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# Sparsity hypotheses for robust estimation of the noise standard deviation in various signal processing applications.

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***Abstract*—This paper concerns the problem of estimating the noise standard deviation in different signal processing applications. The presented estimator derives from recent results in robust statistics based on sparsity hypotheses. More specifically, these theoretical results make the link between a standard problem in robust statistics (the estimation of the noise standard deviation in presence of outliers) and sparsity hypotheses. The estimator derived from these theoretical results can be applied to different signal processing applications where estimation of the noise standard deviation is crucial. In the present paper, we address speech denoising and Orthogonal Frequency Division Multiple Access (OFDMA). A relevant application should also be Communication Electronic Support (CES). For such applications, the algorithm proposed is a relevant alternative to the median absolute deviation (MAD) estimator.**

## I. INTRODUCTION

Sparsity is an important notion because there exist transforms that are sparse in the sense that they make it possible to represent a large class of signals by coefficients that are mostly small except a few ones whose amplitudes are large. In [1] and its continuation [2], the authors have then suggested a formal notion of sparsity, mainly motivated by the necessity to decide whether a wavelet coefficient is significant or not for non-parametric estimation of the signal.

In many signal processing applications where noise is white and Gaussian, the noise standard deviation is often unknown and must be estimated to process the observations. Since very little is generally known about the signals, it is often relevant to estimate the noise standard deviation via a robust estimator such as the median absolute deviation (MAD) estimator [3]. This estimator is very natural when the wavelet transform is used because, for smooth signals, the signal wavelet coefficients are very few among the detail coefficients at the first decomposition level.

The present paper, as a continuation of [2, Section 4], addresses the problem of estimating the noise standard deviation in applications where the number or the amplitudes of the outliers are too large for the MAD estimator to perform well. These applications are speech denoising and Orthogonal Frequency Division Multiple Access (OFDMA). Other application such as Communication Electronic Support (CES) can be expected in forthcoming work. In these different applications, the same algorithm can be used to estimate the noise standard deviation. This algorithm derives from a theoretical result involving sparse hypotheses of the same type as those given in [2].

## II. THEORETICAL BACKGROUND

The estimator presented in this section for the noise standard deviation derives from [4, Theorem 1], which was originally motivated by practical issues in radar and speech processing. This theoretical background relies on a crucial sparsity assumption.

The random vectors and variables introduced below are assumed to be defined on the same probability space $(\Omega, \mathcal{B}, \mathrm{P})$. As usual, $\mathbf{N}$ stands for the set of all natural numbers and we write (a-s) for “almost surely”. Let $Y = (Y_k)_{k \in \mathbf{N}}$ be a sequence of independent observations that are $d$-dimensional random vectors such that $Y_k = \varepsilon_k S_k + X_k$ for $k \in \mathbf{N}$ where: $\epsilon_k$ is a random variable valued in $\{0, 1\}$ that models the presence or the absence of $S_k$; $X_k$ stands for some centred $d$-dimensional Gaussian distributed real random vector with covariance matrix $\sigma_0^2 \mathbf{I}_d$ with $\sigma_0 \neq 0$, $\mathbf{I}_d$ being the $d \times d$ identity matrix. We assume that $\varepsilon_k$, $S_k$ and $X_k$ are independent for every natural number $k$. Basically, the sequence $Y$ models a sequence of independent observations where random signals are either present or absent in independent and additive white Gaussian noise modelled by the sequence $X = (X_k)_{k \in \mathbf{N}}$. Given any natural number $k$, $S_k$ stands for some possible random signal with unknown distribution and $\varepsilon_k$ models the possible occurrence of $S_k$. The two possible values for $\varepsilon_k$ form a hypothesis pair about the distribution of $Y_k$: the null hypothesis is that $\varepsilon_k = 0$ and the alternative one is $\varepsilon_k = 1$.

For any $\tau \in [0, \infty)$ and any $q \in \mathbf{N}$, consider the sample moments

$$\mathcal{M}_\nu(q, \tau) = \frac{1}{q} \sum_{k=1}^{q} \|Y_k\|^\nu \, I(\|Y_k\| \leq \sigma_0 \tau), \nu = 0, 1. \quad (1)$$

where $\| \cdot \|$ stands for the standard Euclidean norm in $\mathbf{R}^d$, $I(A)$ denotes the indicator function of any given event $A$: $I(A)$ assigns 1 to any element of $A$ and 0 to any element of the complementary set $\Omega \setminus A$ of $A$. Assume, for a while, that the random vectors $S_k$ and thus, the random vectors $Y_k$ are i.i.d. According to the strong law of large numbers, $\mathcal{M}_\nu(q,\tau)$ tends to $\mathbb{E}\left[\|Y_k\|^\nu I(\|Y_k\| \leq \sigma_0\tau)\right]$ (a-s) when $q$ tends to $\infty$ so that

$$\frac{\mathcal{M}_1(q,\tau)}{\mathcal{M}_0(q,\tau)} \approx \frac{\mathbb{E}\left[\|Y_k\| I(\|Y_k\| \leq \sigma_0\tau)\right]}{\mathbb{E}\left[I(\|Y_k\| \leq \sigma_0\tau)\right]}, \tag{2}$$

in the sense given by the strong law of large numbers. In the expressions above, $\sigma_0\tau$ plays the role of a threshold height so that we can consider a thresholding test to decide on the value of $\varepsilon_k$ given $Y_k$. This test decides that $\varepsilon_k = 1$ if the norm $\|Y_k\|$ is above $\sigma_0\tau$ and $0$, otherwise. If the norm of $S_k$ is significantly large in comparison to the noise standard deviation, we can expect the existence of a threshold height $\sigma_0\tau$ such that the probability of error of the thresholding test is small. For this threshold, when $\|Y_k\| > \sigma_0\tau$ (resp. $\|Y_k\| \leqslant \sigma_0\tau$), the probability that the signal $S_k$ is present (resp. absent) should be large. Therefore, when the norms of the signals are large enough, it seems reasonable to make the following approximation

$$\begin{aligned}\mathbb{E}\left[\|Y_k\|^\nu I(\|Y_k\| \leq \sigma_0\tau)\right] \approx& \mathbb{E}\left[\|X_k\|^\nu I(\|X_k\| \leq \sigma_0\tau)\right]\\ &\times \mathrm{P}[\varepsilon_k = 0],\end{aligned} \tag{3}$$

for $\nu = 0, 1$. By combining the latter approximation with Eq. (2), it follows that $\mathcal{M}_1(q,\tau)/\mathcal{M}_0(q,\tau)$ should tend to

$$\mathbb{E}\left[\|X_k\| I(\|X_k\| \leq \sigma_0\tau)\right] / \mathbb{E}\left[I(\|X_k\| \leq \sigma_0\tau)\right] = \sigma_0\Phi(\tau) \tag{4}$$

when $q$ and the norms of the signals are large enough. In Eq. (4), $\Phi(\tau) = \Upsilon_1(\tau)/\Upsilon_0(\tau)$ for any $\tau \in [0,\infty)$ with

$$\Upsilon_\alpha(\tau) = \int_0^\tau t^{\alpha+N-1} e^{-t^2/2} \mathrm{d}t$$

for any given $\alpha \in [0,\infty)$. The difficulty in combining Eqs. (2) and (3) relies on the fact that Eq. (2) involves an almost everywhere convergence when $q$ tends to infinity whereas Eq. (3) relates to a convergence when the norms of the signals are large enough. We define the minimum amplitude of the sequence $S = (S_k)_{k\in\mathbf{N}}$ as the supremum $\varrho$ of the set of those $\rho \in [0,\infty]$ such that, for every natural number $k$, $\|S_k\|$ is larger than or equal to $\rho$ (a-s):

$$\varrho = \sup\left\{\rho \in [0,\infty] : \forall k \in \mathbf{N}, \|S_k\| \geq \rho \text{ (a-s)}\right\}. \tag{5}$$

We now assume that the signals are such that $\sup_{k\in\mathbf{N}} \mathbb{E}[\|S_k\|^2]$ is finite so that the signals have finite energy and the energies of these signals are upper-bounded. We also assume that $\mathrm{P}[\varepsilon_k = 1] \leqslant 1/2$ for every $k \in \mathbf{N}$ so that the signals are less absent than present. This second assumption is a sparsity assumption in a wide sense in that it does not impose that the probability of occurrence is small. According to [4, Theorem 1], $\sigma_0$ is the unique positive real number $\sigma$ such that, for every $\beta_0 \in (0,1]$,

$$\lim_{\varrho\to\infty} \left\| \limsup_q \Delta_q(\sigma, \beta\xi(\varrho/\sigma)) \right\|_\infty = 0 \tag{6}$$

uniformly in $\beta \in [\beta_0, 1]$ where, for any pair $(\sigma,\tau)$ of positive real numbers,

$$\Delta_q(\sigma,\tau) = \left| \frac{\displaystyle\sum_{k=1}^q \|Y_k\| I(\|Y_k\| \leq \sigma\tau)}{\displaystyle\sum_{k=1}^q I(\|Y_k\| \leq \sigma\tau)} - \sigma\Phi(\tau) \right|,$$

and $\xi(\rho)$ stands for the unique positive solution for $x$ in the equation ${}_0\mathrm{F}_1(d/2; \rho^2x^2/4) = e^{\rho^2/2}$, where ${}_0\mathrm{F}_1$ is the generalised hypergeometric function [5, p. 275]. It is in the sense of the convergence criterion of Eq. (6) when $\beta = 1$ that $\mathcal{M}_1(q, \xi(\varrho/\sigma_0))/\mathcal{M}_0(q, \xi(\varrho/\sigma_0))$ can be said to tend to $\sigma_0\Phi(\xi(\varrho/\sigma_0))$ when $q$ and $\varrho$ are large enough.

Given $q$ observations $Y_1, \ldots, Y_q$ whose minimum amplitude is $\varrho$, if we choose $L \in \mathbf{N}$ and set $\beta_\ell = \ell/L$ for every $\ell \in \{1,\ldots,L\}$, the foregoing suggests estimating $\sigma_0$ by a possibly local minimum $\overline{\sigma_0}$ of $\sup_{\ell\in\{1,\ldots,L\}} \Delta_q(\sigma, \beta\xi(\varrho/\sigma))$ when $\sigma$ ranges over a suitable search interval, which it is not necessary to specify at this stage. Details about the derivation of this discrete cost are given in [4]. Following the terminology proposed in [4], the estimate $\overline{\sigma_0}$ is called an Essential Supremum Estimate (ESE) of the noise standard deviation. This name follows from the fact that the essential supremum norm plays an important role in proposition Eq. (6) and its generalisation stated in [4, Theorem 1]. Note that any minimisation routine for scalar bounded non-linear functions is suitable. We use the `MATLAB` routine `fminbnd.m` based on parabolic interpolation (see [6]) for this minimisation.

### *A. Complex and Modified Complex ESE*

We now focus on the case of practical relevance where the observation and, thus, the signal and noise, are two-dimensional random vectors or, equivalently, complex random variables. As above, these observations are assumed to be independent, without assuming that these observations are identically distributed. Such observations can be the complex values provided by the standard $I$ and $Q$ decomposition encountered in most receivers in radar, sonar and telecommunication systems; below, these complex observations are those provided by Discrete Fourier Transforms (DFTs). In the two-dimensional case, that is, when $d = 2$, the expression of $\xi$ simplifies: according to [7, Eq. 9.6.47, p. 377], for every $x \in [0,\infty)$, $\mathrm{I}_0(x) = {}_0\mathrm{F}_1(1; x^2/4)$ where $\mathrm{I}_0$ is the zeroth-order modified Bessel function of the first kind; therefore, $\xi(\rho) = \mathrm{I}_0^{-1}(e^{\rho^2/2})/\rho$ for any $\rho \in [0,\infty)$. Note also that, in the two-dimensional case,

$$\Phi(\tau) = \int_0^\tau t^2 \exp(-t^2/2)\mathrm{d}t / (1 - \exp(-\tau^2/2))$$

for $\tau \in [0,\infty)$. Experimental results given in [4, Section 4] suggest that the asymptotic conditions in Eq. (6) can be relaxed in practice. As a consequence, $\varrho$ is set to $0$ in [8] so that $\xi(0) = \sqrt{d}$ and an estimate of $\sigma_0$ is computed as a possibly local minimum $\widetilde{\sigma_0}$ of

$$\sup_{\ell \in \{1,\dots,L\}} \left\{ \left| \frac{\sum_{k=1}^{q} \|Y_k\| \, I(\|Y_k\| \leq \beta_\ell \sigma \sqrt{d})}{\sum_{k=1}^{q} I(\|Y_k\| \leq \beta_\ell \sigma \sqrt{d})} - \sigma \Phi(\beta_\ell \sqrt{d}) \right| \right\}. \tag{7}$$

The estimate $\widetilde{\sigma_0}$ is called the Complex Essential Supremum Estimate (C-ESE).

Although, in contrast to Eq. (6), the C-ESE is computed under non-asymptotic conditions, simulations presented in [8] for two-dimensional random signals with probabilities of presence less than or equal to one half and uniformly distributed on circles centred at the origin with known radii show that $\widetilde{\sigma_0}$ is a reasonably good estimate of $\sigma_0$. However, the heuristic approach of [8] suggests a better estimate for $\sigma_0$. This new estimate, called the Modified C-ESE (MC-ESE) and denoted by $\widehat{\sigma_0}$, is computed on the basis of $\widetilde{\sigma_0}$ by setting

$$\widehat{\sigma_0} = \lambda \sqrt{\frac{\sum_{k=1}^{q} \|Y_k\|^2 \, I(\|Y_k\| \leq \widetilde{\sigma_0} \sqrt{N})}{\sum_{k=1}^{q} I(\|Y_k\| \leq \widetilde{\sigma_0} \sqrt{N})}} \tag{8}$$

where $\lambda$ is some constant to choose. According to the rationale proposed in [8], this constant should be close to 1; experimental results rather suggest to set $\lambda = \sqrt{2}$, which is the value adopted to get the experimental results presented below. Summarizing, given $q$ observations $Y_1, \dots, Y_q$, the MC-ESE of the noise standard deviation is thus obtained according to the following two steps

**[Step 1:]** Compute the C-ESE $\widetilde{\sigma_0}$ as a possibly local minimum of Eq. (7). A search interval for the computation of this minimum is proposed in [4].

**[Step 2:]** The MC-ESE is then obtained according to Eq. (8).

Designed for dealing with signals whose prior probabilities of presence are less than or equal to one half, MC-ESE can be regarded as an alternative to the Median Absolute Deviation (MAD) estimator, which performs poorly when the number or the amplitudes of the outliers are too large, which is the case with the applications addressed below.

## III. Application to OFDMA

Orthogonal Frequency Division Multiple Access (OFDMA) is a promising multiple access technology for new generation wireless networks [9]. In this application, knowledge of the noise standard deviation can be of prime importance because it enables propagation channel estimation improvement, signal detection and is a key decision parameter for adaptive modulation and coding or adaptive power allocation. By using the MC-ESE algorithm, we can perform a blind estimation of the noise standard deviation taking into account the possible time-frequency sparsity of OFDMA signals. This sparse nature occurs in the case of low network load or for systems using segmentation and sectorization [10] so that all subcarriers may not be active at the same time. More specifically, assuming that an OFDMA symbol consists of up to $N$ active subcarriers, the discrete-time baseband equivalent transmitted signal is

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k \in \mathbf{Z}} \sum_{n=0}^{N-1} \varepsilon_{k,n} S_{k,n} e^{2i\pi \frac{n}{N}(t-D-k(N+D))} g(t-k(N+D)),$$

where $t \in \mathbf{Z}$, $S_{k,n}$ is a sequence of random symbols assumed to be centred, independent and identically distributed (i.i.d), $\varepsilon_{k,n}$ represents an i.i.d sequence of random variables valued in $\{0,1\}$ that model the absence or presence of signal activity in a time-frequency slot $(k,n)$. $D$ is the cyclic prefix (CP) length and $g$ is the rectangular pulse shaping filter. Let $\{h(\ell)\}_{\ell=0,\cdots,L}$ be a baseband equivalent discrete-time Rayleigh fading channel impulse response of length $L+1$ with $L+1 < D$. The received samples of the OFDM signal are then expressed as

$$y(t) = e^{-i(2\pi\delta \frac{t-\tau}{N} + \theta)} \sum_{\ell=0}^{L} h(\ell) s(t-\ell-\tau) + x(t)$$

where $\delta$ is the carrier frequency offset, $\theta$ the initial arbitrary carrier phase, $\tau$ the timing offset and $x(t)$ the additive white Gaussian noise such that $x(t) \sim \mathcal{CN}\left(0, \sigma_0^2\right)$.

Let us consider $M$ samples of the received signal $y(t)$. Split this set of observations into $K$ disjoint frames of $N$ samples each such that $M = KN$. Apply an $N$-Discrete Fourier Transform (DFT) on each frame. We obtain a matrix $[Y_{k,n}]_{k \in \{1,\dots,M\}, n \in \{0,\dots,N-1\}}$ of complex values where $k$ is the frame index and $n$ the DFT bin number

$$Y_{k,n} = (1/\sqrt{N}) \sum_{t=0}^{N-1} y(kN+t) e^{-2i\pi nt}.$$

For each frame $k$ and each bin $n$, we assume the random presence of an OFDMA frequency component $\widetilde{S}_{k,n}$. We therefore have $Y_{k,n} = \widetilde{\varepsilon}_{k,n} \widetilde{S}_{k,n} + X_{k,n}$: $\widetilde{\varepsilon}_{k,n} \in \{0,1\}$ indicates whether the OFDMA frequency component $\widetilde{S}_{k,n}$ is present or absent in the $k$th bin of the $n$th frame. The complex random variables $X_{k,n}$ are mutually independent and identically distributed with $X_{k,n} \sim \mathcal{CN}\left(0, \sigma_0^2\right)$. Instead of performing an estimate of $\sigma_0^2$ on the basis of the $M = KN$ values we have, we split the observation set into subsets of $m$ observations each. We then compute an estimate of $\sigma_0^2$ on each subset and then average the estimates thus obtained. The following results are averaged over $1000$ Monte Carlo runs. We consider 512-subcarrier OFDMA systems with $D = 128$. The slot allocation is assumed to be i.i.d. The number of OFDMA symbols available at reception is set to 25. The Signal-to-Noise Ratio (SNR) is defined as SNR(dB) $= 10\log_{10}\left(\mathbb{E}\left[|\varepsilon_{k,n} S_{k,n}|^2\right] / \sigma_0^2\right)$. The propagation channel simulated is a time-invariant discrete-time channel $\{h_k(\ell)\}_{\ell=0,\cdots,L}$ with an exponential decay profile for its non-null component (i.e., $\mathbb{E}[|h_k(\ell)|^2] = Ge^{-\ell/42}$ for $\ell = 0, \cdots, L$ with $\sum_{\ell=0}^{L} \mathbb{E}[|h_k(\ell)|^2] = 1$). $L$ is set to 128. Figure 1 compares the Normalized Mean Square Error (NMSE) of the MC-ESE (with $\lambda = \sqrt{2}$) to that obtained by using the well known MAD estimates. The MC-ESE outperforms the MAD method as the latter is not resistant to large outlier numbers or amplitudes. However, further study of the MC-ESE is required to be fully applicable to OFDMA signals. Theoretical results in [4] are established for any sparsity degree but at this stage of investigation, the MC-ESE implementation is limited to the case where the signal is less present than absent which is not always verified for OFDMA signals.

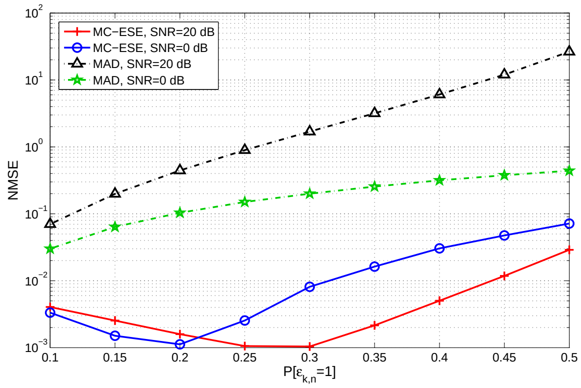


Fig. 1. MAD and MC-ESE performance comparison.

## IV. Application to Wiener filtering in speech processing

Let $s(t)$, $t = 0, \ldots, T-1$, be the samples of some speech corrupted by independent and additive noise $x(t)$, $t = 0, 1, \ldots, T-1$, so that the samples of the observed signal are $y(t) = s(t) + x(t), t = 0, \ldots, T-1$. The standard Wiener filtering based on Malah's decision-directed approach (see [11]) makes it possible to denoise the noisy speech signal. The performance of this filtering depends on our ability to estimate the noise spectrum. The MC-ESE can be used to estimate this spectrum. We begin with the case where noise is white and Gaussian with standard deviation $\sigma_0$. Work in progress concerns the case of coloured noise. When noise is white and Gaussian, the noise standard deviation is estimated as follows. We split the $T$ available samples $y(t)$, $t = 0, 1, \ldots, T-1$, into frames of $N = 2^r$ successive samples each where $r$ is an integer such that $NF_s \approx 20$ms, $F_s$ being the sampling frequency. The frames do not intersect. Let $K$ stand for the number of frames so constructed. Consider the complex values $Y_{k,n}$, $k \in \{1, \ldots, K\}$, $n \in \{0, \ldots, N-1\}$, obtained by DFT on the observed signal in frame $k$. Because of the DFT Hermitian symmetry, we consider $Y_{k,n}$, $k \in \{1, \ldots, K\}, n \in \{0, \ldots, N/2-1\}$ only, where $N$ is assumed to be even. The time-frequency representation of the noisy speech signal is sparse in the sense that the speech time-frequency components are less present than absent. To estimate the noise standard deviation, the MAD estimator is inappropriate because speech components are too numerous. The MC-ESE can be used instead. It is reasonable to model the presence and the absence of a speech time-frequency component $S_{k,n}$ by a discrete random variable $\varepsilon_{k,n}$ valued in $\{0, 1\}$ so that we write $Y_{k,n} = \varepsilon_{k,n} S_{k,n} + X_{k,n}$. We split the observation set $Y_{k,n}$, $k \in \{1, \ldots, K\}$, $n \in \{0, \ldots, N/2-1\}$, into subsets of $m$ observations each and the MC-ESE is used to estimate $\sigma_0$ on the basis of each subset of $m$ observations; the final estimate of the noise standard deviation is obtained by averaging all these estimates returned by the MC-ESE. The average Segmental Signal to Noise Ratios (SSNRs) obtained by denoising 25 sentences of the TIDIGITS database are those of figure 2. The SSNR is the average of the SNR values on short segments.

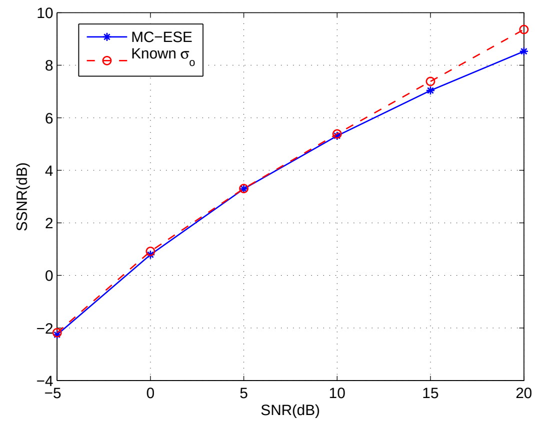


Fig. 2. SSNR improvement for 25 speech signals of the TIDIGITS database additively corrupted by independent AWGN.

## V. Conclusion and perspectives

On the basis of recent results in robust statistics based on sparsity assumptions, we have presented a very promising algorithm, namely, the MC-ESE, for the estimation the noise standard deviation in presence of sparse signals. We have illustrated the relevance of this estimator in two signal processing applications. Some investigations still need to be carried out to further explore the theoretical results developed in [4] in order to extend the MC-ESE estimator to cases where prior probabilities of presence of OFDMA signals can be above 0.5. For speech processing applications, an improvement of the noise standard deviation estimate may be possible by combining the MC-ESE with other algorithms such as the minimum statistics based estimator detailed in [12].

In forthcoming work, the application of the MC-ESE to Communication Electronic Support (CES) will also be addressed. Basically, CES refers to measures taken to gather information intercepted from radio-frequency emissions of non-cooperative communication systems [13]. Current CES systems are based on HF, VHF or UHF acquisitions, which are usually wideband in order to maximise the probability of intercepting the radiated emissions. Signals resulting from this wideband interception are sparse in the time-frequency domain as they are composed, in most cases, of a noisy mixture of few narrowband transmissions. In a non-cooperative context and having little or no prior information on the intercepted signals, the detection of non-cooperative transmissions is usually performed using detectors with constant false alarm rate (CFAR) that require prior knowledge of the noise power. The noise variance is often unknown and must be estimated. The problem is thus very similar to that of OFDMA signals.

Theoretical extensions are required as well. Indeed, the MC-ESE is not designed for asymptotic situations but basically derives from Eq. (6) and [4, Theorem 1], which are theoretical asymptotic results. It then turns out that the good performance measurements of the MC-ESE are explained by neither Eq. (6) nor [4, Theorem 1] only and deserve some theoretical investigations so as to get better insight into the behaviour of the MC-ESE and better analyse its robustness.

Finally, in order to extend the field of applications of the MC-ESE estimator, it would be relevant to study an adaptive implementation of the algorithm for real-time processing of continuous data flows.

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