A quantitative criterion for selecting the optimal sparse representation of dynamic cardiac data in compresses MRI

Muhammad Usman, Philipp G. Batchelor

A QUANTITATIVE CRITERION FOR SELECTING THE OPTIMAL SPARSE REPRESENTATION OF DYNAMIC CARDIAC DATA IN COMPRESSED MRI

Muhammad Usman and Philipp G. Batchelor

Division of Imaging Sciences, Kings College London, United Kingdom
Email: muhammad.3.usman@kcl.ac.uk, philip.batchelor@kcl.ac.uk

ABSTRACT

One of the important performance factors in compressed sensing (CS) reconstructions is the level of sparsity in sparse representation of the signal. The goal in CS is to find the sparsest representation of the underlying signal or image. However, for compressible or nearly sparse signals such as dynamic cardiac MR data, the quantification of sparsity is quite subjective due to issues such as dropped SNR or low contrast to noise ratio (CNR) in sparse domains such as x-f space or temporal difference domains. Hence, we need a criterion to compare different sparse representations of compressible signals. In this paper, we define a model that can fit the decay of practical compressible signals and as an application; we verify that this model can be used as a basis for selecting the optimal sparse representation of dynamic cardiac MR data.

Index Terms— Compressed Sensing (CS), x-f space, Sparsity, Temporal difference

1. INTRODUCTION

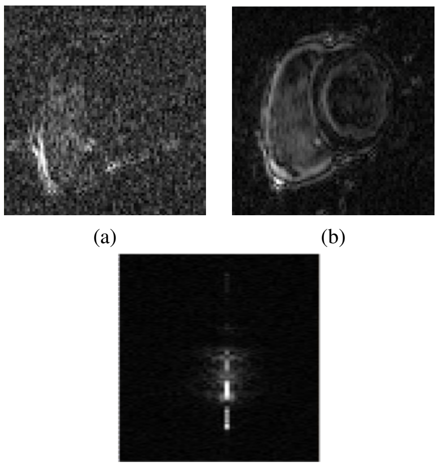
Most of the energy in a compressible signal lies in few significant coefficients in its sparse representation and the amplitudes of the remaining coefficients are closer to 0. If we sort the coefficients of a compressible signal in decreasing amplitudes, the degree of 'sparseness' of the signal can be quantified by the decay rate at which the amplitude of coefficients is decreasing. For some signals, this decay can be modeled by an exponential law($|c_i| = ae^{-bi}; b \geq 0, a > 0$). Alternately, we can use a power law as proposed in [1]. For a signal of length N with sorted coefficient amplitudes $|c_0| \geq |c_1| \geq .... \geq |c_{N-1}|$, the power law approximates the amplitude of the i^{th} coefficient as $|c_i| = C.i^{-r} \ \ r \geq 0$, where r is the decay rate and C is a positive constant. For different sparse representations of a compressible signal, we do curve fitting governed by power law and decaying exponential law to the coefficients sorted in decreasing amplitudes. The sparse representation corresponding to the power or exponential curve having the highest decay factor r is chosen to be the optimal representation.

2. METHOD

Our procedure for finding the optimal representation of dynamic cardiac data is as follows: Two sets of dynamic cardiac data of size ($n_f \times n_p \times n_t$, n_f: number of frequency encoding indices, n_p: number of phase encoding indices, n_t: number of time frames) (224x155x50) and (336x178x48) are acquired with Philips MRI scanner 1.5 T, SSFP sequence, FOV 430x320 mm^2 and TE/TR: 1.46/3 ms. The sparse representations to be tested are two difference frame representations[2] and x-f space [3]. The difference frame representation is obtained by subtracting the test frame (frame to be reconstructed) from the reference frame; where the reference frame can be any time frame (example: first time frame) or the composite frame obtained by taking the temporal average of all time frames. Suppose $I(t_i)$ is the i^{th} time frame in a sequence of n_t dynamic cardiac time frames and I_{ref} is the reference frame, then the sparse representation obtained by using the difference operator is given as $I_{diff}(t_i) = I(t_i) - I_{ref}, 1 \leq i \leq n_t$ where $I_{ref} = \frac{1}{n_t}\sum_{k=1}^{n_t} I(t_k)$, if the reference frame is composite frame and $I_{ref} = I(t_1)$, if the reference frame is first time frame. The x-f space representation is obtained by taking the Fourier transform of dynamic cardiac data along the temporal dimension. For each sparse representation, we fit the exponential and power curves to transform coefficients sorted in decreasing amplitudes. For difference frame representation, the mean decay factor is obtained by taking the average of decay factors estimated for all n_t time frames.For x-f space representation, the decay factors are estimated for x-f space corresponding to all n_f frequency encoding indices and averaged to get the mean decay factor. To test whether the optimal representation selected on the basis of decay factor values also gives better results practically, we perform the CS reconstruction on randomly under-sampled data with different under-sampling factors. Our CS reconstructions are based on minimizing the l_1 norm as proposed in [4].

3. RESULTS

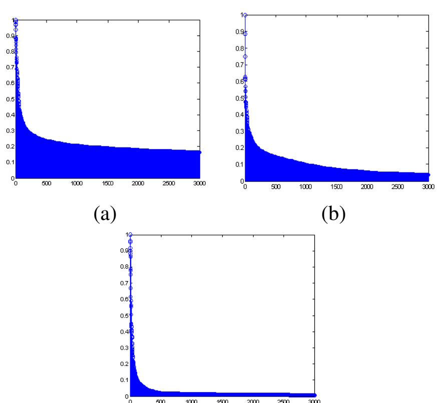
The three sparse representations of fully-sampled dynamic cardiac data i.e. difference frame with first frame as reference, difference frame with composite frame as reference and x-f space centered around $f = 0$ (horizontal axis: temporal frequency f, vertical axis: spatial location x) are shown in Fig.1(a), (b) and (c) respectively. In Fig.1(a) and (b), the test image used is the second time frame. For each sparse



(a) (b) (c)

Fig. 1: Three sparse representations of actual (fully-sampled) dynamic cardiac data

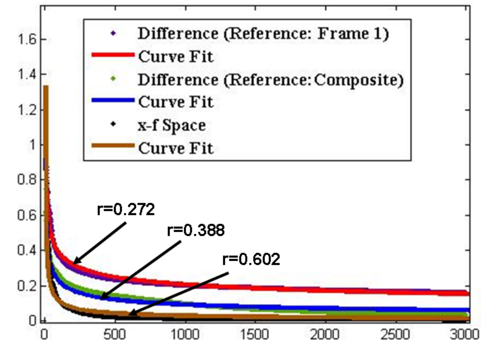
representation, we order the coefficients in decreasing magnitudes. The normalized ordered coefficient profiles for the three sparse representations are shown in Fig.2(a), b) and (c) respectively. Governed by the power law and the decaying ex-



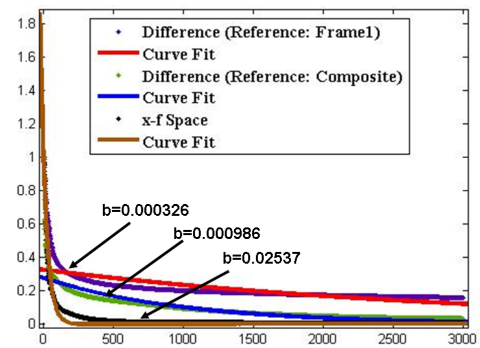
(a) (b) (c)

Fig. 2: Normalized ordered coefficient profile for sparse representations of dynamic cardiac data

ponential law , the curves fitted to the sorted coefficients for each sparse representation are shown in Fig.3(a) and Fig.3(b) respectively with corresponding estimated decay factor values. The goodness of fit parameters for curve fitting by power law and exponential law are shown in tables in Fig.4(a) and Fig.4(b) respectively. The parameter values corresponding to power-law curve fit indicate that the power law is a quite reasonable fit for dynamic cardiac data sparse representations. The overall mean decay factors estimated from the power



(a) Power-law curve fit with decay factor r



(b) Exponential law curve fit with decay factor b

Fig. 3: Power law and exponential law curve fitting to sorted coefficients

decay curve fitting for the three sparse representations are 0.337, 0.396 and 0.598 respectively. The CS reconstructions from under-sampled data with under-sampling factor of 3 are shown in Fig.5(a),(b) and (c) for each (fully-sampled) sparse representation in Fig.1 (a), (b) and (c) respectively. Of all the three sparse representations shown in Fig. 1(a), (b) and (c), the x-f space has the highest mean decay factor (0.598) and results in nearly exact CS reconstruction [Fig.5(c)]. The difference frame obtained by using first time frame as reference and second time frame as test frame results in much sparser signal. However, due to low CNR in some regions in difference frame representation, the CS reconstruction results in Fig.5(a) are worse than the results for other sparse representations. This effect can be seen in lower mean decay factor value for this sparse representation.

4. CONCLUSION AND FUTURE WORKS

The power law is a quite good fit for coefficients in sparse representations of dynamic cardiac data and can be used to select the optimal sparse representation of dynamic cardiac

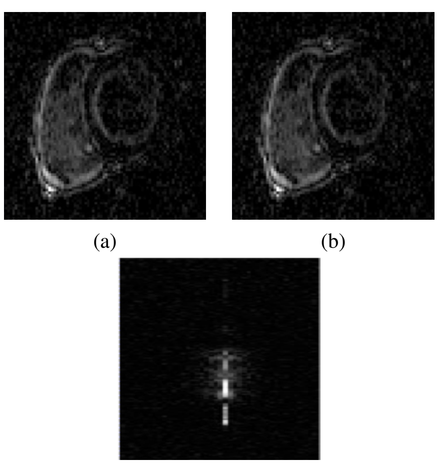
Sparse Representation	Difference (Reference: 1st frame)	Difference (Reference: Composite)	x-f Space
SSE	0.7498	1.3	2.511
Adjusted R-square	0.9605	0.9186	0.8302
RMS Error	0.0158	0.02082	0.02894

(a) Power-law curve fit

Sparse Representation	Difference (Reference: 1st frame)	Difference (Reference: Composite)	x-f Space
SSE	10.18	3.3	1.362
Adjusted R-square	0.46	0.7934	0.9079
RMS Error	0.061	0.03319	0.0232

(b) Exponential law curve fit

Fig. 4: Goodness of fit parameters for power law and exponential law curve fit



(a) (b)

(c)

Fig. 5: CS reconstruction for three sparse representations with under-sampling factor of 3

data. For exactly sparse signal of length N, Candès has shown a linear relationship between the required number of measurements M for exact reconstruction and the signal sparsity S given by $M \geq CS\ log\ N$, where C is a constant [5]. The potential work includes developing a similar expression for compressible signals as a function of decay factor.

5. REFERENCES

[1] P. Jost et al., EUSIPCO 2008, LTS-CONF-2008-149 [2] J. Ji et al., ISBI 2008, pp 1613-1616 [3] S. Kozerke et al., MRM 2004; 52:19-26 [4] E. van den Berg et al. , TR-2008-01, Jan 2008 [5] Candès et al., IEEE Trans. On Information Theory, pp. 489-509, 2006.