet, we propose a switching controller design. In our system, two modes are considered, which switch according to the value of time-delay. The mode 1 is active if the time-delay belongs to $[\sigma_{min}^1, \sigma_{max}^1]$ and the mode 2 if in $[\sigma_{min}^2, \sigma_{max}^2]$, where $\sigma_{max}^1 = \sigma_{min}^2$.
Of course, for greater delay values, the performance cannot be guaranteed anymore and an alternative solution has to be considered. In our system, we give a command for the robot to stop until the communication comes back to a sufficient quality.
In the following, it is assumed that despite the nonsymmetric of the delays ($\delta_{con}(t) \neq \delta_{obs}(t)$), the two delays belong to the same interval.

2.4 Description of the closed-loop system

Consider the Slave as a linear system. It is described by the following form:

$$\begin{cases} \dot{x}(t) = & Ax(t) + Bu(t - \delta_{coni}(t)), \\ y(t) = & Cx(t), \end{cases} \tag{1}$$

where $i = 1, 2$, $\delta_{con1}(t) \in [\sigma_1, \sigma_2]$, $\delta_{con2}(t) \in [\sigma_2, \sigma_3]$ and $\dot{\delta}_{coni}(t) \leq 1$.
This allows for using of a polytopic formulation of the variable delays (Seuret A. *et al.*, 2006). In order to guarantee the closed-loop performance whatever the delay variation, the exponential stability with the rate α must be achieved. In other words, there must be a real $\kappa \geq 1$ so that the solution $x(t;t_0,\phi)$ starting at any time t_0 from any initial function ϕ satisfies:$\|x(t;t_0,\phi)\| \leq \kappa \|\phi\|_c e^{-\alpha(t-t_0)}$. In this paper, it is achieved using the following state feedback and observer:

$$u(t) = -K_i \hat{x} + k y_c, \tag{2}$$

$$\begin{cases} \dot{\hat{x}}(t) = & A\hat{x}(t) + Bu(t - \hat{\delta}_{coni}(t)) - L_i(y(t - \delta_{obsi}(t)) - \hat{y}(t - \delta_{obsi}(t))), \\ \hat{y}(t) = & C\hat{x}(t), \end{cases} \tag{3}$$

$i = 1, 2$ corresponding the two switching periods. y_c being the desired set point and k a gain ensuring an unitary static gain for the closed loop.
Because of the separation principle, one can divide the analysis of the global stabilization into two smaller problems: the observer design and the controller design.

3 A switched systems approach: arbitrary switching

Note that in the system, there are no more buffers, which means the Master is not supposed to know the time of packet applied. The separation principle do not hold in this case. To solve the problem, the control gains and the observer gains are computed as if there was a separation principle (section 3.1.3 and 3.1.4) for some performances (guaranteed by the exponential convergence). This means that they are computed as if there is no packet lost and the Master-to-Slave delay is known, $\hat{\delta}_{coni}(t) = \delta_{coni}(t)$.
Then the global stability of the system is checked by considering both observation and control problem for the given gains (section 3.2).

3.1 Stabilization for the case without packet loss

3.1.1 Asymptotic stability of the switched systems

Consider the switched system

$$\dot{x}(t) = Ax(t) + \chi(\tau)A_1 x(t - \tau_1(t)) + (1 - \chi(\tau))A_2 x(t - \tau_2(t)), \tag{4}$$

where $\tau_i \in [\sigma_i, \sigma_{i+1}],\ i = 1,2$ and where $\chi : R \to \{0,1\}$ is the characteristic function of $[\sigma_1, \sigma_2]$

$$\chi(s) = \begin{cases} 1, \text{ if } s \in [\sigma_1, \sigma_2] \\ 0, \text{ otherwise.} \end{cases}$$

Consider the following Lyapunov functional:

$$\begin{array}{l} V(t,x_t,\dot{x}_t) = x^T(t)Px(t) + \sum_{i=0}^{2} \int_{t-\sigma_{i+1}}^{t} x^T(s)S_i x(s)ds \\ + \sum_{i=0}^{2} (\sigma_{i+1} - \sigma_i) \int_{-\sigma_{i+1}}^{-\sigma_i} \int_{t+\theta}^{t} \dot{x}^T(s)R_i\dot{x}(s)dsd\theta \end{array} \tag{5}$$

where $\sigma_0 = 0$, $P > 0$ and $R_i, S_i \geq 0$.
Differentiating V, we find

$$\begin{array}{l} \dot{V}(t,x_t,\dot{x}_t) \leq 2x^T(t)P\dot{x}(t) + \dot{x}^T(t)\sum_{i=0}^{2}(\sigma_{i+1} - \sigma_i)^2 R_i)\dot{x}(t) \\ -\sum_{i=0}^{2}(\sigma_{i+1} - \sigma_i)\int_{t-\sigma_{i+1}}^{t-\sigma_i} \dot{x}^T(s)R_i\dot{x}(s)ds + x^T(t)\sum_{i=0}^{2} S_i x(t) - \sum_{i=0}^{2} x^T(t-\sigma_i)S_i x(t-\sigma_i). \end{array} \tag{6}$$

We start with the case of $\chi = 1$, i.e. of $\tau \in [\sigma_1, \sigma_2]$. We apply the Jensen's inequality (Gu K. *et al.*, 2003)

$$\int_{b(t)}^{a(t)} \dot{x}^T(s)R_i\dot{x}(s)ds \geq \frac{1}{\Delta}\int_{b(t)}^{a(t)} \dot{x}^T(s)dsR_i \int_{b(t)}^{a(t)} \dot{x}(s)ds$$

Where $b(t) < a(t) < 0$ and $\Delta = \max(a(t)) - \min(b(t))$. Then, denoting

$$v_{j1} = \int_{t-\tau(t)}^{t-\sigma_j} \dot{x}(s)ds, \quad v_{j2} = \int_{t-\sigma_{j+1}}^{t-\tau(t)} \dot{x}(s)ds, \tag{7}$$

we obtain

$$\begin{array}{l} \dot{V}(t,x_t,\dot{x}_t) \leq x^T(t)P\dot{x}(t) + \dot{x}^T(t)\sum_{i=0}^{2}(\sigma_{i+1} - \sigma_i)^2 R_i)\dot{x}(t) + \sum_{i=0}^{2} x^T(t)S_i x(t) \\ -\sum_{i=0}^{2} x^T(t-\sigma_{i+1})S_i x(t-\sigma_{i+1}) - [x(t) - x(t-\sigma_1)]^T R_0[x(t) - x(t-\sigma_1)] \\ -[x(t-\sigma_2) - x(t-\sigma_3)]^T R_2[x(t-\sigma_2) - x(t-\sigma_3)] - v_{11}^T R_1 v_{11} - v_{12}^T R_1 v_{12}. \end{array} \tag{8}$$

We use further the descriptor method (Fridman E. and Shaked U., 2001), where the right-hand side of the expression

$$0 = 2[x^T(t)P_2^T + \dot{x}^T(t)P_3^T][Ax(t) + A_j x(t-\sigma_j) - A_j v_{j1} - \dot{x}(t)], \tag{9}$$

with some $n \times n$-matrices P_2, P_3 is added into the right-hand side of (8). We also add free weighting matrices of (He Y. *et al.*, 2004)

$$0 = 2[x^T(t)Y_{j1}^T + \dot{x}^T(t)Y_{j2}^T][x(t-\sigma_{j+1}) + v_{j1} + v_{j2} - x(t-\sigma_j)]. \tag{10}$$

Setting $\eta_j(t) = col\{x(t), \dot{x}(t), x(t-\sigma_1), x(t-\sigma_2), v_{j1}, v_{j2}, x(t-\sigma_3)\}$, we obtain that

$$\dot{V}(t,x_t,\dot{x}_t) \leq \eta_1^T(t)\Phi|_{\chi=1}\eta_1(t) < 0, \tag{11}$$

if the LMI

$$\Phi|_{\chi=1} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & R_0 + P_2^T A_1 - Y_{11}^T & Y_{11}^T & Y_{11}^T - P_2^T A_1 & Y_{11}^T & 0 \\ * & \Phi_{22} & P_3^T A_1 - Y_{12}^T & Y_{12}^T & Y_{12}^T - P_3^T A_1 & Y_{12}^T & 0 \\ * & * & -(S_0 + R_0) & 0 & 0 & 0 & 0 \\ * & * & * & -(S_1 + R_2) & 0 & 0 & R_2 \\ * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & -(S_2 + R_2) \end{bmatrix} < 0 \tag{12}$$

holds, where

$$\begin{array}{l} \Phi_{11} = A^T P_2 + P_2^T A + \sum_{i=0}^2 S_i - R_0, \\ \Phi_{12} = P - P_2^T + A^T P_3,\ \Phi_{22} = -P_3 - P_3^T + \sum_{i=0}^2 (\sigma_{i+1} - \sigma_i)^2 R_i. \end{array} \tag{13}$$

For $\chi = 0$, i.e. for $\tau \in [\sigma_2, \sigma_3]$ applying the same arguments and representation with $j = 2$ and $i = 0, 1$ we obtain the LMI

$$\Phi|_{\chi=0} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & R_0 & P_2^T A_2 - Y_{21}^T & Y_{21}^T - P_2^T A_2 & Y_{21}^T & Y_{21}^T \\ * & \Phi_{22} & 0 & P_3^T A_2 - Y_{22}^T & Y_{22}^T - P_3^T A_2 & Y_{22}^T & Y_{22}^T \\ * & * & -(S_0 + R_0 + R_1) & R_1 & 0 & 0 & 0 \\ * & * & * & -(S_1 + R_1) & 0 & 0 & 0 \\ * & * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & * & -S_2 \end{bmatrix} < 0 \tag{14}$$

Then , along the switched system (4)

$$\dot{V}(t, x^t, \dot{x}^t) \leq \chi(\tau)\eta_1^T(t)\Phi|_{\chi=1}\eta_1(t) + (1 - \chi(\tau))\eta_2^T(t)\Phi|_{\chi=0}\eta_2(t) < 0, \tag{15}$$

Thus, the following result is obtained.

Theorem 1 *Let there exist $n \times n$-matrices $P > 0$, $R_i > 0, S_i > 0, i = 0, 1, 2$, P_2,P_3, Y_{j1} and Y_{j2}, $j = 1, 2$ such that the LMIs (12), (14) with notations given in (13) are feasible. Then system (4) is asymptotically stable for all fast-varying delays $\tau_j \in [\sigma_j, \sigma_{j+1}]$, $j = 1, 2$.*

3.1.2 Exponential stability of the switched systems

According to (Niculescu S.-I. *et al.*, 1998), for a real $\alpha > 0$, the system (4) is said to be α-stable, or 'exponentially stable with the rate α', if there exits a scalar $F \geq 1$ such that the solution $x(t; t_0, \phi)$ satisfies:

$$|x(t; t_0, \phi)| \leq F|\phi|e^{-\alpha(t-t_0)}. \tag{16}$$

Substituting the new variable $x_\alpha(t) = e^{\alpha t}x(t)$ with $\alpha > 0$ in (4), we get:

$$\begin{aligned} \dot{x}_\alpha(t) = \quad & (A + \alpha I)x_\alpha(t) + \chi(\tau)e^{\alpha\tau_1(t)}A_1 x_\alpha(t - \tau_1(t)) \\ & + (1 - \chi(\tau))e^{\alpha\tau_2(t)}A_2 x_\alpha(t - \tau_2(t)). \end{aligned} \tag{17}$$

Considering the case $\chi = 1$, as $e^{\alpha\sigma_1} \leq e^{\alpha\tau_1(t)} \leq e^{\alpha\sigma_2}$, we can rewrite the equation (17) in the following polytopic form:

$$\begin{aligned} \dot{x}_\alpha(t) = \quad & \sum_{i=1}^{2} \lambda_i(t)\{(A+\alpha I)x_\alpha(t) + e^{\alpha\sigma_1}A_1x_\alpha(t-\tau_1(t)) \\ & + e^{\alpha\sigma_2}A_1x_\alpha(t-\tau_1(t))\}. \end{aligned} \tag{18}$$

for i=1,2, $\lambda_i(t) \geq 0$, $\sum_{i=1}^{2} \lambda_i(t) = 1$.
The case $\chi = 0$ can be rewritten in the same way.

Theorem 2 *If the system (17) can be proved to be asymptotically stable by the theorem (1), then the system (4) is exponentially stable.*

3.1.3 Observer design

We define the error vector between the estimated state $\hat{x}(t)$ and the present system state $x(t)$ as $e(t) = x(t) - \hat{x}(t)$. From (1) and (3), this error is ruled by:

$$\dot{e}(t) = \quad Ae(t) + L_iCe(t - \delta_{obsi}(t)). \tag{19}$$

Applying the theorem (1) and (2), we choose $P_3 = \varepsilon P_2$, where ε is the adaptable parameter to obtain the best result, and $W_i = P_2^T L_i$, i=1,2, then we get following results.

Theorem 3 *Suppose that, for some positive scalars α and ε, there exists $n \times n$ matrices $0 < P$, S_k, $R_k(k = 0,1,2)$, and matrices P_2, Y_{11}, Y_{12}, Y_{21}, Y_{22}, W_1, W_2 with appropriate dimensions such that the following LMI conditions are satisfied:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & R_0 + \beta_{ij}W_iC - Y_{11}^T & Y_{11}^T & Y_{11}^T - \beta_{ij}W_iC & Y_{11}^T & 0 \\ * & \Phi_{22} & \varepsilon\beta_{ij}W_iC - Y_{12}^T & Y_{12}^T & Y_{12}^T - \varepsilon\beta_{ij}W_iC & Y_{12}^T & 0 \\ * & * & -(S_0 + R_0) & 0 & 0 & 0 & 0 \\ * & * & * & -(S_1 + R_2) & 0 & 0 & R_2 \\ * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & -(S_2 + R_2) \end{bmatrix} < 0 \tag{20}$$

for $i = 1$, $j = 1,2$;

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & R_0 & \beta_{ij}W_iC - Y_{21}^T & Y_{21}^T - \beta_{ij}W_iC & Y_{21}^T & Y_{21}^T \\ * & \Phi_{22} & 0 & \varepsilon\beta_{ij}W_iC - Y_{22}^T & Y_{22}^T - \varepsilon\beta_{ij}W_iC & Y_{22}^T & Y_{22}^T \\ * & * & -(S_0 + R_0 + R_1) & R_1 & 0 & 0 & 0 \\ * & * & * & -(S_1 + R_1) & 0 & 0 & 0 \\ * & * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & * & -S_2 \end{bmatrix} < 0 \tag{21}$$

for $i = 2$, $j = 1,2$ hold, where

$$\begin{aligned} &\Phi_{11} = (A+\alpha_i I)^T P_2 + P_2^T(A+\alpha_i I) + \textstyle\sum_{k=0}^{2} S_k - R_0, \\ &\Phi_{12} = P - P_2^T + \varepsilon(A+\alpha_i I)^T P_2, \\ &\Phi_{22} = -\varepsilon P_2 - \varepsilon P_2^T + \textstyle\sum_{k=0}^{2}(\sigma_{k+1} - \sigma_k)^2 R_k. \end{aligned} \tag{22}$$

β_{ij} are defined by:

$$\beta_{11} = e^{\alpha_1\sigma_1}, \quad \beta_{12} = e^{\alpha_1\sigma_2} \quad \beta_{21} = e^{\alpha_2\sigma_2}, \quad \beta_{22} = e^{\alpha_2\sigma_3} \tag{23}$$

Then, the gain: $L_i = (P_2^T)^{-1} W_i,\ i = 1,2$, *makes* $\hat{x}(t)$ *of observer (3) exponentially converge to the vector* $x(t)$ *of equation 1, with a decay rate* α *for an output delay* $\delta_{obsi}(t) = \delta_{obsi} + \eta_{obsi}(t), |\eta_{obsi}(t)| \leq \mu_{obsi}$.

3.1.4 Controller design

We first consider a controller $u = K_i x$, $i = 1,2$, *i.e.* the ideal situation $e(t) = 0$, $x(t) = \hat{x}(t)$ and:

$$\dot{x}(t) = Ax(t) + BK_i x(t - \delta_{coni}(t)). \tag{24}$$

Applying the theorem (1) and (2), choosing $\bar{P}_3 = \varepsilon \bar{P}_2$, applying the bijective transformation $M_i = K_i \bar{P}_2$, i=1,2, and some matrix manipulations (congruence), then we get following results.

Theorem 4 *Suppose that, for some positive scalars* α *and* ε, *there exists* $n \times n$ *matrices* $0 < \bar{P}$, $\bar{S}_k$, $\bar{R}_k (k = 0,1,2)$, *and matrices* $\bar{P}_2$, $\bar{Y}_{11}$, $\bar{Y}_{12}$, $\bar{Y}_{21}$, $\bar{Y}_{22}$, M_1, M_2 *with appropriate dimensions such that the following LMI conditions are satisfied:*

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{R}_0 + \beta_{ij} BM_i - \bar{Y}_{11}^T & \bar{Y}_{11}^T & \bar{Y}_{11}^T - \beta_{ij} BM_i & \bar{Y}_{11}^T & 0 \\ * & \bar{\Phi}_{22} & \varepsilon \beta_{ij} BM_i - \bar{Y}_{12}^T & \bar{Y}_{12}^T & \bar{Y}_{12}^T - \varepsilon \beta_{ij} BM_i & \bar{Y}_{12}^T & 0 \\ * & * & -(\bar{S}_0 + \bar{R}_0) & 0 & 0 & 0 & 0 \\ * & * & * & -(\bar{S}_1 + \bar{R}_2) & 0 & 0 & \bar{R}_2 \\ * & * & * & * & -\bar{R}_1 & 0 & 0 \\ * & * & * & * & * & -\bar{R}_1 & 0 \\ * & * & * & * & * & * & -(\bar{S}_2 + \bar{R}_2) \end{bmatrix} < 0 \tag{25}$$

for $i = 1$, $j = 1,2$;

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{R}_0 & \beta_{ij} BM_i - \bar{Y}_{21}^T & \bar{Y}_{21}^T - \beta_{ij} BM_i & \bar{Y}_{21}^T & \bar{Y}_{21}^T \\ * & \bar{\Phi}_{22} & 0 & \varepsilon \beta_{ij} BM_i - \bar{Y}_{22}^T & \bar{Y}_{22}^T - \varepsilon \beta_{ij} BM_i & \bar{Y}_{22}^T & \bar{Y}_{22}^T \\ * & * & -(\bar{S}_0 + \bar{R}_0 + \bar{R}_1) & \bar{R}_1 & 0 & 0 & 0 \\ * & * & * & -(\bar{S}_1 + \bar{R}_1) & 0 & 0 & 0 \\ * & * & * & * & -\bar{R}_2 & 0 & 0 \\ * & * & * & * & * & -\bar{R}_2 & 0 \\ * & * & * & * & * & * & -\bar{S}_2 \end{bmatrix} < 0 \tag{26}$$

for $i = 2$, $j = 1,2$ *hold, where*

$$\begin{array}{l} \bar{\Phi}_{11} = \bar{P}_2 (A + \alpha_i I)^T + (A + \alpha_i I) \bar{P}_2^T + \sum_{k=0}^2 \bar{S}_k - \bar{R}_0, \\ \bar{\Phi}_{12} = \bar{P} - \bar{P}_2^T + \varepsilon \bar{P}_2 (A + \alpha_i I)^T, \\ \bar{\Phi}_{22} = -\varepsilon \bar{P}_2 - \varepsilon \bar{P}_2^T + \sum_{k=0}^2 (\sigma_{k+1} - \sigma_k)^2 \bar{R}_k. \end{array} \tag{27}$$

β_{ij} *(i,j=1,2) are defined by (23),*
Then, the gain: $K_i = M_i (\bar{P}_2)^{-1},\ i = 1,2$, *exponentially stabilizes the system (24) with the decay rate* α *for all the delay* $\delta_1(t)$.

3.2 Stabilization with packet loss

In this part, the maximum number of packet loss is considered for both zones, while the system stability is still guaranteed. In this case, $\hat{\delta}_{coni}(t) \neq \delta_{coni}(t)$ in the equation (3), *i.e.* $\sigma_i + NT \leq \hat{\delta}_{coni}(t) \leq \sigma_{i+1} + NT$, $i = 1,2$ corresponding to the two zones. Adapting the theorem (Seuret A. and Richard J.-P., 2008) for both zones, then the maximum number of packet loss is obtained.

Theorem 5 *For given K_i and L_i, for q representing the subscript x or e, suppose there exist positive definite matrices $P_{q1}, S_q, R_{qa}, S_{xe}$ and R_{xe} and matrices of size $n \times n$: P_{q2}, P_{q3}, Z_{ql} for $l = 1,2,3$, $Y_{ql'}$ for $l' = 1,2$, such that the following LMI's hold:*

$$\begin{bmatrix} \Theta_x & P_x^T \begin{bmatrix} 0 \\ BK_i \end{bmatrix} - \begin{bmatrix} Y_{x1}^T \\ Y_{x2}^T \end{bmatrix} & \mu_i P_x^T \begin{bmatrix} 0 \\ BK_i \end{bmatrix} & P_x^T \begin{bmatrix} 0 \\ BK_i \end{bmatrix} & \mu_i P_x^T \begin{bmatrix} 0 \\ BK_i \end{bmatrix} \\ * & -S_x & 0 & 0 & 0 \\ * & * & -(\mu_i R_{xa}) & 0 & 0 \\ * & * & * & -S_{xe} & 0 \\ * & * & * & * & -\mu_i R_{xe} \end{bmatrix} < 0 \qquad (28)$$

$$\begin{bmatrix} \Theta_e & P_e^T \begin{bmatrix} 0 \\ L_i C \end{bmatrix} - \begin{bmatrix} Y_{e1}^T \\ Y_{e2}^T \end{bmatrix} & \mu_i P_e^T \begin{bmatrix} 0 \\ L_i C \end{bmatrix} & P_e^T \begin{bmatrix} 0 \\ \beta_i BK_i \end{bmatrix} & \mu_i P_e^T \begin{bmatrix} 0 \\ \beta_i BK_i \end{bmatrix} \\ * & -S_e + S_{xe} & 0 & 0 & 0 \\ * & * & -(\mu_i R_{ea}) & 0 & 0 \\ * & * & * & -\beta_i R_{ep} & 0 \\ * & * & * & * & -\beta_i R_{xp} \end{bmatrix} < 0 \qquad (29)$$

$$\begin{bmatrix} R_q & Y_{q1} & Y_{q2} \\ * & Z_{q1} & Z_{q2} \\ * & * & Z_{q3} \end{bmatrix} \geq 0, \qquad (30)$$

$q \in \{x, e\}$, where $\beta_i = 2\mu_i$, $\mu_i = \sigma_{i+1} - \sigma_i + NT, \delta_i = \sigma_{i+1} + \sigma_i + NT$,

$$P_q = \begin{bmatrix} P_{q1} & 0 \\ P_{q2} & P_{q3} \end{bmatrix} \quad \Theta_x = \Theta_x^n + \begin{bmatrix} 0 & 0 \\ 0 & 2\mu_i * R_{xp} \end{bmatrix} \qquad (31)$$

$$\Theta_e = \Theta_e^n + \begin{bmatrix} 0 & 0 \\ 0 & 2\mu_i * (R_{ep} + R_{xe}) \end{bmatrix} \qquad (32)$$

$$\Theta_q^n = P_q^T \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P_q + \begin{bmatrix} S_q + Y_{q1} + Y_{q1}^T + \delta_i Z_{q1} & Y_{q2} + \delta_i Z_{q2} \\ * & \delta_i R_q + 2\mu_i R_{qa} + \delta_i Z_{q3} \end{bmatrix} \qquad (33)$$

Then, the global remote system is asymptotically stable.

4 Experiment

The experiments are done on two computers separated from about 40 kilometers away (Fig. 2). The Master program runs on the remote computer with an advanced computing capability, the Slave one on the local one which also communicates with the Miabot by Bluetooth. The local time-delay of Bluetooth is treated as constant one and is added into the total time-delay.
According to the identification of the Miabot, when the speed is no more than 2m/sec, it can be treated as a linear system and the model is showed in equation (34).

$$\begin{cases} \dot{x}(t) = & \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.024 \end{bmatrix} u(t - \delta_{con}(t)) \\ y(t) = & \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{cases} \qquad (34)$$

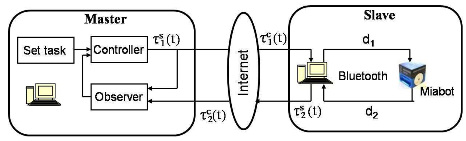

Figure 2: Structure of the global system

Considering two zones of delay with [0.01, 0.09[and [0.09, 0.2]. It means that the gains switch when the delay crosses the value of 0.09*sec*. The controller gains are computed using theorem 3 and 4. Then the global stability (observer + controller) is checked using theorem 5. The maximum exponential convergence ensuring the global stability are: $\alpha_1^c = \alpha_1^o = 3.1$, $\alpha_2^c = \alpha_2^o = 1$. The maximum numbers of packets lost for the two modes are 0 and 3 respectively while the system stays stable. Note that no packet losses have to be considered when the delay is small.
The gains K_i and L_i $(i = 1, 2)$ are:

$$\begin{bmatrix} L_1 & L_2 \end{bmatrix} = \begin{bmatrix} -3.02 & -1.37 \\ -0.27 & -0.36 \end{bmatrix}, \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -1311 & -129 \\ -682 & -68 \end{bmatrix}, \quad (35)$$

4.1 Results of remote experiment

The result is shown in Fig.3, in which the blue curve represents the set values; the black and red represent respectively the robot's estimated position and speed; the green corresponds to the real position of the Miabot. Fig.4 illustrate the corresponding switched control signals from Master to Slave. The red curve is the real control while the green and the black ones are the controls calculated respectively for the two subsystems. We can see the switch points according to the values of time-delay. Fig.5 depicts the variable time-delays and also the switching signals.
On Fig.3, one can notice that when the time-delay is smaller than 90*ms*, only the first mode is active, *i.e.* only the gains K_1 and L_1 are active. In this case, the system is quickly stabilized. While when the time-delay becomes bigger than 90*ms*, certain performances are guaranteed. As it is clearly shown in Fig.3, the global stability of the closed loop is maintained despite the assumption that the time-delay of Bluetooth is treated as a constant one.

5 Conclusion

In addition to some fundamental results, an experimental platform has been developed to illustrate the results of the network-based control theory. This platform is able to control a Slave robot through a network and joins skills in automatic control, computer science and networks.The experimental results confirm the theory: 1) The exponential

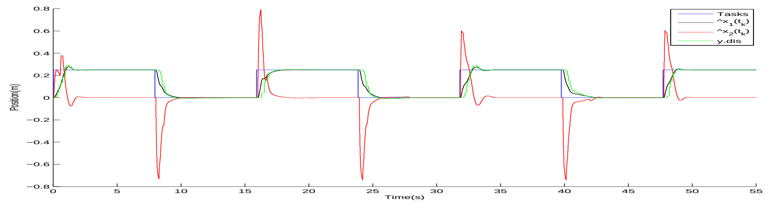

Figure 3: Results of remote experiment

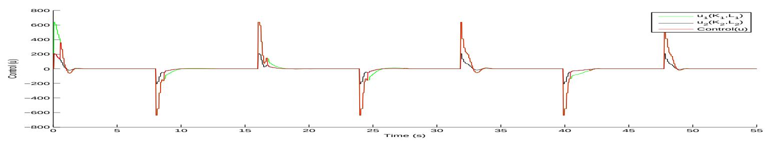

Figure 4: The corresponding switched control

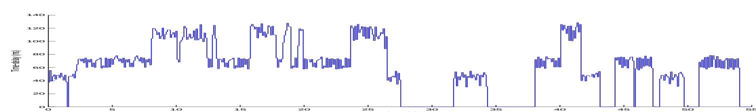

Figure 5: The corresponding variable time-delays and signals of switching

stability is obtained in both the time-delay zones and the *uniform* stability is guaranteed whatever the zone switches. 2) As there is no buffer in the side of the Slave, the control is directly applied to the Miabot, which is more efficient than the system with buffer (Jiang W.-J. *et al.*, 2008).

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