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***Using Bypass to Tighten WCET Estimates for
Multi-Core Processors with Shared Instruction
Caches***

Damien Hardy — Thomas Piquet — Isabelle Puaut

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Using Bypass to Tighten WCET Estimates for Multi-Core Processors with Shared Instruction Caches

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Abstract: Multi-core chips have been increasingly adopted by the microprocessor industry. For real-time systems to exploit multi-core architectures, it is required to obtain both tight and safe estimates of worst-case execution times (WCETs). Estimating WCETs for multi-core platforms is very challenging because of the possible interferences between cores due to shared hardware resources such as shared caches, memory bus, etc.

This paper proposes a compile-time approach to reduce shared instruction cache interferences between cores to tighten WCET estimations. Unlike [29], which accounts for all possible conflicts caused by tasks running on the other cores when estimating the WCET of a task, our approach drastically reduces the amount of inter-core interferences. This is done by controlling the contents of the shared instruction cache(s), by caching only blocks statically known as reused. Experimental results demonstrate the practicality of our approach.

Key-words: WCET, hard real time systems, multi-core processors, memory hierarchy, static analysis, abstract interpretation.

Utilisation du mécanisme de bypass pour réduire l'estimation du pire temps d'exécution pour les processeurs multi-cœur avec caches d'instructions partagés

Résumé : Avec l'arrivée de matériel complexe dans les systèmes temps-réel embarqués (processeurs avec des fonctions d'amélioration des performances tel que les pipelines, les hiérarchies de caches, les multi-cœurs), de nombreux processeurs multi-cœurs ont maintenant des caches partagés. Ainsi, considérer les caches partagés lors de la validation du comportement temporel des systèmes temps-réel, en particulier lors de l'estimation d'une borne supérieure du pire temps d'exécution des tâches s'exécutant sur le système devient nécessaire.

Dans ce rapport, nous présentons d'une part une approche d'analyse statique pour les caches d'instructions partagés en présence de tâches interférentes s'exécutant simultanément sur d'autres cœurs. D'autre part, nous proposons une méthode utilisant un mécanisme de bypass pour réduire le nombre d'interférences dans les caches partagés afin de réduire l'estimation du pire temps d'exécution. Cette approche se base sur l'estimation statique de non réutilisation de blocs de cache pouvant alors ne pas être mis dans le(s) cache(s) partagé(s), réduisant ainsi le nombre d'interférences dans les caches partagés.

Nous montrons que notre méthode permet la plupart du temps une estimation du pire temps d'exécution plus précise en utilisant notre approche de bypass. Une évaluation du temps de calcul est réalisée montrant que l'analyse est effectuée en un temps raisonnable.

Mots-clés : pire temps d'exécution, temps-réel strict, processeurs multi-cœur, hiérarchie mémoire, analyse statique, interprétation abstraite.

1 Introduction

In real-time systems it is crucial to prove that the execution of a task meets its deadline in all execution situations, including the worst-case. This proof needs an estimation of the worst-case execution time (WCET) of any task in the system. WCET estimates have to be safe, i.e. larger than or equal to any possible execution time. Moreover, they have to be tight, i.e. as close as possible to the actual worst-case execution time, to correctly dimension the resources required by the system. WCET estimation methods have to account for all possible flows in a program and determine the longest execution path (so-called *high-level analysis*). They also have to account for the hardware the program is running on, through a *low-level analysis*. A number of static analysis methods have been designed in the last two decades at both levels, mainly for mono-core processors [27].

Multi-cores have been increasingly adopted for both desktop and now embedded applications. However, for multi-core architectures to be used in hard real-time systems, it is required to obtain both tight and safe estimates of WCETs. This is a very challenging task because of the possible interferences between cores due to shared hardware resources such as shared caches. WCET estimation for multi-core platforms has been the subject of very few studies. We present in this paper a new WCET estimation method for multi-core platforms with shared instruction caches. The proposed method provides tight WCET estimates, through a control of the contents of the shared instruction cache(s), more precisely by caching only the blocks statically known to be reused.

Related work. Many WCET estimation methods have been designed in the last two decades (see [27] for a survey). The mostly used static WCET computation technique, called IPET (Implicit Path Enumeration Technique) such as [14] estimates the WCET through the resolution of an Integer Linear Programming (ILP) problem constraining the execution frequencies of the program basic blocks.

Static WCET estimation methods need a low-level analysis phase to determine the worst-case timing behavior of the microarchitectural components: pipelines and out-of-order execution [4, 11], branch predictors [1] and caches. Regarding cache memories on mono-core architectures, two main classes of approaches have been proposed: *static cache simulation* [16, 17], based on dataflow analysis, and the methods described in [6, 25, 7], based on abstract interpretation. Both classes of methods provide for every memory reference a classification of the outcome of the reference in the worst-case execution scenario (e.g. *always-hit*, *always-miss*, *first-miss*, etc.). These methods, originally designed for code only, and for direct-mapped or set-associative caches with a Least Recently Used (LRU) replacement policy, have been later extended to other replacement policies [10], data and unified caches [26], and caches hierarchies [9]. Related techniques estimate the worst-case delay to reload the cache contents after a preemption by a higher priority task [18, 23].

Very few studies have considered WCET analysis for multi-core platforms so far. The method described in [29] estimates the WCET in presence of shared caches on multi-cores by analysing *inter-thread cache conflicts*. They assume a dual-core architecture with a private L1 instruction cache and a shared L2 instruction cache. The method aims at analysing the WCET of a real-time task (*rtt*), running on one core, during the execution of a non real-time task (*nrnt*) running on the other core. The method detects conflicts in the shared L2 instruction cache, and integrates such con-

flicts by changing the cache classification of the concerned program blocks of the *rtt*. The classification of a program block in the *rtt* located in a loop, which would have been classified as a *hit* if the task was executed alone, is changed to *always-except-one* if none of the conflicting blocks in the *nrtt* is located in a loop. This method might underestimate the WCET of the *rtt* when different cache blocks of the *nrtt* evict a *rtt* cache block several times during the execution of the loop, or if the *nrtt* executes several times while the *rtt* executes. More generally, the method described in [29] is expected to lack scalability with respect to the task size and number of tasks, because every conflict with every other task has to be considered. Our proposed method, like [29] identifies all inter-task interferences due to cache sharing for the sake of safety. However, it is complemented by a *bypass* method of the shared instruction cache(s), caching in the shared cache(s) only the blocks statically known to be reused, allowing to drastically reduce the amount of inter-core interferences.

A very different approach for multi-cores with shared instruction caches is proposed in [24] and is based on the combined use of cache *locking*, i.e. temporarily disabling cache replacement, and *partitioning*, i.e. partitioning the cache among the tasks or cores. The objective of such a joint use of locking and partitioning is to completely avoid intra-task and inter-task conflicts, which then do not need to be analysed. Different combinations of partitioning (per-core/per-task) and locking (static/dynamic) are experimented. With partitioning approaches, interferences caused by shared caches are avoided, thus having a positive impact on the WCET of every task; on the other hand, partitioning comes at the cost of a lower volume of cache available per task/core, having a negative impact on the WCET. In contrast to [24], our approach does neither lock nor partition the shared instruction cache(s). Experiments would be required to assess the respective merits of [24] compared with our approach.

Related cache bypass techniques approaches have already been proposed in [5, 3, 19] for instruction and data caches on mono-core and multi-core architectures. These studies show the presence of cache blocks that are not reused, called *single-usage* blocks. These blocks generate wasteful conflicts in the cache, which lead to a deteriorated behavior of this cache. For instance, in [19], they measured for the second level cache of a superscalar processor that in average 33% of the accesses are made to a cache block that will not be reused (with the SPEC CPU 2000 benchmark). To reduce the conflicts generated by these accesses, they propose to bypass cache blocks that are not reused from a cache level. In [19], both the detection and the bypass of single-usage block are dynamic and hardware implemented. The dynamic approach by [19] reduces significantly conflicts in the shared cache of a multi-core architecture, hence the miss rate of the cache is decreased. In [5], they suggest a static (using profiling) and a dynamic solution to select the cache blocks to bypass. All these related studies aim at reducing the average-case execution time, while we focus on a reduction of worst-case execution time estimates. Furthermore, in our case, detection of single-usage blocks is done at compile-time.

Our approach also has some links with the mono-core approach described in [12], designed for single-level data caches. In [12] accesses to data are classified as either *hard-to-predict* or *easy-to-predict* at compile-time, and only *easy-to-predict* data are cached in the L1 data cache. Our approach also implements selective caching based on information computed at compile-time, the relevant information being the reuse of instructions in the shared instruction cache(s) in our case.

Contributions. Our first contribution in this paper is the proposal of a safe WCET estimation method for multi-core architectures with shared instruction cache(s). The proposed method is more general than [29] in the sense that it supports multiple levels of shared caches, set-associative caches and an arbitrary number of real-time tasks and cores competing for the shared caches.

Our second contribution is the static identification of single-usage (not reused) program blocks in shared instruction caches, thanks to static analysis of the program code.

Finally, our third and main contribution consists in a compiler-directed bypass scheme, which, from the static knowledge of single-usage blocks, allows a drastic reduction of inter-task and intra-task interferences, and thus a tighter WCET estimate.

Paper outline. The rest of the paper is organized as follows. Section 2 presents the assumptions our analysis is based on, regarding the target architecture and task scheduling. Section 3 presents a safe WCET estimation method for multi-core architectures with shared instruction caches that considers all interferences for the shared instruction caches. Section 4 then presents the main contribution of the paper, a bypass technique to decrease interferences due to instruction cache sharing, and thus to tighten the WCET estimate. Experimental results are given in Section 5.

2 Assumptions

A multi-core architecture is assumed. Each core has a private first-level (L1) instruction cache, followed by instruction cache levels with at least one shared cache. The caches are set-associative. Each level of the cache hierarchy is non-inclusive:

- A piece of information is searched for in the cache of level ℓ if and only if a cache miss occurred when searching it in the cache of level $\ell - 1$. Cache of level 1 is always accessed.
- Except if the bypass mechanism presented in Section 4 is used, every time a cache miss occurs at cache level ℓ , the entire cache block containing the missing piece of information is always loaded into the cache of level ℓ .
- There are no actions on the cache contents (i.e. invalidations, lookups/modifications) other than the ones mentioned above.

Our study concentrates on instruction caches; it is assumed that the shared caches do not contain data, i.e. in a first step towards a general solution, data is assumed to either be non-cacheable, or the caches are partitioned between code and data.

Our method assumes a LRU (Least Recently Used) cache replacement policy. Furthermore, an architecture without timing anomalies [13] is assumed. The access time variability to main memory and shared caches, due to bus contention, is supposed to be bounded and known, by using for instance *Time Division Multiple Access (TDMA)* like in [22].

Figure 1 illustrates two different supported architectures.

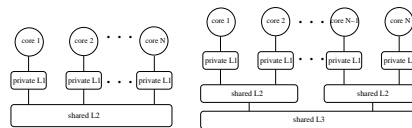


Figure 1: Two examples of supported architectures

Regarding scheduling, it is assumed that a job does not migrate between cores at run-time. Migrations are allowed between job instances only. No further assumption is made on task scheduling, implying that any part of an interfering task may be executed simultaneously with the analysed task and may thus pollute the shared cache(s). This assumption was made in a first approach to keep WCET estimation and schedulability independent activities, as traditionally done when temporally validating real-time software. We do not attempt to explore joint WCET estimation and scheduling, which is left for future work. Tasks are independent (i.e. do not synchronize with each other), but might share code, such as libraries (see paragraph 3.2.3).

The focus in this paper is to estimate the WCET of a hard-real time task running on a core, in isolation from the tasks running on the same core, but suffering indirect interferences because of cache sharing from tasks running on the other cores. The computation of cache-related preemption delay due to intra-core interferences is considered out of the scope of this paper.

3 WCET Analysis on multi-cores with multiple levels of instruction caches

In this section, we describe a safe WCET estimation method in presence of shared instruction caches. Paragraph 3.1 first presents our base WCET estimation method for multi-level caches on mono-core processors, initially presented in [9]. Paragraph 3.2 then extends the base method to cope with interfering tasks running on the other cores. No attempt is made in this section to reduce the volume of such inter-task interferences, which will be the subject of Section 4.

3.1 Static multi-level cache analysis for mono-core processors [9]

The cache analysis is applied successively on each level of the cache hierarchy, from the first cache level to the main memory. The analysis is contextual in the sense that it is applied for every call context of functions (functions are virtually inlined). The references considered by the analysis of cache level ℓ depend on the outcome of the analysis of cache level $\ell - 1$ to consider the filtering of memory accesses between cache levels, as depicted in Figure 2 and detailed below.

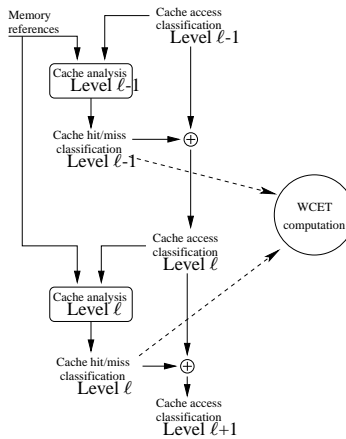


Figure 2: Multi-level cache analysis on a mono-core processor

The outcome of the static cache analysis for every cache level ℓ is a *Cache Hit/Miss Classification (CHMC)* for each reference, determining the worst-case behavior of the reference with respect to cache level ℓ :

- *always-miss* (AM): the reference will always result in a cache miss,
- *always-hit* (AH): the reference will always result in a cache hit,
- *first-miss* (FM): the reference could neither be classified as hit nor as miss the first time it occurs but will result in cache hit afterwards,
- *not-classified* (NC): in all other cases.

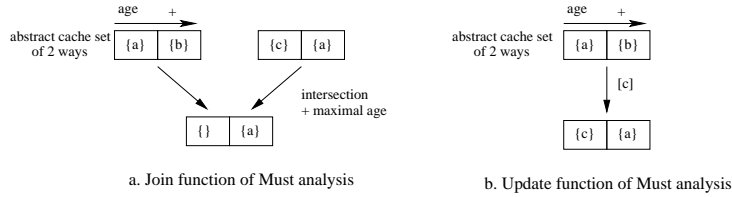


Figure 3: *Join* and *Update* functions for the Must analysis with LRU replacement

At every level ℓ , a *Cache Access Classification (CAC)* determines if an access may occur or not at level ℓ , and thus should be considered by the static cache analysis of that level. There is a *CAC*, noted $CAC_{r,\ell,c}$ for every reference r , cache level ℓ , and call context c ¹. The *CAC* defines four categories for each reference, cache level, and call context:

- *A* (Always): the access always occurs at the cache level.
- *N* (Never): the access never occurs at the cache level.
- *U – N* (Uncertain-Never): the access could occur or not the first time but next accesses will never occur at the cache level. This category, subset of the *U* category described below, was added to our original cache analysis published in [9], because it allows a more precise identification of single-usage blocks.
- *U* (Uncertain) when the access cannot be classified in the three above categories.

The cache analysis at every cache level is based on a state-of-the-art single-level cache analysis [25], based on abstract interpretation. The method is based on three separate fixpoint analyses applied on the program control flow graph, for every call context:

- a *Must* analysis determines if a memory block is always present in the cache at a given point: if so, the block is classified *always-hit (AH)*;
- a *Persistence* analysis determines if a memory block will not be evicted after it has been first loaded; the classification of such blocks is *first-miss (FM)*.
- a *May* analysis determines if a memory block may be in the cache at a given point: if not, the block is classified *always-miss (AM)*. Otherwise, if neither detected as always present by the *Must* analysis nor as persistent by the *Persistence* analysis, the block is classified *not classified (NC)*;

Abstract cache states (ACS) are computed for every basic block according to the semantics of the analysis and the cache replacement policy by using functions (*Update* and *Join*) in the abstract domain. *Update* models the impact on the ACS of every

¹The call context c will be omitted from the formulas when the concept of call context is not relevant.

reference inside a basic block; *Join* merges two ACS at convergence points in the control flow graph (e.g. at the end of conditional constructs).

Figure 3 gives an example of an ACS of a 2-way set-associative cache with LRU replacement policy on a *Must* analysis (only one cache set is depicted). An *age* is associated to every cache block of a set. The smaller the block age the more recent the access to the block. For the *Must* and *Persistence* analyses, each memory block is represented only once in the ACS, with its maximum age. It means that its actual age at run-time will always be lower than or equal to its age in the ACS.

At every cache level ℓ , the three analyses (*Must*, *May*, *Persistence*) consider all references r guaranteed to occur at level ℓ ($CAC_{r,\ell} = A$). References with $CAC_{r,\ell} = N$ are not analysed. Regarding uncertain references ($CAC_{r,\ell} = U$ or $CAC_{r,\ell} = U - N$), for the sake of safety, the ACS is obtained by exploring the two possibilities ($CAC_{r,\ell} = A$ and $CAC_{r,\ell} = N$) and merging the results using the *Join* function. For all references r , $CAC_{r,1} = A$, meaning that the L1 cache is always accessed.

The CAC of a reference r for a cache level ℓ depends on CHMC of r at level $\ell - 1$ and the CAC of r at level $\ell - 1$ (see Figure 2). Table 1 shows all the possible cases of computation of $CAC_{r,\ell}$ from $CHMC_{r,\ell-1}$ and $CAC_{r,\ell-1}$.

$CAC_{r,\ell-1} \backslash CHMC_{r,\ell-1}$	AM	AH	FM	NC
A	A	N	U-N	U
N	N	N	N	N
U-N	U-N	N	U-N	U-N
U	U	N	U-N	U

Table 1: Cache access classification for level ℓ ($CAC_{r,\ell}$)

The CHMC of reference r is used to compute the cache contribution to the WCET of that reference, which can be included in well-known WCET computation methods [14, 21].

3.2 Static multi-level cache analysis for multi-cores

Compared with its execution in isolation, the execution of a task on a multi-core architecture with shared cache(s) may introduce some extra misses in the shared cache resulting from the interfering tasks. In term of static analysis, it means that some accesses previously classified as *always-hit* (or *first-miss*) using a mono-core cache analysis (without considering the interfering tasks running on the other cores) may have to be changed into *first-miss* or *not classified*. The cache analysis method presented in this paragraph can be seen as a safe extension of the method presented in [29] to set-associative caches, multiple levels of shared caches and an arbitrary number of tasks/cores competing for the shared caches.

Compared to the mono-core analysis described in paragraph 3.1, considering inter-core interferences for shared cache level(s) requires to change the static cache analysis for the shared cache level(s), keeping intact the analysis of the private cache level(s). As illustrated in Figure 4, the analysis of a shared cache level ℓ estimates the worst-case number of conflicts per cache set due to tasks running on the other cores and then computes a cache classification to account for these conflicts.

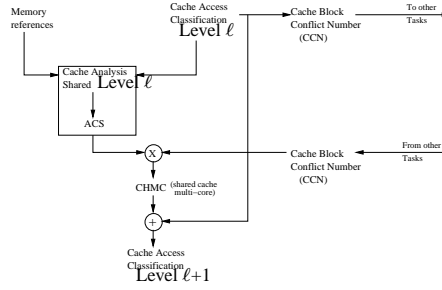


Figure 4: Cache analysis on a multi-core processor for a shared cache level ℓ

3.2.1 Estimation of interferences

The problem of determining the worst-case number of inter-tasks blocks conflicts which occur in a cache set of a shared cache can be represented as a flow problem². Since this class of problems is NP-complete [8], we propose an heuristic to compute a safe upper bound of the number of interfering blocks for each cache set. This number for a set s is named hereafter *cache block conflict number*, $CCN(s)$.

For each shared cache level ℓ , the static cache analysis applied to each interfering task, provides information about which references may occur at level ℓ through the CAC. Any reference which may occur at level ℓ ($CAC_{r,\ell} \neq N$) is considered as interfering, regardless of the time when this access may occur to stay independent from the scheduling. $CCN(s)$ is then the sum, for all interfering tasks of all the different interfering blocks.

3.2.2 Accounting for inter-core interferences in cache classification

The number of conflicts per set $CCN(s)$ is used, together with the Abstract Cache States (ACS) provided by the cache analysis, to determine a new CHMC accounting for inter-task interferences.

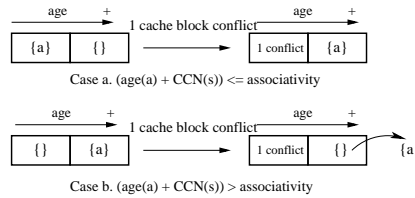


Figure 5: Accounting for inter-core interferences

The ACS produced by the *Must* analysis keeps the oldest age of a cache block in the cache set. Accounting for interfering tasks implies that in the worst-case this age has to be increased by $CCN(s)$, with s the cache set. If this corrected age is still less or equal than the degree of associativity then the block will be ensured to be in the cache, otherwise the cache block is considered absent from the *Must* ACS. Figure 5 illustrates both cases. The same procedure is applied to the *Persistence* analysis. Conversely for the *May* analysis, which determines the cache blocks which may be in the cache, no modification is required.

Because of inter-core interferences, the CHMC of a reference on a multi-core platform may be more pessimistic than its equivalent in the mono-core case. Thus, indi-

²each task can be modeled as a flow and is linked with the shared resources to the other tasks.

rectly, the CAC of the next cache level might also differ. For our approach to be safe for multiple levels of shared caches, we analyse for each task the shared cache levels in sequence and change the resulting CACs before analysing the next shared cache level.

3.2.3 Code sharing

To take into account the effect of code sharing between tasks due to shared libraries, we divide $CCN(s)$ into $CCN_{private}(s)$ and $CCN_{shared}(s, sb)$. The ranges of addresses of the shared libraries are assumed to be given as a parameter of the analysis.

$CCN_{private}(s)$ represents the number of conflicts per set due to the private code of interfering tasks, and is computed as before. Conversely, $CCN_{shared}(s, sb)$ represents the number of conflicts for a shared block sb mapped to cache set s and is used to determine the number of effective conflicts with sb . $CCN_{shared}(s, sb)$ is computed in two steps.

- The first step determines the set $Shared(s)$ of shared interfering cache blocks mapped to set s . Because of code sharing, each block belonging to a shared library and used by an interfering task has to be considered only once. Thus, $Shared(s)$ is defined as the union, for all interfering tasks, of the set of used shared cache blocks mapped to set s ($CAC_{r,\ell} \neq N$).
- The second step to compute $CCN_{shared}(s, sb)$ stems from the fact that the analysed task may also use some block in $Shared(s)$. Regarding *Must* analysis, at every program point, a shared block is considered as conflicting with block sb present in ACS_{Must} only if its age in ACS_{Must} is strictly higher than $age(sb)$:

$$CCN_{shared}(s, sb) = | Shared(s) \setminus \{b \in ACS_{Must}, age(b) \leq age(sb)\} |$$

Similarly, regarding *Persistence* analysis:

$$CCN_{shared}(s, sb) = | Shared(s) \setminus \{b \in ACS_{Persistence}, age(b) \leq age(sb)\} |$$

Finally, for both the *Must* and *Persistence* analysis, the formula which determines if a cache block sb is evicted is:

$$age(sb) + CCN_{private}(s) + CCN_{shared}(s, sb) > associativity$$

Similarly to a system without code sharing, the *May* analysis needs not be modified.

4 Interference reduction using bypass of static single-usage blocks

Upon a cache miss, the conventional operation mode of a cache hierarchy is to retrieve the missing block from lower levels and to store them into all upper hierarchy levels. However, it is difficult to assert that storing the block into intermediate levels will be really useful. In some cases, a block stored in the cache after a miss may not be accessed again before its eviction. Such blocks, named single-usage blocks, contribute to the well known cache pollution phenomenon [19].

Static cache analysis methods have the ability to estimate single usage blocks at compile-time. The main contribution of this paper is to estimate such *static single*

usage (SSU) blocks, and force the bypass of such blocks from the shared cache(s), in order to reduce pollution in shared caches and thus to tighten the WCET estimates.

Figure 6 illustrates the modifications in the WCET estimation procedure of Section 3.2 to identify SSU blocks and to account for the bypass of such blocks. These modifications concerning shared caches levels only are detailed below.

4.1 Identification of Static Single Usage (SSU) blocks

For a given shared cache level ℓ , a *static multiple usage block* is defined as a block statically known to be accessed multiple times and still present in the shared cache when reused, in at least one execution context. Any other block is termed *static single usage (SSU) block*.

SSU blocks are estimated for every task taken in isolation, with no specific treatment for shared code, using the CHMC and CAC (upper part of Figure 6). More formally, we define the SSU identification function, which returns *true* if the analysed cache block is a SSU block, as follows:

$$\bigwedge_{c \in \text{contexts}} \bigwedge_{r \in \text{cache block}} f_{SSU}(CAC_{r,\ell,c}, CHMC_{r,\ell,c})$$

with f_{SSU} defined in Table 2.

$CAC_{r,\ell,c} \backslash CHMC_{r,\ell,c}$	AM	AH	FM	NC
A	true	false	false	true
N	true	true	true	true
U-N	true	false	true	true
U	true	false	false	true

Table 2: $f_{SSU}(CAC_{r,\ell,c}, CHMC_{r,\ell,c})$ for level ℓ

Function f_{SSU} returns true if a reference in a given context is not guaranteed to be reused, and false otherwise (CHMC=AH or FM). Blocks with $CAC_{r,\ell,c} = U - N$ and $CHMC_{r,\ell,c} = FM$ are classified as SSU blocks because there are known to be accessed only once, by definition of the U-N CAC category.

4.2 Static cache analysis on multi-cores with bypass of SSU blocks

The lower part of Figure 6 shows the integration of SSU blocks in the shared cache analysis. SSU blocks r are marked as bypassed ($CAC_{r,\ell} = BP$). During the cache analysis of every cache level ℓ , accesses to BP blocks are semantically equivalent to never accessed blocks (whose $CAC_{r,\ell} = N$). Conflict number $CCN(s)$ is computed like in Section 3.2 except that blocks whose $CAC = BP$ are not considered as interfering. The *CHMC* of BP accesses is set to *always-miss*. No modification of the cache analysis internals is required.

To take into account the bypass information when analysing multiple levels of shared caches, when analysing a shared cache level ℓ a safe CAC has to be propagated to the next cache level. Since blocks marked as BP only impact the current cache level ℓ , the original CAC (before its replacement by BP) is propagated to cache level $\ell + 1$.

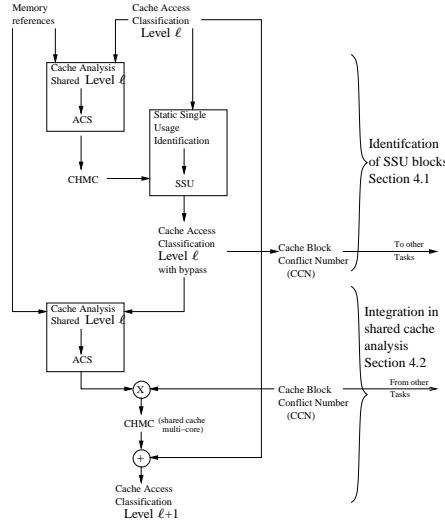


Figure 6: Cache analysis on a multi-core processor using bypass, for a shared cache level ℓ

4.3 Implementation

A straightforward way to implement our bypass approach is to use the scheme described in [20] where instructions have a dedicated bit to control their cacheability. After the estimation of SSU blocks, this bit can be set at compile-time without any modification of the code memory layout. For multiple levels of shared caches, this solution requires n bits where n represents the number of shared cache levels (at most two bits for standard architectures).

Other alternative but more complex implementations suggested in [15] are to distinguish cached instructions from uncached instructions by addresses, or to dynamically turn on/off instruction caching. Nevertheless, these implementations need more compiler support because of the heaviest code restructuring required.

5 Experimental results

In this section, the benefits of using the bypass mechanism proposed in Section 4 to reduce inter-task and intra-task interferences are evaluated. We first describe the experimental conditions (§ 5.1) and then we give and analyse experimental results for a 2-level cache hierarchy with a shared L2 instruction cache (§ 5.2).

The performance metrics used to evaluate our proposal are the hit ratios in the L1 and L2 cache along the worst-case execution path, obtained using static analysis. No comparisons with measured values are given, because generating the worst-case interference for the shared L2 cache is extremely difficult to achieve due to the impact of the tasks timing and scheduling on the actual interferences for the shared L2 cache [28].

5.1 Experimental setup

Cache analysis and WCET estimation. The experiments were conducted on MIPS R2000/R3000 binary code compiled with gcc 4.1 with no optimization and with the

default linker memory layout. The WCETs of tasks are computed by the Heptane timing analyzer [2], more precisely its Implicit Path Enumeration Technique (IPET). The analysis is context sensitive (functions are analysed in each different calling context). To separate the effect of the caches from those of the other parts of the processor micro-architecture, WCET estimation only takes into account the contribution of instruction caches to the WCET. The effects of other architectural features are not considered. In particular, timing anomalies caused by interactions between caches and pipelines, as defined in [13] are disregarded. The cache classification *not-classified* is thus assumed to have the same worst-case behavior as *always-miss* during the WCET computation in our experiments. The cache analysis starts with an empty cache state. WCET estimation accounts for the CACs and CHMCs of all cache levels. For space consideration, WCET computation is not detailed here, interested readers are referred to [9].

Name	Description	Code size (bytes)
crc	Cyclic redundancy check computation	1432
qurt	Root computation of quadratic equations	1928
lms	LMS adaptive signal enhancement	2828
fdct	Fast Discrete Cosine Transform	3468
fft	Fast Fourier Transform	3536
minver	Inversion of floating point 3x3 matrix	4408
adpcm	Adaptive pulse code modulation algorithm	7740
statemate	Automatically generated code by STARC (STAtchart Real-time-Code generator)	8900

Table 3: Benchmark characteristics

Benchmarks. The experiments were conducted on eight benchmarks (see Table 3 for the applications characteristics). All benchmarks are maintained by Mälardalen WCET research group³.

Cache hierarchy. The results are obtained on a 2-level cache hierarchy composed of a private 4-way L1 cache of 1KB with a cache block size of 32B and a shared 8-way L2 cache of 4KB with a cache block size of 32B. Cache sizes are small compared to usual cache sizes in multi-core architectures. However, there are no large-enough public real-time benchmarks available to experiment our proposal. As a consequence, we have selected quite small commonly used real-time benchmarks and adjusted cache sizes such that the benchmarks do not fit entirely in the caches. All caches are implementing a LRU replacement policy. Latencies of 1 cycle (respectively 10 and 100 cycles) are assumed for the L1 cache (respectively the L2 cache and the main memory).

5.2 Results for a multi-core architecture with a shared L2 instruction cache

5.2.1 Impact of L2 bypass in a system without inter-core interference

An interesting expected side effect of the proposed bypass approach is that it allows a decrease of the WCET estimate of a task without any concurrent task competing for

³<http://www.mrtc.mdh.se/projects/wcet/benchmarks.html>

the L2 cache, compared with the WCET estimate of the same task without any bypass. Indeed, all non-reused program blocks in a shared cache level are bypassed. As a consequence, this can avoid intra-task conflicts for shared cache blocks.

This phenomenon is quantified in Table 4, by considering a task executing on one core without any interfering task running on the other cores competing for the shared L2 cache.

For every benchmark (column 1), we examine the worst-case static hit ratio ($\frac{\text{number of hits}}{\text{number of accesses}}$) for the analysed task in the L1 cache (fraction of hits in the L1 cache along the worst-case execution path, column 2). Columns 3 and 4 give the worst-case static hit ratio in the L2 cache respectively with and without the L2 bypass scheme proposed in Section 4. The number of accesses and the number of hits at each level of the cache hierarchy are computed using the frequency of basic blocks returned by the IPET WCET computation. Finally, column 5 gives the percentage of bypassed blocks in the L2 cache ($\frac{\text{number of bypassed cache blocks}}{\text{number of cache blocks}}$).

Benchmark	L1 hit ratio	L2 hit ratio no bypass	L2 hit ratio bypass	bypass ratio
minver	93.99%	39.76%	39.76%	94.92%
adpcm	89.74%	33.60%	33.96%	88.02%
fdct	87.25%	84.03%	84.03%	5.5%
statemate	83.40%	0.72%	1.21%	98.92%
fft	88.76%	1.97%	12.50%	92.79%
crc	93.10%	98.97%	98.97%	88.89%
lms	87.24%	0.61%	0.61%	94.38%
qurt	93.57%	12.56%	12.56%	98.36%

Table 4: Impact of L2 bypass on WCET estimation with no interference for the L2 cache.

The worst-case hit ratio in the L1 cache indicates the percentage of references statically known to hit in the L1 cache. The higher the hit ratio, the lower the number of references to the shared L2 cache. The percentage of bypassed blocks given in column 5 indicates how much pollution is avoided in the L2 cache. The higher the bypass ratio, the higher the expected reduction of inter-core interference.

As expected, our bypass scheme provides worst-case hit-ratios in the shared L2 cache always larger than or equal to when no bypass mechanism is used. For three benchmarks (*statemate*, *adpcm*, *fft*), the worst-case hit ratio is strictly larger than when not using L2 bypass. In these cases, not storing SSU blocks in the L2 instruction cache may allow to detect the reuse of blocks which were previously classified as misses (because they were in conflict with a SSU block). In the best case (*fft*), the worst-case hit ratio is multiplied by a factor 10, which significantly reduces the task WCET.

In term of percentage of bypassed cache blocks, the ratio is for all tasks but *fdct* high, meaning that the degree of pollution in the shared L2 cache is important. For those applications, reducing L2 cache pollution is expected to drastically reduce inter-core interference. For *fdct*, the percentage of bypassed blocks is low, explained by the code structure, made of two large loops whose code do not fit into the L1 cache but entirely fit in the L2 cache. Note that the percentage of SSU blocks is much higher than when single-usage blocks are detected dynamically, such as in [19], reporting an average number of single-usage blocks of 33% for the SPEC CPU 2000 benchmarks. This difference comes from the fact that static cache analysis, for the sake of safety, underestimates the set of reused blocks compared to real executions.

5.2.2 Multi-core cache analysis with and without bypass

In this paragraph an architecture with two cores is considered. We estimate the WCET of a task running on one core, competing for the shared L2 cache with one of the eight benchmarks of Table 3 running on the other core, in a context without code sharing between tasks.

The results are presented in Table 5. For each benchmark, the results without bypass of the L2 cache (first line) and with bypass (second line) are given. For each configuration, we give the worst-case hit ratio in the L2 cache for the analysed task obtained: (i) without interfering task running on the other core (see also Table 4) (ii) with the interfering task having the lowest amount of interference; (iii) with an average of the worst-case hit ratio with each of the eight tasks successively competing for the shared L2 cache ; (iv) with the interfering task having the highest amount of interference.

Bench.	bypass	no-interf	multi-core		
		alone	weakest adversary	average of adversaries	highest adversary
minver	no	39.76%	39.76%	24.85%	0%
	yes	39.76%	39.76%	39.76%	39.76%
adpcm	no	33.60%	33.26%	20.60%	0%
	yes	33.96%	33.96%	33.76%	32.90%
fdct	no	84.03%	84.03%	24.88%	0%
	yes	84.03%	84.03%	73.32%	7.34%
statemate	no	0.72%	0.72%	0.19%	0%
	yes	1.21%	1.21%	1.21%	1.21%
fft	no	1.97%	1.97%	1.23%	0%
	yes	12.50%	12.50%	12.50%	12.50%
crc	no	98.97%	98.97%	61.85%	0%
	yes	98.97%	98.97%	98.97%	98.97%
lms	no	0.61%	0.61%	0.38%	0%
	yes	0.61%	0.61%	0.61%	0.61%
quirt	no	12.56%	12.56%	7.85%	0%
	yes	12.56%	12.56%	12.56%	12.56%

Table 5: Estimated worst-case L2 hit ratio of the analysed task (bypass vs no bypass).

With the weakest adversary task (column *weakest adversary*), the increase of worst-case hit ratio compared with a system without L2 bypass is generally slim.

In average (column *average of adversaries*), the increase of worst-case hit ratio compared with a system without L2 bypass is significant.

In the worst-case (column *highest adversary*), without bypass, the multi-core cache analysis always results in a hit ratio of 0% in the shared L2 cache. This hit ratio of 0% not always occurs with the same competing task. Moreover, for all benchmarks, the highest adversary is not unique; several adversary tasks result in a hit ratio of 0%. Said differently, the worst-case hit ratio in a system without L2 bypass is extremely poor. This demonstrates the pessimism of methods such as the base method presented in Section 3 and the approach described in [29], that consider all interferences between cores without any mechanism to decrease inter-core interference. If we now compare the worst-case hit ratio in the L2 when bypassing *SSU* blocks with the worst-case hit ratio without interference, the value is the same for six out of the eight considered benchmarks, and very close for *adpcm* (1.04%). In contrast, for *fdct* the difference is significant (76,69%). This case occurs when *fdct* competes with itself for the shared L2 cache (recall that no code sharing is assumed in this paragraph), and results in a high volume of L2 cache consumed by *fdct*. When *fdct* is not competing with itself

anymore, the worst-case hit ratio becomes in the L2 cache 75.06% and the difference of hit ratio in the L2 cache compared to an interference free situation is then less than 9%.

In summary, using our bypass scheme, the amount of inter-core interferences is drastically reduced and the multi-core WCET is generally very close to the WCET without interferences for the shared L2 cache.

5.2.3 Scalability issues

The scalability of our bypass scheme is analysed by considering all benchmarks, each running on a distinct core, as interfering tasks of the analysed task. In this configuration the cumulated size of all interfering tasks is around 8 times bigger than the size of the L2 shared instruction cache.

The results are presented in Table 6. The *fdct* task which needs the higher volume of L2 capacity even in the case of our bypass approach, is left out in the left column and considered in the right column. For each configuration, we give the worst-case inter-core miss ratio resulting from the interferences with the tasks running on the other cores ($\frac{L2missInterf - L2missNoInterf}{L2missInterf}$).

Bench.	no-interf	all interfering tasks except fdct		all interfering tasks including fdct	
		L2 hit ratio	L2 inter-core miss ratio	L2 hit ratio	L2 inter-core miss ratio
minver	39.76%	38.55%	1.96%	0%	39.76%
adpcm	33.96%	21.97%	0.84%	15.59%	21.77%
fdct	84.03%	66.08%	52.92%	1.63%	83.77%
statemate	1.21%	1.16%	0.05%	0%	1.21%
fft	12.50%	6.82%	6.10%	0.88%	11.73%
crc	98.97%	98.97%	0%	0%	98.97%
lms	0.61%	0.61%	0%	0%	0.61%
qurt	12.56%	12.56%	0%	0%	12.56%

Table 6: Estimated worst-case L2 hit ratio and inter-core miss ratio of the analysed task (bypass, 7-8 interfering tasks).

When task *fdct* is left out, for a majority of benchmarks (*minver*, *statemate*, *crc*, *lms*, *qurt*) the decrease of the worst-case hit ratio in the L2 cache is low, compared with the worst-case hit ratio in the L2 cache with no adversary task running on the other cores. For the other benchmarks, the decrease of worst-case hit ratio in the L2 cache is larger. However, for all benchmarks, there are still hits in the L2 cache when using bypass. In contrast, no hit in the L2 cache can be guaranteed even with one single task running on the other core when no bypass is used, as previously shown in Table 5.

In contrast, when *fdct* is kept, the decrease of the worst-case hit ratio in the L2 cache is significant because *fdct* consumes a large percentage of the L2 cache capacity. With this kind of task, the proposed bypass approach is not sufficient to decrease the amount of interference for the L2 cache. Additional methods such as cache partitioning have to be used to isolate such cache consuming tasks from the other tasks.

5.2.4 Code Sharing.

The impact of considering code sharing (paragraph 3.2.3) is evaluated in Table 7. Due to the difficulties to find different degrees of shared code in real-time benchmarks, the evaluation was achieved with a single task, running on 2 or 3 cores, and considering

in the analysis that a certain percentage of its code was shared by the task instances. Table 7 shows the worst-case hit ratio in the L2 cache for the *fdct* task, running on one core, when one (respectively two) instance(s) of *fdct* are running on the other one or two cores. The amount of shared code between the instances is varied between 0% (no code sharing) to 100% (all the code is shared between the two instances). A percentage of $x\%$ indicates that the first $\frac{x * code_size}{100}$ bytes of code are shared between the competing task instances.

% of shared code	1 interfering instance		2 interfering instances	
	no bypass L2 hit ratio	bypass L2 hit ratio	no bypass L2 hit ratio	bypass L2 hit ratio
0	0 %	7.34%	0 %	0%
10	1.63 %	9.79%	0 %	0%
20	17.95 %	29.37%	0 %	0%
30	35.08 %	39.16%	0 %	1.63%
40	36.71 %	39.16%	0 %	1.63%
50	36.71 %	39.16%	0 %	1.63%
60	41.61 %	54.66%	6.53 %	15.50%
70	44.06 %	57.93%	6.53 %	22.03%
80	71.79 %	74.24%	25.29 %	50.58%
90	84.03 %	84.03%	55.48 %	57.93%
100	84.03 %	84.03%	84.03 %	84.03%

Table 7: Estimated worst-case L2 hit ratio of *fdct* with code sharing (no bypass/bypass, 1-2 *fdct* interfering instances).

The table shows that code sharing tightens WCET estimation as compared to a system in which code sharing is not considered.

5.2.5 Analysis time

In term of computation time, the most time consuming situation of our experiments was described in paragraph 5.2.3. In this situation, up to nine tasks were analysed to determine the blocks to be bypassed for each of them, followed by a WCET estimation of the analysed task to account for inter-core interferences and bypassed cache blocks. The whole process always took less than 3 minutes on a Intel Core 2 Duo E6700 (2.66 GHz) with 2 GB of RAM.

6 Conclusions and future work

Estimating WCETs for multi-core platforms is very challenging because of the possible interferences between cores due to shared hardware resources such as shared caches. We have proposed in this paper a technique to reduce the amount of inter-task interferences, achieved by caching in the shared instruction caches(s) only blocks statically known as reused. Experimental results have shown that our approach allows to drastically reduce the WCET of tasks compared to methods which consider all inter-core conflicts and do not attempt to reduce their amount.

Our ongoing work is to extend our approach to data and unified caches. Another direction for future research is to use bypass for non-shared caches as well, in order to further reduce intra-task conflicts. Another direction would be to explore joint WCET estimation and scheduling to avoid some inter-task conflicts. Finally, a last direction would be to compare our proposal with cache locking and partitioning schemes, which avoid inter-task interference at the cost of a reduced cache volume per task/core.

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