# Reconstruction of Undersampled Cardiac Cine MRI data Using Compressive Sensing Principles

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***Abstract*—Reduction of MRI data acquisition time is an important goal in the MRI field. Undersampling k-t space is a solution to reduce acquisition time. MR images may have sparse or compressible presentations in appropriate transform domains, such as wavelets. According to the Compressive Sensing (CS) theory, they can be recovered from randomly undersampled k-t space data that guarantees incoherency between sparsifying transform and sampling operator. However, pure random k-space sampling can be more time-consuming than full k-space sampling because of MRI principles. In this paper, we propose a new method based on hidden Markov models (HMM) to undersample k-space along the phase-encoding direction. To this end, we cluster extracted features of each k-space line by fuzzy c-means (FCM) method and consider the resulting class labels as the states of a Markov chain. Then we train a HMM and find the related transition matrix to each k-space line. We choose the lines having more non-diagonal transition matrices to sample data along them. We reconstruct the image by minimizing the L1 norm subject to data consistency using conjugate-gradient method and use simulation to set the parameters of the proposed method (e.g., iteration number). We apply our method to reconstruct undersampled Cardiac Cine MRI data with and without sparsifying transform, successfully. The use of fuzzy clustering as an intermediate tool to study complicated phenomena by HMM, applicability to non-dynamic MRI data and simplicity can be accounted as the specifications of the proposed method.**

## I. Introduction

Some time-consuming applications of MRI such as: fMRI, Cardiac and specroscopic imaging developed in the recent years. Thus it seems necessary to find some solutions to increase MRI speed without losing the quality. On the other hand, the hardware solution to reduce MRI data acquisition time, using more powerful gradient amplifiers, is limited by technical and biological considerations. Therefore, the researchers have searched some methods to use intrinsic redundancy and correlation of MRI data in k-t space and reconstruct images without complete sampling of k-t space. We can divide these approaches into the following three categories [1]:
1. Exploiting correlations in k-space, such as parallel imaging [2], [3];
2. Exploiting temporal correlations, such as UNFOLD [4]; and
3. Exploiting correlations in both k-space and time domain, such as k-t BLAST [1].

We should emphasize that in the mentioned methods, the images signal to noise ratio (SNR) decrease by the increase in speed. In this paper, we use CS to decrease the mentioned drawbacks.

In the next Section, we describe the used concepts in this article such as Compressive Sensing (CS) [5], [6] and hidden Markov models (HMMs). In the third Section, we attempt to detail the proposed method. Throughout the fuzzy clustering, will study the k-space lines by use of HMMs and then select the ones having more variations in time. Afterwards, we reconstruct the desired images by CS concepts. In the fourth Section, we present some results of successful application of the proposed method to reconstruct undersampled Cardiac Cine MRI data. Finally, we present the strengths and drawbacks of the proposed method and give a suggestion to improve it.

## II. Theory

In this Section, we introduce the theory of compressive sensing [7] then review two major attempts to speed up MRI using CS principles. Finally, we learn about training HMMs based on some observations.

### A. Compressive Sensing

Let $x \in \mathrm{R}^N$ and the matrix $\Psi = [\psi_1, \psi_2, ..., \psi_N]$ be a basis for $\mathrm{R}^N$. We say that $x$ is $K-sparese$ if we have:

$$x = \sum_{i=1}^{K} \theta_i \Psi_i \tag{1}$$

and $K \ll N^2$. Consider also an $M \times N$ $measurement$ $matrix$ $\Phi, M \ll N$ where the rows of $\Phi$ are incoherent with the columns of $\Psi$. Incoherenency between rows of measurement matrix and basis vectors means that we need all of the vectors in second set to expand each of the vectors in first set and vice versa. Compressive Sensing theory, states that only $cK$ incoherent $measurements : y = \Phi x$ are required to reconstruct signal $x$, with high probability. It is important to know that the $c$ ratio is always bigger than 1 and in the most of scientific applications is a number between 3 and 4. Incoherency is an essential condition for this theory to be successful. For example, if the rows of the measurement matrix be i.i.d. Gaussians random vectors, such a matrix is incoherent with any other fixed matrix, with high probability. The problem of using CS in MRI is exactly lying here, because random sampling of k-t space can be more time-consuming than complete sampling. Now, we want to review two proposed methods dealing with CS in MRI.

### B. Compressive Sensing in MRI

In [8], some of the k-t space lines along the phase-encoding have been skipped to decrease data acquisition time. To this end, the authors have been suggested using constrained random sampling pattern, as follows:
Sampling points on a regular undersampling pattern are randomly shifted along the phase-encoding direction by 1, 0 or +1 with same probability.

Their proposed method has been yielded to improved results in comparison with pure random selecting of some k-t space lines along the phase-encoding direction and particularly proposed for dynamic MRI. As we know, dynamic MR images are naturally sparse in temporal frequency domain. Thus we don't need to any sparsifying transform if we reconstruct desired images in temporal frequency domain. Need to high SNR data and time consuming calculations are the drawbacks of their method.

In [9], at first, a relation has been introduced to measure the coherency between sampling pattern and the basis of sparsifying transform. Then the incoherency of the common sampling patterns has been evaluated by this relation. The author has shown that the pure random sampling has the best result and random selecting of k-space lines has the worst record among the others. After that a general relation, has been proposed for non-linear reconstruction of undersampled MRI data based on CS concepts. We describe this relation further, in the third Section of the paper.

### C. Hidden Markov Models

Markov chains [10] are mathematical descriptions of Markov models with a discrete set of states. Markov chains are characterized by:
- A set of states $\{1, 2, ..., M\}$,
- An $M \times M$ transition matrix (T) whose (i,j) entry is the probability of a transition from state i to state j,
- A set of possible outputs or emissions, $\{s_1, s_2, ..., s_N\}$,
- An $M \times N$ emission matrix (E) whose (i,k) entry gives the probability of emitting symbol $s_k$ given that the model is in state i.

A hidden Markov model is one in which we observe a sequence of emissions, but do not know the sequence of states the model went through to generate the emissions. In this paper, we use the Baum-Welch algorithm [10] to estimate the HMM parameters (T and E) for time variations of k-space lines, after fuzzy clustering.

## III. Proposed Method

It was mentioned that speeding up of MRI by undersampling k-t space contributes to decrease in SNR of the reconstructed images. On the other hand, the reconstruction step in CS is robust to noise. Furthermore, MR images may have sparse or compressible presentations in appropriate transform domains, such as wavelets. Also, use of CS in MRI does not need any specific hardware changes. Therefore, in this research we focus on increasing MRI speed by the idea of CS. We limit the proposed method, to Cartesian sampling pattern in order to simplicity and applicability. Different steps of the proposed method, are described below:
1. Preprocessing: Detaching of the organ under study from available images and making k-t space corresponding to each array coil channels.
2. Feature extraction of each k-t space lines along the phase-encoding direction. These features are: average, standard deviation, median and maximum.
3. Clustering the time samples of each k-space lines (using the features described above and for each channel of array coil) by fuzzy c-means algorithm. The number of clusters is assumed to be a quarter of the number of time samples.
4. Training hidden Markov model for each k-space lines using Baum-Welch algorithm. The results of above clustering (for each channel) are considered as training sequences for the HMM describing time behavior of each k-space line.
5. Using the transition matrix came from above, to decide about selecting or rejecting a k-space line to sample. We assume that the lines which have more non-diagonal transition matrix have more information in them. Thus they are more adequate to be sampled.
6. According to [9], image reconstruction by the following constrained L1 norm minimization:

$$\left\{ \begin{array}{c} min.\|\Psi m\|_1 + \lambda TV(m) \\ s.t.\|F_u m - y\|_2 \leq \epsilon \end{array} \right. \tag{2}$$

It is done by means of conjugate gradient and use of available codes at www.stanford.edu/∼mlustig. It should be noted that in (2), $m$ denotes the desired image, $\Psi$ is the sparsifying transform, $F_u$ is the undersampled Fourier operator, y is the collected samples of k-space and $\epsilon$ shows the strength of noise. A total variation term has been added to the object of minimization to yield improved result. The first row in (2) shows the sparsity of the desired image and second row guarantees the consistency of collected data and final image. In the following Section, we present the results of the above method.

## IV. Results

The mentioned approach was applied to two Cardiac Cine MRI datasets, from an 8-channel coil. The results of fuzzy clustering and training of HMMs for 4 k-space lines are presented in Fig. 1 and Fig. 2 respectively. It must be noted that the k-space lines which have been finally selected to sample, have more non-diagonal transition matrices, as shown in Fig. 2. The original (desired) Cardiac image, resulted by zero padding and inverse Fourier transform, reconstructed image by randomly selected k-space lines and reconstructed image by proposed sampling mask, are shown in Fig. 3. The image related to random mask has poor quality in comparison with the result of proposed method. The reduction factor for these images is 2 and we have not used any sparsifying transform.

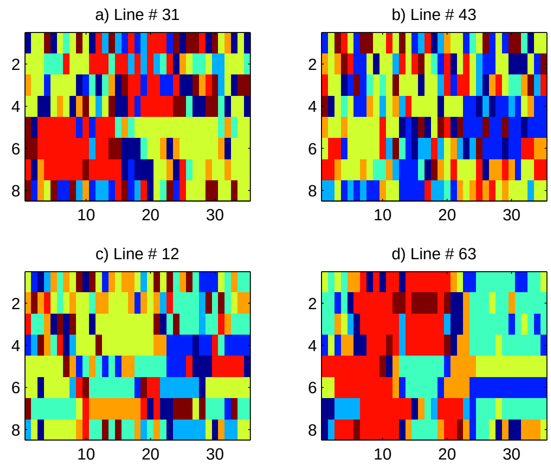


Fig. 1. a-d) Clustering results for four different k-space lines. The vertical axis refers to 8 coil channels and the horizontal one shows 35 time samples.

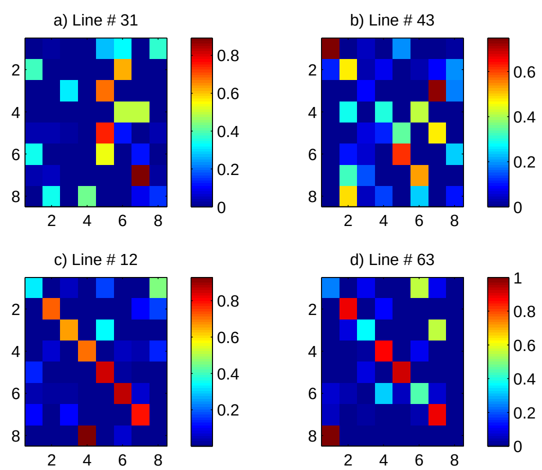


Fig. 2. The resulting transition matrices for different k-space lines, by Baum-Welch algorithm. a) and b) refer to the lines that were selected to sample and c) and d) refer to the lines that were not selected.

For higher undersampling (speed up) factors, we have to use a sparsifying transform.

Fig. 4 shows the original image and some undersampled Cardiac images. They have been reconstructed by wavelet transform (Daubechies-4) as the sparsifying transform and sampling pattern according to the proposed method. We have added selecting the central k-space lines along the phase encoding direction to our method, as a constraint.

## V. Discussion and Conclusion

In this article, we presented an approach to increase Cardiac Cine MRI speed by undersampling k-t space. To this end, we studied time behavior of k-space lines along phase-encoding direction by HMM and selected the ones that have large time variations. We used CS principles to reconstruct desired images. We successfully applied our method to reconstruct half sampled Cardiac Cine MRI, without any sparsifying transform. It is possible to earn higher speeding up factors by using sparsifying transform and adding the central k-space lines to the selected lines by our method.

The use of fuzzy clustering as an intermediate tool to study complicated phenomena by HMM, applicability to non-dynamic MRI data and simplicity can be accounted as the specifications of our method. It must be noted that we assumed to have access to all of the k-t space data to develop our algorithm. However, in practice we want to know the most important part of k-t space data to reconstruct acceptable images. On the other hand, we can use the resulting pattern for an individual or a group for the others by assuming similarity of MRI data of an organ in different individuals. Fig. 5 shows

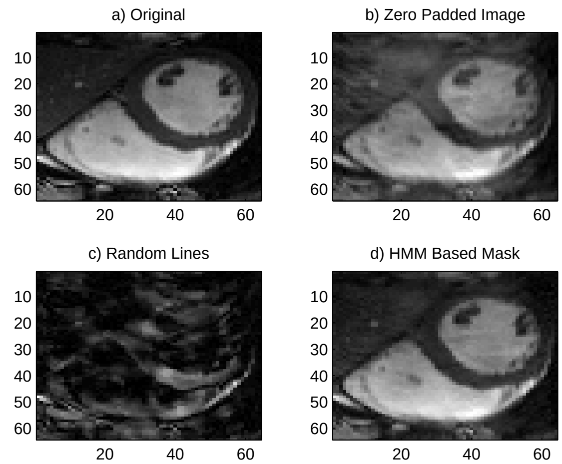


Fig. 3. a) desired image, b) image resulted by zero-padding and inverse Fourier transform, c) image resulted by random selection of some k-space lines, d) image resulted by selecting some k-space lines according to the proposed method.

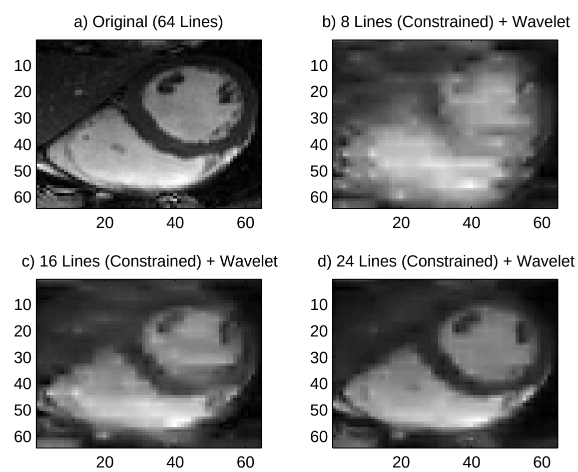


Fig. 4. a) desired image, b), c) and d) resulting images for different reduction factors, using wavelet transform as the sparsifying transform and selecting some k-space lines according the proposed method (constrained).

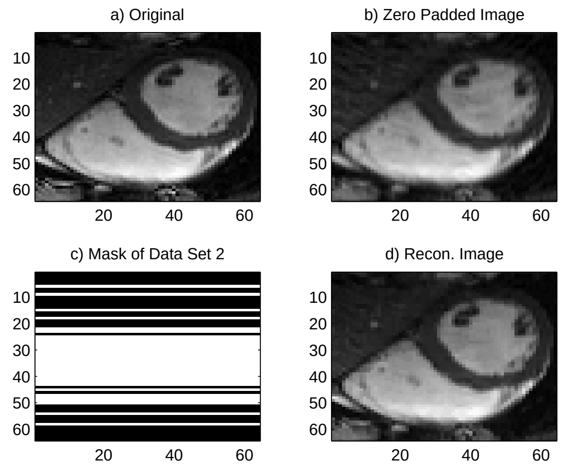


Fig. 5. a) desired image, b) image resulted by zero-padding and inverse Fourier transform, c) extracted mask of dataset 2 by the proposed method, d) reconstructed image by selection of some k-space lines according to the shown mask.

the reconstructed image using the k-space of dataset 1 and sampling pattern extracted from dataset 2.

In the proposed method, we did not use the correlation between two adjacent k-space lines to decide about selecting them or not. However, it may possible to use Distributed Compressive Sensing (DCS) [11] to enter this correlation and get improved results.

## ACKNOWLEDGMENT

The authors would like to thank Dr. Peter Kellman for providing the Cine Cardiac MRI data.

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