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Change point detection with application to the identification of a switching process

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Abstract

This paper deals with the change point detection problem with application to filtering and on-line identification of simple hybrid systems. The proposed method allows to obtain fast estimators for the unknown switch times and parameters. Numerical simulations with noisy data are provided.

1 Introduction

This paper dwells on change detection problems and fast identification techniques (non asymptotic) initially proposed by M. Fliess & H. Sira-Ramirez [1] in the framework of nonlinear and finite dimensional models. While most of the existing estimation techniques have been done in a probabilistic setting [3], this approach is based on a new standpoint which is based on differential algebra.

The paper is organized as follows. Section 2 focuses on change detection and filtering problem for noisy piece-wise constant signals. A two step procedure is proposed in which on-line switching time estimates are first provided and used in a second phase for filtering issues. Section 3 is devoted to an application to switched linear systems identification. Most of the results are performed in a distributional framework using usual definitions and basic properties described below.

Distribution Framework We recall here some standard definitions and results from distribution theory [6], and fix the notations to be used in the sequel. The space of C^∞-functions having compact support in an open subset Ω of $\mathbb{R}$ is denoted by $\mathcal{D}(\Omega)$, and $\mathcal{D}'(\Omega)$ is the space of distributions on Ω, i.e., the space of continuous linear functionals on $\mathcal{D}(\Omega)$. For $T \in \mathcal{D}'$, $\langle T, \varphi \rangle$ denotes the real number which linearly and continuously depends on $\varphi \in \mathcal{D}$. This number is defined as $\langle T, \varphi \rangle = \int_{-\infty}^{\infty} f.\varphi$ for a locally bounded function $T = f$. For the Dirac distribution $T = \delta$ and its derivative $T = \dot{\delta}$, the functional is defined as $\langle T, \varphi \rangle = \varphi(0)$ and $\langle T, \varphi \rangle = \dot{\varphi}(0)$ respectively.

The complement of the largest open subset of Ω in which a distribution T vanishes is called the support of T and is written $\operatorname{supp} T$. Write $\mathcal{D}'_+$ the space of distributions with support contained in $[0, \infty)$. It is an algebra with respect to convolution with identity δ, the Dirac distribution. For $T, S \in \mathcal{D}'_+$, the convolution product is defined as $\langle T * S, \varphi \rangle = \langle T(x).S(y), \varphi(x+y) \rangle$, and can be identified with the familiar convolution product $(T * S)(t) = \int_{-\infty}^{\infty} T(\theta)S(t-\theta)d\theta$ in case of locally bounded functions T and S.

Functions are considered through the distributions they define and are therefore indefinitely differentiable. Hence, if y is a continuous function except at a point a with a finite jump σ_a, its derivative writes $\dot{y} = dy/dt + \sigma_a\, \delta_a$, where dy/dt is the distribution stemming from the usual derivative of y. Derivation, integration and translation can be formed from the convolution products $\dot{y} = \delta^{(1)} * y, \quad \int y = H * y, \quad y(t-\tau) = \delta_\tau * y$, where $\delta^{(1)}$ (or $\dot{\delta}$) is the derivative of the Dirac distribution, and H is the familiar Heaviside step function. For $S, T \in \mathcal{D}'_+$, $\operatorname{supp}\ S * T \subset \operatorname{supp}\ S + \operatorname{supp}\ T$, where the sum in the right hand side is defined by $\{x + y\,;\, x \in \operatorname{supp}\ S,\ y \in \operatorname{supp}\ T\}$. Finally, with no danger of confusion, we shall denote $T(s)$, $s \in \mathbb{C}$, the Laplace transform of T.

2 Change point detection

We propose in this section a new tool in order to overcome the classical trade-off between having stable but slowly adapting estimates and agile but noise sensitive estimators. The approach is based on an on-line two step procedure in which the estimated switching times are reintroduced in a parameter estimation algorithm. Although generalization to higher order signals is straightforward, we focus on a piece-wise constant signal $y(t)$ described in (1), with the corresponding switching time function $s(t)$:

$$y(t) = \sum_{i=0}^{\infty} a_i\, \chi_{[t_i, t_{i+1}]}(t), \quad s(t) = \sum_{i=0}^{\infty} t_i\, \chi_{[t_i, t_{i+1}]}(t), \tag{1}$$

where $\chi_{[X]}(t)$ denotes the characteristic function of the set X. In the distribution sense, a first order derivation of y and a multiplication by a smooth function e result respectively in:

$$\dot{y}(t) = \sum_{i=0}^{\infty} \sigma_i\, \delta(t - t_i), \quad \text{and} \quad e\, \dot{y}(t) = \sum_{i=0}^{\infty} e(t_i)\, \sigma_i\, \delta(t - t_i), \tag{2}$$

where the σ_i $i = 1, 2 \cdots$ denote the unknown jumps of y at t_iA convolution with a regular function α leads to the following on line available data z and v defined respectively by (3) and (4). Note that the third term in both equations shows how measurement derivatives can be avoided.

$$z = \alpha * \dot{y} = \dot{\alpha} * y = \sum_{i=0}^{\infty} \sigma_i \, \alpha(t - t_i), \tag{3}$$

$$v = \alpha * e \, \dot{y} = \dot{\alpha} * e \, y - \alpha * \dot{e} \, y = \sum_{i=0}^{\infty} e(t_i) \, \sigma_i \, \alpha(t - t_i). \tag{4}$$

Next, and owing to the support of a convolution product, a appropriate support for α allows us to derive a linear relation between the formed signals, whose coefficient is nothing but the switching time function s. More precisely,

$$\operatorname{supp} \alpha \subset \left[0, \min_i (t_{i+1} - ti)\right] \Rightarrow v(t) = \left(\sum_{i=0}^{\infty} e(t_i) \, \chi_{[t_i, t_{i+1}]}(t)\right) \, z(t) \tag{5}$$

$$\text{and} \quad e(t) = t \Rightarrow v(t) = s(t) \, z(t). \tag{6}$$

A direct approach for the estimation of $s(t)$ would consist in forming the ratio $s(t) = \frac{v(t)}{z(t)}$. Nevertheless, this simple procedure raises the problem of the indetermination of the ratio due to the compactness of the support of α. To avoid such indetermination and the use of some fixed and a priori threshold, the following asymptotic algorithm is proposed:

$$\dot{\hat{s}} = -z^2 \hat{s} + z \, v. \tag{7}$$

An explicit solution of this differential equation can be easily obtained using standard mathematical tools (i.e. homogeneous solution and variation of constant formula). It is worthy of note that from (7), the dependence of z with respect to the unknown jumps σ_i makes the rate of convergence uncontrollable, but this rate can yet be modified using a tunable gain K in the design of α. Finally, this latter estimate $\hat{s}(t)$ is re-used to adapt the size of a sliding window assigned to the filtering of the measurements y. For this example, a filtered estimation y_e of the noisy signal y is simply obtained using its mean value within a sliding window that does not contain any switch, namely:

$$y_e(t) = \frac{1}{t - \hat{s}(t)} \int_{\hat{s}(t)}^{t} y(\theta) \, d\theta. \tag{8}$$

A simulation result is depicted in Figure 1 and shows combined agility and robustness properties of the estimator with respect to both rapid changes and noisy data. Note that in a noisy context, the non linearities in the estimation algorithm of $\hat{s}(t)$ may introduce bias in the switching time estimates. However, it is worthy of note that these noise effects do not directly affect the estimation y. This is not a surprising result since any lower bound that does not contain switch time is appropriate for the filter.

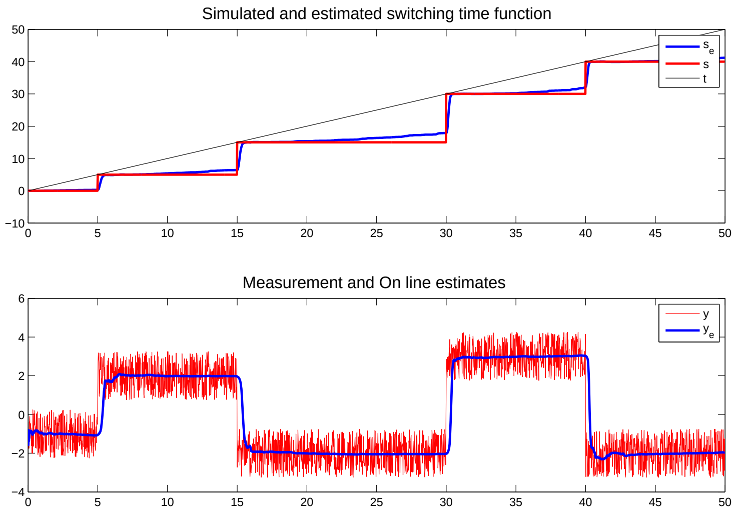

Figure 1: Estimated and simulated switch time function $s(t)$ (top). Noisy and and filtered signal (bottom).

3 Application to the identification of a simple switching process

Many works have been devoted to state estimation of hybrid systems (see e.g. [4]), while parameters estimation, in case of piecewise affine systems, is rather viewed as the problem of reconstructing the piecewise affine map [2, 5]. In all cases, these techniques are based on a probabilistic setting (particle filtering, clustering or classification). Based on the former technique, this section aims at suggesting new tools that can be viewed as a first step toward a more general approach for a class of switched systems. The process under consideration is described by the switched linear system:

$$\dot{y} + a\,y = k\,u, \quad a(t) = \sum_i a_i\,\chi_{[\tau_i,\tau_{i+1}]}(t), \tag{9}$$

where $\tau_0 = 0 < \tau_1 < \tau_2 ...$ denote the switching times. Note that the switching rule (external or state dependent) is not taken into account here. The gain k is assumed to be known and we focus on the on-line estimation of both the switching times and the discontinuous parameter a. Let us first observe that in case of an estimation of the restriction of a to any interval (t_0, t) that does not contain switch times, we recover the linear case already treated [1]. In such case, a multiplication of (9) by t which cancels the possible nonzero initial condition term, followed by an integration by part easily results in the non asymptotic estimation:

$$\hat{a}_{|(t_0,t)} = \frac{k\int_{t_0}^{t}\theta u d\theta - ty + t_0 y(t_0) + \int_{t_0}^{t} y d\theta}{\int_{t_0}^{t}\theta y d\theta}, \quad (t_0,t) \not\ni \tau_i, \quad i = 1,2,\cdots \tag{10}$$

This estimation can be extended to $\mathbb{R}_+ \setminus \{\tau_i\}$ if the lower bound t_0 is replaced by any function $\tilde{s}(t)$ satisfying:

$$s(t) \leq \tilde{s}(t) < t, \quad \text{with} \quad s(t) = \sum_i \tau_i \, \chi_{[\tau_i,\tau_{i+1}]}(t), \tag{11}$$

yielding the estimate of the time varying parameter:

$$\hat{a} = \frac{k\int_{\tilde{s}(t)}^{t}\theta u d\theta - ty + \tilde{s}(t) y(\tilde{s}(t)) + \int_{\tilde{s}(t)}^{t} y d\theta}{\int_{\tilde{s}(t)}^{t}\theta y d\theta} \tag{12}$$

Therefore, the remaining task will consist in the sequel in providing an estimate $\hat{s}$ of the switching function s given in (11). The identification algorithm (12) will be based on this estimation. In order to avoid additional singularities and indetermination at 0, we shall assume that u is continuous and y is positive. We therefore get from (9):

$$a = k\frac{u}{y} - \frac{\dot{y}}{y} = kz - \dot{w}, \quad z := u/y, \quad w := \log(y). \tag{13}$$

As a piece wise constant function, a admits the following singular derivative:

$$\dot{a} = k\dot{z} - \ddot{w} = \sum_i \sigma_i \delta_{\tau_i}, \tag{14}$$

where the σ_i $i = 1, 2\cdots$ denote the unknown jumps of a at τ_i, while σ_0 contains possible additional terms due to the jumps of z and w at 0. The estimation principle for the commutation times is not quite different from that of the previous section. Let us denote:

$$z_0 = z, \; z_i = t^i z, \quad w_0 = w, \; w_i = t^i w. \tag{15}$$

Recalling the properties $\dot{\alpha} * x = \alpha * \dot{x}$, $t^k \delta_\tau = \tau^k \delta_\tau$, some simple manipulations yield from (14):

$$\begin{aligned} \alpha * (t\dot{a}) &= \dot{\alpha} * z_1 - \alpha * z - \ddot{\alpha} * w_1 + 2\dot{\alpha} * w \\ &= \sum_i \sigma_i \tau_i \; (\delta_{\tau_i} * \alpha), \end{aligned} \tag{16}$$

$$\begin{aligned} \alpha * (t^2\dot{a}) &= \dot{\alpha} * z_2 - 2\alpha * z_1 - \ddot{\alpha} * w_2 + 4\dot{\alpha} * w_1 - 2\alpha * w_0 \\ &= \sum_i \sigma_i \tau_i^2 \; (\delta_{\tau_i} * \alpha). \end{aligned} \tag{17}$$

The causality requirement implies here that the selected function α is twice differentiable with support $\subset (0, \infty)$. For such α, and although the jumps σ_i of a are by nature unknown, the left hand side members of these equations are available quantities whose manipulation will allow the switching times estimation. By considering a smooth function α with support within $[0, \Delta]$ with $\Delta = \min_i(t_{i+1} - ti)$, one therefore gets a non asymptotic but local switches identification algorithm:

$$d(t) = s(t)\, n(t). \tag{18}$$

Here again, and although the simple ratio (18) may be sufficient for off-line applications, we can use the on-line algorithm (7) based on the new entries n and d, and in which neither a priori thresholds nor initialization procedures are required.

Figure 2 (top) show the input/output trajectories of the plant subject to an input $u = .3 * (5 + cos(.35 * t). * (2 + sin(.13 * t)))$ an a time varying coefficient a given in (9) with $\{a_i\} = \{0.2, 1, 0.2, 1.2, 0.6\}$ and $\{\tau_i\} = \{0, 3, 5.5, 8, 10\}$ s. Figure 2(center) shows the time history of the switching time estimator $\hat{s}(t)$, as well as the simulated switch function $s(t) = \sum_i \tau_i \, \chi_{[\tau_i, \tau_{i+1}]}(t)$, and the reference time t (dashed). The simulated (dashed) and estimated coefficient a in the noisy context are depicted in Figure 2 (bottom).

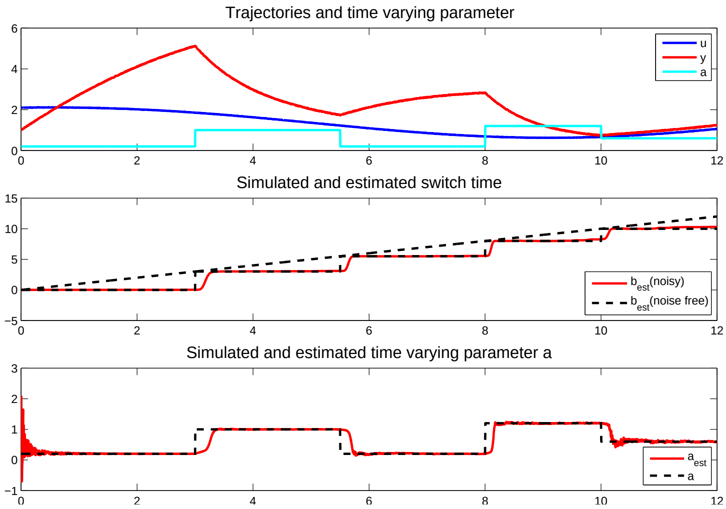

Figure 2:Trajectories(top). Switching time estimator $\hat{b}$(center). Simulated and estimated time varying parameter a (bottom)

4 conclusion

This paper has presented new tools for the change point detection, filtering and estimation of simple hybrid systems. For the hybrid systems, the ability to estimate parameters in a small time interval allows us to consider for further works the simultaneous on-line identification and control issues. Multivariable systems with partial state measurements as well as the extension to discrete-time processes are under active investigations

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