Reception State Estimation of GNSS satellites in urban environment using particle filtering

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***Abstract*— The reception state of a satellite is an unavailable information for Global Navigation Satellite System receivers. His knowledge or estimation can be used to evaluate the pseudorange error. This article deals with the problem using three reception states: direct reception, alternate reception and blocked situation. This parameter, estimated using a Dirichlet distribution, is included in a particle filtering algorithm to improve the GNSS position in urban area. The algorithm takes into account two observation noise models depending on the reception of each satellite. Gaussian probability distribution is used with a direct path whereas a Gaussian mixture model is used in the alternate case.**

Keywords: Satellite navigation, Position estimation, Dirichlet distribution, Particle filtering.

I. Introduction

Main transport applications with Global Navigation Satellite Systems are used in dense urban area. The consequences of environmental obstructions are a lack of the position service and multipath phenomena come out and degrade in particular the accuracy of the position. Solutions are currently used to decrease the influence of the multipath on the accuracy and the availability of GNSS systems. This paper focuses on filtering methods and presents the estimation of the reception state of each satellite. The originality of the approach is to adapt the error model in the filtering process to the reception condition of each satellite signal. A state model of reception of each pseudorange is indicated by a variable. The variable will be helpful to choose a model of all observation errors when multipath are involved in satellite reception process. This parameter takes three values, one for direct path (Line Of Sight: LOS), one for alternate case (Non LOS) and another for no reception state. Because of the geometric position of satellites and the obstacle situation (buildings, trees...) in relation to the mobile position, the value of the parameter changes randomly during the time. For an alternate path reception, the probability distribution of the errors is unspecified because of multipaths. This distribution is then modelled by a Gaussian mixture model. This modelling allows us to model the overall reception process which switches between the observation's models corresponding with the three states of reception (LOS, NLOS and no reception). In this paper, we will recall the typical errors caused by the multipaths in typical urban canyon environments [1]. In the third part, algorithms performed will be described with an emphasis on the multi-sensors (multi-satellites) situation. A particle filtering algorithm adaptated to the new conditions. We have chosen to describe a pseudorange error modelling in the urban area assuming that the error distributions in urban environment can be approximated by a mixture of Gaussian. Statistical studies will be performed in order to provide the best distribution error model for a typical urban canyon over time. Simulation results will illustrate it.

II. Three satellite reception state

In an urban environment, the satellite signal can be received with or without reflexion according either to the position of the receiver, the satellites or the obstacles close to the receiver. These propagation phenomena set up a signal delay on the reception and thus make a geometrical bias on the pseudorange (estimated distance between satellite and receiver). These errors have different characteristics according to the reception state of the satellite considered. We assume two cases of reception .

A. Direct ray

When the satellite signal is received without reflexion or diffraction, the pseudorange is correctly estimated. This is the case of reception in an open sky "free of obstacles" environment for example. The distribution of the pseudorange errors is then Gaussian [2]. The Kalman filter and its alternatives (EKF and UKF) used for linear or nonlinear systems with Gaussian noises are then well adapted for this "Line Of Sight"case of reception.

B. Alternate path

This state occurs when the signal is received after reflexions or diffractions on obstacles. We assume that there is not any direct ray. This will be called the alternate path or the Non Line of Sight (NLOS) state. The distribution of the errors in this case is unknown (unspecified). It is related to external parameters such as obstacles density around the GNSS receiver, their height, distance to the mobile,$\cdots$

C. No reception

The satellite is completely unavailable or his signal is very weak to be received. The satellite may be under the horizon (negative elvation angle) or its signal may be *blocked* by the obstacles although it is above the horizon.

D. Evolution and observation models

The filtering of the noise (or the errors) that we propose is based on the principles of nonlinear filtering. We consider the state sequence $\{\mathbf{x}_t; t \in \mathbb{N}\}$ composed of the 3D dynamic user parameters. The observations $\mathbf{y}_t$ are the pseudoranges given from N expected satellites. $\mathbf{v}_t$ and $\mathbf{w}_t$ are respectively the process and measurement noises. The state of reception of each pseudorange $\mathbf{y}_t$ is modelled by a discrete variable $\mathbf{r}$. The three values represent the LOS, NLOS and blocked cases. Because of the geometric position of satellites and obstacles (buildings,$\cdots$) surrounding the mobile, the value of this variable randomly. In the following, the $\mathbf{r}$ variable have to take tree values 0, 1 and 2 representing the tree hypothetic reception states of each satellite.

$$\mathbf{r}_{t,i} = j \tag{1}$$

where $j \in \{0, 1, 2\}$ and $i = 1, \cdots, N_t$: number of satellites.

$$\mathbf{r}_{t,i} = \begin{cases} 0 & \text{for } \textit{blocked} \text{ situation} \\ 1 & \text{for } \textit{direct} \text{ ray} \\ 2 & \text{for } \textit{alternate} \text{ path} \end{cases}$$

The equations of states and measurements are expressed as follows:

- Equation of evolution process, dynamics parameters of the mobile:

$$\mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{v}_{t-1}) \tag{2}$$

- Observation equation (pseudoranges):

$$\mathbf{y}_t = h(\mathbf{x}_t, \mathbf{w}_t) \tag{3}$$

The observation noises $\mathbf{w}_t$ will be modelled according to each satellite reception state as described above. The pseudoranges $\mathbf{y}_t$ observed are related to the hidden state s $\mathbf{x}_t$ by a nonlinear equation (3). This equation comes to:

$$\mathbf{y}_t \approx \begin{cases} 0 & \text{for } \mathbf{r}_{t,i} = 0 \\ h(\mathbf{x}_t) + \mathcal{N}\big(u_{t,i}, \Sigma^2_{t,i}\big) & \text{for } \mathbf{r}_{t,i} = 1 \\ h(\mathbf{x}_t) + \displaystyle\sum_{j=1}^{J} \pi_j(\mathbf{x}_t)\, \mathcal{N}\big(u_j(\mathbf{x}_t), \Sigma^2_j(\mathbf{x}_t)\big) & \text{for } \mathbf{r}_{t,i} = 2 \end{cases}$$

Where $\mathcal{N}(u(.), \Sigma^2(.))$ is the normal error distribution with mean $u(.)$ and variance $\Sigma^2(.)$ for satellite i.
In case of an alternate path, the pseudorange error has a gaussian mixture distribution with J components. Each component has a mean $u(.)$, a variance $\Sigma^2(.)$ and a parameter $\pi(.)$ weighting its contribution in the mixture. The three variables depend on state vector $\mathbf{x}_t$.

III. Particle filtering algorithm

A. Probability distribution functions

Particle filters are Sequential Monte Carlo Methods which have been employed in many signal processing areas envolving estimation methods [3] [7]. In positioning and navigation, the study in [4] is one of them. We will use a bayesian estimation of the two unknown variables according to the evolution model (2). $\mathbf{v}_t$ is a centered white gaussian noise. The a priori pdf for the evolution process is then written: $p(\mathbf{x}_t/\mathbf{x}_{t-1}, \mathbf{r}_{t-1,i}, \mathbf{y}_{t-1,i})$. At time $t = 0$, the pdf is assumed to be $p_0(\mathbf{x}_0)$.

The *a priori* density of the reception state variables is:

$$\begin{aligned} Pr(\mathbf{r}_{t,i} = 0) &= \alpha_{t,i,0} \\ Pr(\mathbf{r}_{t,i} = 1) &= \alpha_{t,i,1} \\ Pr(\mathbf{r}_{t,i} = 2) &= 1 - (\alpha_{t,i,0} + \alpha_{t,i,1}) \end{aligned}$$

And then,

$$Pr(\mathbf{r}_t/\alpha_t) = \prod_{i=1}^{N} Pr(\mathbf{r}_{t,i}/\alpha_{t,i}) \tag{4}$$

$$\tilde{\mathbf{x}}^{(k)}_t \sim q(\mathbf{x}_t/\mathbf{x}^{(k)}_{t-1}, \tilde{\mathbf{r}}^{(k)}_{t-1,i}, \mathbf{y}_{t-1,i}) \tag{5}$$

$$\tilde{\mathbf{r}}^{(k)}_{t,i} \sim q(\mathbf{r}_{t,i}/\mathbf{x}^{(k)}_{t-1}, \alpha^{(k)}_{t-1,i}, \mathbf{y}_{t-1,i}) \tag{6}$$

$$\tilde{\alpha}^{(k)}_{t,i} \sim q(\alpha_{t,i}/\alpha^{(k)}_{t-1,i}, \tilde{\mathbf{r}}^{(k)}_{t-1,i}, \sigma) \tag{7}$$

Where $q(\cdots/\cdots)$ is the importance distribution. The optimal importance distribution in (5) is $p(\mathbf{x}_t/\mathbf{x}^{(k)}_{t-1}, \tilde{\mathbf{r}}^{(k)}_{t-1,i}, \mathbf{y}_{t-1,i})$. In this work, we make use of an approximation obtained with Extended Kalman filtering. For more information on how to choose the importance distribution, [4] and [5] give more details.

Caron in [6] demonstrates particle filter algorithms for switching observation models specially in an asynchronous case. Appling this theory to our specified reception states, the equation (6) showing the importance distribution can be rewritten as follows:

- for $\mathbf{r}_{t,i} = 0$,

$$q\big(\mathbf{r}_{t,i}/\mathbf{x}^{(k)}_{t-1}, \alpha^{(k)}_{t-1,i}, \mathbf{y}_{t-1,i}\big) = \frac{\alpha^{(k)}_{t-1,i,0}\, p_0(y_{t,i})}{\displaystyle\sum_{j=0}^{2} \alpha^{(k)}_{t-1,i,j}\, p_j(y_{t,i})} \tag{8}$$

- for $\mathbf{r}_{t,i} = 1$,

$$q\big(\mathbf{r}_{t,i}/\mathbf{x}^{(k)}_{t-1}, \alpha^{(k)}_{t-1,i}, \mathbf{y}_{t-1,i}\big) = \frac{\alpha^{(k)}_{t-1,i,1}\, \mathcal{N}\Big(\big(y_{t,i} - h_{t,i,1}(\hat{x}^{(k)}_{t|t-1})\big), {\Sigma^2_{t,i,1}}^{(k)}\Big)}{\displaystyle\sum_{j=0}^{2} \alpha^{(k)}_{t-1,i,j}\, \mathcal{N}\Big(\big(y_{t,i} - h_{t,i,j}(\hat{x}^{(k)}_{t|t-1})\big), {\Sigma^2_{t,i,j}}^{(k)}\Big)} \tag{9}$$

- for $\mathbf{r}_{t,i} = 2$,

$$q\big(\mathbf{r}_{t,i}/\mathbf{x}_{t-1}^{(k)}, \alpha_{t-1,i}^{(k)}, \mathbf{y}_{t-1,i}\big) =$$
$$\frac{\alpha_{t-1,i,2}^{(k)} \sum_{m=1}^{M} \pi(x_{t,i})\, \mathcal{N}\Big(\big(y_{t,i} - h_m(\hat{x}_{t|t-1}^{(k)})\big), \Sigma_m^{2\ (k)}\Big)}{\sum_{j=0}^{2} \alpha_{t-1,i,j}^{(k)} \sum_{m=1}^{M} \pi(x_{t,i})\, \mathcal{N}\Big(\big(y_{t,i} - h_m(\hat{x}_{t|t-1}^{(k)})\big), \Sigma_m^{2\ (k)}\Big)} \tag{10}$$

Then, the probabilities $\alpha_{t,i}$ in (7), the optimal importance distribution $q(\alpha_{t,i}/\alpha_{t-1,i}^{(k)}, \tilde{\mathbf{r}}_{t-1}^{(k)}, \sigma)$ is given by a Dirichlet distribution for each satellite.

$$q(\alpha_{t,i}/\alpha_{t-1,i}^{(k)}, \tilde{\mathbf{r}}_{t-1}^{(k)}, \sigma) = \mathcal{D}\big((\sigma+1)\, {\alpha'}_{t-1,i}^{(k)}\big) \tag{11}$$

where ${\alpha'}_{t-1,i}^{(k)} = \frac{\sigma}{\sigma+1}\alpha_{t-1,i}^{(k)} + \frac{1}{\sigma+1}\delta_{r_{t,i}^{(k)}}(j)$ for $j \in 0,1,2$, and σ is a fixed value related to the evolution of $\alpha_{t,i}$ parameter.

B. Particle filter algorithm with observation switching models

This algorithm is based to the particle filtering algorithms developped in [5].

- Initialization
  For $k = 1, \cdots K$, (K: number of particles)
  - draw $\mathbf{x}_0^{(k)} \sim p_0(\mathbf{x}_0)$
  - draw $\alpha_{0,i}^{(k)} \sim p_0(\alpha_{0,i})$
  - assign initial particle weights: $w_0^k \leftarrow \frac{1}{K}$
- Iteration
  For $t = 1, 2, \cdots$
  - Notice N_t
    For $k = 1, \cdots K$,
  - draw the evolution vector
    $\tilde{\mathbf{x}}_t^{(k)} \sim q(\mathbf{x}_t/\mathbf{x}_{t-1}^{(k)}, \tilde{\mathbf{r}}_{t-1,i}^{(k)}, \mathbf{y}_{t-1,i})$
  - For satellite $i = 1, \cdots N_t$
    - draw the reception state variable
      $\tilde{\mathbf{r}}_{t,i}^{(k)} \sim q(\mathbf{r}_{t,i}/\mathbf{x}_{t-1}^{(k)}, \alpha_{t-1,i}^{(k)}, \mathbf{y}_{t-1,i})$
    - draw the probabilities
      $\tilde{\alpha}_{t,i}^{(k)} \sim q(\alpha_{t,i}/\alpha_{t-1,i}^{(k)}, \tilde{\mathbf{r}}_{t-1,i}^{(k)}, \sigma)$

    $k = 1, \cdots K$, update the weights

$$\begin{aligned} \tilde{w}_t^k \quad \propto \quad & w_{t-1}^k \frac{p(\mathbf{y}_{t,i}/\tilde{\mathbf{x}}_{t-1}^{(k)}, \tilde{\mathbf{r}}_{t-1,i}^{(k)}) p(\tilde{\mathbf{x}}_t^{(k)}/\mathbf{x}_{t-1}^{(k)})}{q(\tilde{\mathbf{x}}_t^{(k)}/\mathbf{x}_{t-1}^{(k)}, \tilde{\mathbf{r}}_{t-1,i}^{(k)}, \mathbf{y}_{t-1,i})} \\ & \times \frac{p(\mathbf{r}_{t,i}^{(k)}/\alpha_{t,i}^{(k)})}{q(\tilde{\mathbf{r}}_{t,i}^{(k)}/\mathbf{x}_{t-1}^{(k)}, \alpha_{t-1,i}^{(k)}, \mathbf{y}_{t-1,i})} \\ & \times \frac{p(\alpha_{t,i}^{(k)}/\alpha_{t-1,i}^{(k)})}{q(\tilde{\alpha}_{t,i}/\alpha_{t-1,i}^{(k)}, \tilde{\mathbf{r}}_{t-1,i}^{(k)}, \sigma)} \end{aligned} \tag{12}$$

  - Normalize the weights to get $\sum_{k=1}^{K} \tilde{w}_t^{(k)} = 1$
- Resampling
  This step is usefull to reduce degeneracy of the particle filter algorithm. Over time, some particles get more and more small weights and their contribution is not significant. To avoid this phenomenon, those particles are aliminated with resampling methods [8], [9] and [10].
  - Evaluate $N_{eff} = \frac{1}{\sum_{k=1}^{K} \big(\tilde{w}_t^{(k)}\big)^2}$
    A threshold N_t is defined and compared to N_{eff}. Resampling will occur when $N_{eff} \leq N_t$ and the value of particle weights come to $w_t^k \leftarrow \frac{1}{K}$
- replace particles: $\tilde{\mathbf{x}}_{t,i}^{(k)} \leftarrow \mathbf{x}_{t,i}^{(k)}$
- replace particles: $\tilde{\alpha}_{t,i}^{(k)} \leftarrow \alpha_{t,i}^{(k)}$.

The pdf used to sample the particles are those expressed in (5), (6) and (7). The estimated reception state is: $\mathbf{r}_{t,i} = \tilde{\mathbf{r}}_{t,i}^{(k)} * \tilde{w}_t^k$. The next paragraph shows the results for illustrating the algorithm performance.

IV. Simulations

Simulations have been performed with a 3D model of the city of Rouen, north of France (Latitude: 49.4497 Longitude: 1.08262). The track simulated corresponds to a public transport bus line that crosses the downtown and is about 8230 meters long. With ERGOSPACE®a 3D ray-tracing tool, we assume that a maximum of 3 reflections can be processed by the receiver. Data were collected every 1 second. The tool provides pseudo-range values for each satellite function of time and satellite states of reception. These true states are compared with the states estimated by the algorithm carried out. The fig.2) and fig.2 draw the error estimation along the bus run. The show that the PF algorithm is more accuralate than the Kalman or the EKF algorithm. We notice that the raised error peaks on the fig.2, especially with KF, are due to turning points on the track. The errors seem to be higher than those on the current GNSS receiver. The reason is that the pseudoranges used in this simulation to estimate the GNSS position are raw data which before smoothing and squaring process.

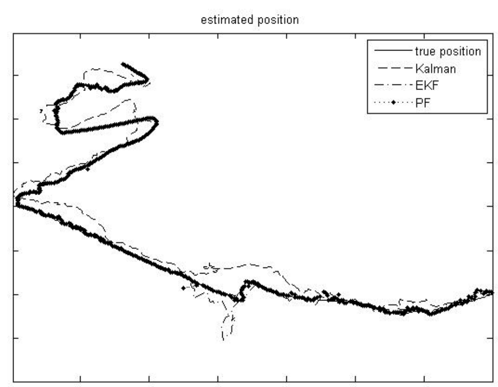

Fig. 1. Estimated positions on the transport bus line

Table I summarizes the best performances of particle filter methods compared to the other filtering techniques. Nevertheless, its main shortcoming remains its processing time.

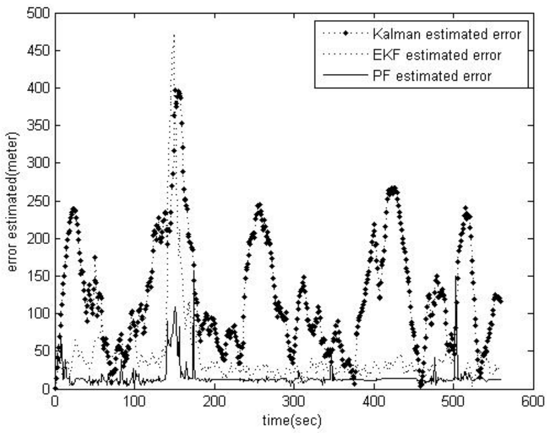

Fig. 2. Comparison of estimated errors

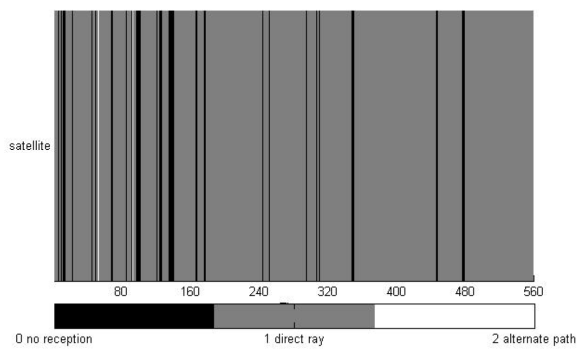

Fig. 3. Estimated reception state

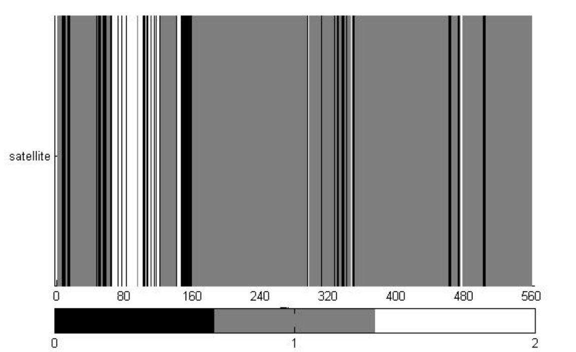

Fig. 4. True reception state

	Processing time(sec)	Error mean(m)	Error Variance
KF	0.81	57.1	5909.9
EKF	0.40	57.7	2165.7
PF	362.6	18.6	972.8

TABLE I

COMPARISON OF THE PF ALGORITHM WITH KALMAN FILTER AND EXTENDED KALMAN FILTER

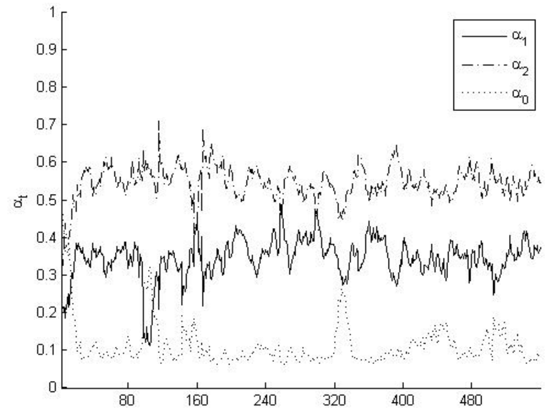

Fig. 5. Evolution of α_t

V. CONCLUSIONS

Multipath in urban environment remains a subject of research especially in position and navigation systems. This study presented a solution for estimating the state reception of GNSS satellite in urban environments. Using Gaussian or Gaussian Mixture techniques, error distributions can be modelled depending on the reception state of satellites. In the particle filtering process, a switching model is developed for the three observation models corresponding to reception states (LOS, NLOS and no reception). Errors in LOS situation are modelled by a simple one Gaussian distribution. But when NLOS case appears, errors have a Gaussian Mixture. With this complete dynamic model of the observed errors, we have introduced a discrete variable determine the reception situation is concerned. This variable has a probability distribution estimated using Dirichlet distribution.

REFERENCES

[1] D-F. Nahimana, J. Marais and E. Duflos, "Pseudo-range error modelling in urban environment" in *Proceedings of the European Navigation Conference*, Geneva, May 29-31, 2007. Switzerland.

[2] M-S. Grewal, L-R. Weill, A-P. Andrews, *Global Positioning Systems, Inertial Navigation, and Integration* Wiley & Sons

[3] A. Doucet, S. Godsill, and C. Andrieu, "On sequential Monte Carlo sampling methods for Bayesian filtering", *Statistical Computing*, vol. 10, no. 3, pp. 197-208, 2000.

[4] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, and P. Nordlund, "Particle filters for positioning, navigation, and tracking", *IEEE Transactions on Signal Processing*, vol. 50, pp. 425-435, Feb. 2002.

[5] A. Doucet, N. de Freitas, and N. Gordon, *Sequential Monte Carlo Methods in practice.* Springer-Verlag, 2001.

[6] F. Caron, M. Davy, E. Duflos, P. Vanheeghe, "Particle Filtering for Multisensor Data Fusion With Switching Observation Models: Application to Land Vehicle Positioning." *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. 55, Issue: 6 Part1, pp. 2703-2719. ISSN: 1053-587X, June 2007

[7] S. Arulampalam, S. Maskell, N. J. Gordon, and T. Clapp, "A Tutorial on Particle Filters for On-line Non-linear/Non-Gaussian Bayesian Tracking", *IEEE Transactions of Signal Processing*, Vol. 50(2), pp. 174-188, February 2002.

[8] J. Carpenter, P. Clifford and P. Fearnhead, "An improved particle filter for non-linear problems". *IEE proceedings on Radar, Sonar and Navigation*, 146, pp. 2-7, February 1999

[9] G. Kitagawa, ”Monte Carlo filter and smoother for non-Gaussian non-linear state space models”. *Journal of Computational and Graphical Statistics*, Vol. 5(1), pp. 1-25, 1996.
[10] J. S. Liu and R. Chen, ”Sequential Monte Carlo methods for dynamic systems”. *Journal of the American Statistical Association*, Vol. 93(443), pp. 1032-1044, Sept. 1998.