Time-Space Opportunistic Routing in Wireless Ad Hoc Networks, Algorithms and Performance

François Baccelli, Bartlomiej Blaszczyszyn, Paul Muhlethaler

Time-Space Opportunistic Routing in Wireless Ad Hoc Networks Algorithms and Performance

François Baccelli[1], Bartłomiej Błaszczyszyn[2] and Paul Mühlethaler[3]

Abstract—**In classical routing strategies for wireless ad-hoc (mobile or mesh) networks packets are transmitted on a pre-defined route that is usually obtained by a shortest path routing protocol. In this paper we review some recent ideas concerning a new routing technique which is *opportunistic* in the sense that each packet at each hop on its (specific) route from an origin to a destination takes advantage of the actual pattern of nodes that captured its recent (re)transmission in order to choose the next relay. The paper focuses both on the distributed algorithms allowing such a routing technique to work and on the evaluation of the gain in performance it brings compared to classical mechanisms. On the algorithmic side, we show that it is possible to implement this opportunistic technique in such a way that the current transmitter of a given packet does not need to know its next relay a priori, but the nodes that capture this transmission (if any) perform a *self selection* procedure to chose the packet relay node and acknowledge the transmitter. We also show that this routing technique works well with various medium access protocols (such as Aloha, CSMA, TDMA). Finally, we show that the above relay self selection procedure can be optimized in the sense that it is the node that optimizes some given utility criterion (e.g. minimize the remaining distance to the final destination) which is chosen as the relay. The performance evaluation part is based on stochastic geometry and combines simulation a analytical models. The main result is that such opportunistic schemes very significantly outperform classical routing schemes when properly optimized and provided at least a small number of nodes in the network know their geographical positions exactly.**

I. Introduction

Routing is the process of selecting paths in a network along which to send network traffic. In packet switching networks routing directs packet forwarding — the transit of logically addressed packets from their source toward their ultimate destination through intermediate nodes. Prior to this, in such networks the nodes usually exchange control packets containing the network topology information that allow each node to find its next relay towards any destination in the connected part of the network. Once the paths (routes) are established in the network, another part of the data communication protocol, called *Medium Access Control (MAC) layer*, is responsible for moving data packets on their paths by organizing simultaneous transmissions in the network.

In wireless ad hoc networks routing is confronted with relatively frequent changes in the network topology. Indeed, in mobile ad hoc networks the nodes may go on and off, as well as change their geographical locations. Besides, the variability of radio channel conditions (so called fading) makes the network topology vary even in networks where the geographic pattern of nodes is relatively static (such as in mesh networks).

Many studies have been carried out to cope with this problem. Existing solutions are frequently subdivided into two classes: reactive protocols and proactive protocols. Proactive protocols are mostly based on existing routing protocols developed for wired networks. The emphasis in these protocols is usually put on reducing the control overhead as they have to be run more often to follow the varying network topology. Reactive protocols, on the other hand, use routes which are built on demand. A source node wishing to obtain a route to a destination node floods the network with a request packet. When the diffusion of this packet reaches the destination, the route can be established.

In this paper, which surveys and complements two recent conference papers [1], [2] of the authors, we consider another class of routing strategies where the relay can be defined at each hop of each packet, depending on the local configuration of simultaneous transmitters. In contrast to wired networks, this configuration essentially determines the feasibility of transmissions

[1]INRIA/ENS, 45 rue d'Ulm, 75005 Paris FRANCE; Francois.Baccelli@ens.fr,

[2]INRIA/ENS and Math. Inst., Univ. of Wrocław, 45 rue d'Ulm, 75005 Paris FRANCE; Bartek.Blaszczyszyn@ens.fr,

[3]INRIA, Le Chesnay, FRANCE; Paul.Muhlethaler@inria.fr

on individual links in wireless networks. This strategy, which we call *opportunistic routing*, and which merges the functionality of routing and MAC layer, has already been shown beneficial in the sense that it usually offers a smaller delay to carry a packet from origin to destination compared to classical routing schemes.

In this paper we describe also a very efficient way to implement opportunistic routing utilizing a *relay self selection technique*. In this procedure, the current emitter of a given packet does not need to know its next relay a priori, but the nodes that capture this transmission (if any) perform a self selection to chose the unique packet relay node and acknowledge the emitter. This technique will be shown compatible with various MAC protocols implemented in wireless networks as e.g. CSMA or Aloha.

One of the goals of this papers is also to evaluate the performance of opportunistic routing. For this, we introduce a realistic model to carry out simulations which allow for an extensive comparison of shortest path and opportunistic routing. Our numerical results reveal some interesting properties related to the jointly optimal tuning of the Aloha MAC and opportunistic routing.

Last but not least, we propose a mathematical framework based on the theory of *stochastic geometry* that allows us to confirm and further study the properties of the opportunistic routing revealed by simulations. Stochastic geometry, which is now a rich branch of applied probability intrinsically related to the theory of point processes, allows one to study random phenomena on the plane or in higher dimension. When applied to communication networks, it provides a natural way of defining and computing macroscopic properties of such networks, by some averaging over all potential geometrical patterns for the nodes, in the same way as queuing theory provides averaged response times or congestion over all potential arrival patterns within a given parametric class. In the point-to-point routing case, the main geometric objects are the (long) paths from a given source node to a destination node, where the relay nodes are picked form some realization of a homogeneous Poisson point process of the plane.

The paper is organized as follows. Section II reviews the existing routing mechanisms ranging from conventional routing algorithms to more recent schemes such geographic routing and opportunistic routing. Section III describes the optimized self selection scheme. This self selection which uses signaling bursts and short slots of carrier sensing can be seen as an improved CSMA scheme. Section IV describes the model for the performance evaluation of opportunistic routing. This model is used for simulations as well as for the mathematical analysis. Section V presents the main observations obtained by simulation. The simulations are carried out both with Aloha and CSMA. Section VI provides a mathematical framework for the analysis of opportunistic routing. This framework allows one to better understand a few observations obtained in Section V.

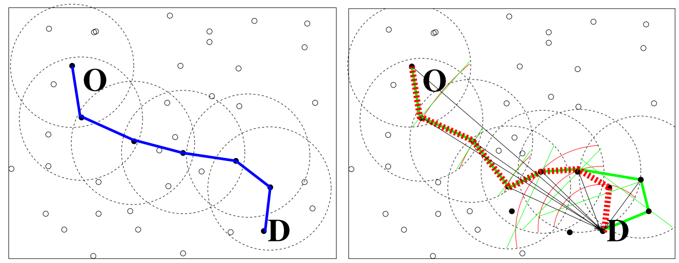

Fig. 1. **Left:** Shortest path (smallest number of hops) from O to D, with neighborhoods defined by discs of fixed radius (maximum transmission range). **Right:** Local greedy geographical routing maximizing the progression towards the destination (the abscissa in the direction towards; green solid line) or minimizing the remaining distance to destination (dashed red line) with the same transmission ranges; these two geometric criteria may give different relays close to the destination.

II. From Shortest-Path to Opportunistic Routing for Wireless Networks — State of the Art

Routing protocols are distributed algorithms that find routes for all pairs of origin and destination nodes (O-D pairs). Usually in a multi-hop network, once a route has been found for an O-D pair, all the packets of this O-D pair follow this route as long as the network topology remains unchanged.

A. Conventional proactive routing

In proactive protocols such as OLSR [3] the computation of routes is based on the exchange of control packets sent by the routing protocol. Using the topology information carried in the control packets, each node can find its next relay towards any destination in the (connected part of the) network. The most prominent algorithm that builds the shortest routes (with the smallest number of hops) is Dijkstra's algorithm [4]. Figure 1 (Left) depicts the shortest path from O to D assuming that the neighborhood of a node is identified via some maximum transmission range parameter: neighbors of a node are all the nodes at a distance smaller than this parameter.

An important problem in conventional proactive routing is that the convergence time of a Dijkstra-like algorithm for finding routes, as well as the routing state of each node (the next relay for any destination in the network), increases considerably as soon as the network has a large number of nodes.

B. Reactive routing

One way to reduce the routing state of nodes is to build routes on demand as in AODV [5]: a source node wishing to obtain a route to a destination node floods the network with a request packet. When the diffusion of this request packet reaches the destination, the backtracking of its tree allows the required route to be established. However this solution does not essentially reduce the complexity of the algorithm, which has to be run each time a source is looking for a destination.

C. Local geographic routing

Coping with scalability problems has been the primary goal of geographic routing [6]. A reduction in complexity, however, comes at the cost of knowing the positions of the nodes that are used to determine the routes to the destinations. More precisely, in geographic routing, instead of running an algorithm to find some globally optimal routes (e.g. the shortest ones) over the whole network, the successive hops of a path are constructed incrementally making "local", "greedy" choices of the next relays according to the geographical locations of the neighbouring nodes. For instance, Takagi and Kleinrock in 1984 [7] proposed to choose the next relay in such a way that it maximizes the (geometric) progression towards the destination: the node with the largest abscissa towards the destination is chosen. Alternatively, one can minimize the remaining distance to destination, and the neighboring node that is closest to the destination serves as the next relay; see [8], [9]. Figure 1 (Right) depicts the paths produced by the local greedy routing with these two geometric criteria.

In all the routing schemes that we describe above, if a route is established between an origin and a destination (proactively or on demand, via a global or a greedy search algorithm) then all the packets of this given O-D pair flow are sent through the same relays. This task is carried out at the MAC layer.

D. Opportunistic routing

Reactive and local geographic routing has paved the way for a new type of routing technique in which the routes are not constructed proactively in the network and where the relays of a given O-D flow are not fixed in advance. In this technique, called opportunistic routing, the relays are chosen dynamically at each hop of each packet, among the nodes which have received the packet transmission. This choice can be also optimized by taking the geographical locations of receivers into account (see e.g. [10], [11], [12], [13], cf. Figure 2). It has been shown that this strategy, which involves both

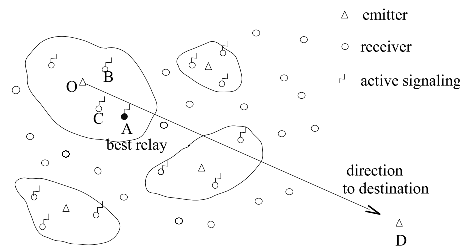

Fig. 2. Opportunistic routing. Clouds around each transmitter depict nodes that capture (receive correctly) the packet. For the top-left transmission we show the relay that maximizes progress to the destination, which will hence be selected as the next relay for this packet.

the routing and the MAC layer of the network is able to reduce the mean time required to carry a packet from the origin to the destination compared to the shortest path routing technique; see [6], [1]). Note that this metric is more fundamental than that of the number of hops in the route (optimized by the shortest path route), which does not include the time which is wasted in unsuccessful attempts to make a particular hop.

E. Performance comparison

[6], [1] show that opportunistic routing can reduce the mean delay required to carry a packet from the origin to the destination compared to shortest path routing. To address this question, as in [1], we use a Signal to Interference Ratio criterion for successful packet reception. This model is justified by many used modulation techniques and has an information theoretic basis. We also assume that the locations of network nodes are the points of some homogeneous Poisson point process. We carry out simulations to compare opportunistic routing with shortest path routing, both combined with Aloha or CSMA.

F. Implementation of opportunistic routing

Opportunistic routing does however come with several technical difficulties which are discussed below.

1) Relay self selection: The major difficulty is how to let the transmitter of a given packet know about its current receivers and chose an optimal one as its relay. This problem can be solved using a *relay self selection* technique. In this case, the transmitter of the packet does not know its next relay a priori, but the nodes that capture this transmission (if any) perform a self selection technique to chose the unique packet relay node and acknowledge the transmitter. To the best of the authors' knowledge, the idea of self selection of relays in opportunistic routing was first presented in [11] and [12]. The contribution presented in [13] also uses this idea. The relay self selection technique proposed in the present article has already been described in [14] in the section on implementation issues, but the primary focus of [14] was the optimization of the slotted Aloha MAC in the context of opportunistic routing. Our relay self selection procedure is optimized in the sense that it is the node that optimizes some given geometric utility criterion (e.g. minimize the remaining distance to the destination) which is chosen as the relay.

2) MAC and routing interplay: Another difficulty involved in opportunistic routing consists in merging the functions of two, traditionally separated, network layers. In particular, we have to know whether this technique can be used with various existing MAC solutions. The techniques presented in [11] and [13] assume a IEEE 802.11 type MAC where the acknowledgment scheme is modified to allow for the relay selection. On the other hand, [12], [14] assume slotted Aloha. In the present article we show how the relay self selection scheme can be used with various MAC techniques: these schemes may be controlled access schemes such as Time Division Multiple Access (TDMA) or random access schemes such as Aloha or CSMA.

3) Node positioning: As already stated, geographic routing, which is used in our self selection procedure, requires knowledge of the nodes geographic positions. Nevertheless, we will show that it is sufficient that a small number of nodes in the network know their positions exactly, e.g. using GPS, and provide this information to the remaining nodes, for the proposed technique to work well and outperform conventional routing techniques.

III. The optimized relay self selection scheme via signaling bursts

In opportunistic routing with relay self selection the transmitter of the packet does not know its next relay a priori, but there is a self selection of this relay among the nodes that capture this transmission. In wireless communications a simple way of electing a winner and letting it transmit is to use a backoff mechanism. Suppose that the receivers of the tagged packet pick independently random times before trying to forward it and that the receiver with the smallest delay initiates the transmission. Other packet holders hearing this transmission resign and discard the packet. Implementing this mechanism would lead to a random choice of one of the nodes among the current receivers (holders) of the packet as its relay. Of course it is natural to prefer a self selection mechanism that elects the relay in some locally optimal manner; e.g. the one that maximizes the packet's progression towards the destination or minimizes the remaining distance to the destination.

A. Preferential backoff

More generally, let us assume that each receiver can objectively evaluate its rank on some universal scale. [3] The problem is thus the following: how can we select the packet receiver with the highest rank through a distributed algorithm. The requirement is that when this algorithm runs on a node, it only knows the rank of this node. One solution is to assign backoff times according to the node's rank: the higher the rank, the shorter the delay. This would make the optimal receiver the first node that starts forwarding the packet. However, a reasonable self selection mechanism must prevent all the other nodes that participate in the selection process from relaying it. This requires that the retransmission of the packet by the best relay be heard by the other potential relays — a condition that cannot be completely guaranteed since the potential relays may be far from each other. Also the radio conditions (including interfering signals) may change when the backoff time has elapsed. In addition to this problem, it is unclear whether the linear selection of a backoff technique would be sufficiently powerful to discriminate between the potential relays. In particular, if the network is dense, one may have to foresee a large backoff window in order to accommodate a large number of potential relays. Finally, the self selection mechanism must acknowledge the previous transmitter of the packet.

B. The signaling bursts

In order to cope with the above requirements we propose a more powerful technique to elect a winner, using *signaling bursts with logarithmic coding of the rank* [15]. This technique, which was first introduced for the HiPERLAN type 1 standard [16], assumes that after the original packet transmission and before its relaying, in the so-called *active signaling phase*, each node that has captured the packet transmits an acknowledgment made up of a short signaling burst. This acknowledgment has two goals: first it allows the best relay to be selected and second it allows the sender to know that the packet has been received and will be relayed by some node.

The burst is composed of a sequence of intervals of the same length in which a given receiver can either transmit

[3] For example, to optimize the progression towards the destination, the rank of a receiver can be taken equal to the abscissa of its location in the coordinates system originated at the transmitter, with the x axis pointing to the destination. In the other geographic criterion evoked in Section II-C, which aims at minimizing the remaining distance to the destination, the rank could be minus the distance to the destination node.

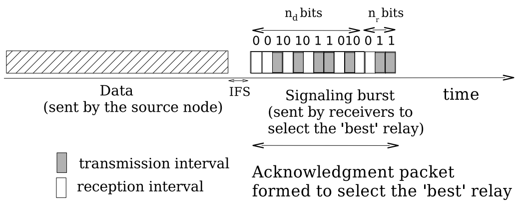

Fig. 3. Structure of the acknowledgment packet to select the 'best' relay towards the destination.

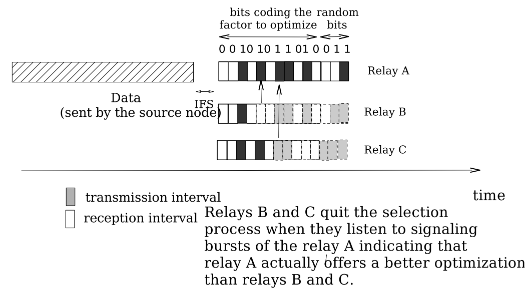

Fig. 4. Example of the 'best' relay using the acknowledgment scheme

or listen (see Figure 3). In order to describe the structure of the burst, let us represent it by the binary sequence, where 0 denotes a listening interval and 1 denotes a transmission interval. Each node participating in the self selection process computes this binary sequence (and thus determines the form of its burst) as follows.

- The first n_d bits encode the rank of the node in base 2 (we recall that the self selection should designate one node with the highest rank as the relay).
- Optionally one may implement the next n_r bits selected at random to discriminate between nodes having (almost) the same rank.
- Finally the last bit is always set to 1. This bit, as we will see, provides the acknowledgment of the successful self selection of the relay.

After having computed the form of their bursts, the nodes start transmitting them simultaneously applying the following rule: *if a given node detects a signal from another node during any of its listening intervals, it quits the selection process*; i.e. it stops transmitting during the entire remaining part of the active signaling phase (cf. Figure 4).

It can easily be checked that if all the nodes participating in the self selection process remain in their communication regions, then at the end of the first n_d bits of the burst, the only nodes (if any) which stay in the competition will have equal rank, the highest among all the participating nodes. This stems from the construction of the signaling bursts: the detection of a transmission during a listening interval implies that a better relay is taking part in the competition.

An example of a relay selection operation is shown in Figure 4, which corresponds to the relay selection around O as shown in Figure 2. Three nodes have captured the transmission sent by O: nodes A, B and C. Node A has the highest rank and thus is selected as the best relay using the active signaling scheme.

The next n_r bits of the burst randomly select one of the nodes with the highest rank, if there are more than one. Finally, this unique winner (if any) of the self selection process will transmit at the last interval of the burst. Thus, if the previous transmitter (the node that sent the packet for which the relay is to be selected) cannot detect a signal in this interval it infers that its packet has not been received or that the selection process between potential relays has failed. In this situation, it has to retransmit the packet.

C. Some implementation issues

Let us now discuss the "real" circumstances, in which the above "ideal" self selection process may fail. Probably the most important of these is interference from other transmitters in the network which are not participating in the self selection process. To cope with this problem, a spreading technique can be used: a unique (CDMA-like) binary code of much higher frequency can be provided in the previous transmission of the packet, to be used by all the receivers during the active signaling burst. All the nodes participating in this self-selection will modulate their bursts (binary multiply) before transmitting. This will protect the communication in this active signaling burst from other ongoing communications. Note that we *do not* suggest using this code for the subsequent data packet retransmission by the elected best relay; the given MAC used will take care of it.

Another problem is how to determine n_d in order for the signaling burst to be able to correctly discriminate between nodes. Let us assume, for example, that the rank is some geometric distance; e.g. progression. Then $n_d = 13$ will allow distances up to about 8 km to be coded with the precision of 1m. Whether this is a sufficient tuning depends on the maximal transmission range in the given network.

D. Relay self selection and multiple access schemes

The relay self selection technique that we have described above can operate with various access schemes: both with controlled schemes (in which access is granted by the protocol in such a way that there are no collisions) and with random access schemes, since the protocol incorporates an acknowledgment mechanism. We discuss some possible choices below.

1) TDMA: In this protocol time slots are assigned to network nodes in such a manner that there are no collisions. It is, however, difficult to use such a scheme in ad hoc networks since attributing time slots in these dynamic networks is extremely complex.

2) CSMA/CA: These protocols have been widely used in wireless networks. In these random access protocols, the channel is sensed prior to any transmission to be sure that it is not used. Above a given threshold, called the carrier sense threshold, the channel is assumed to be occupied whereas below this threshold, the channel is assumed to be free. When a collision occurs, a simple backoff technique is used to schedule the re-transmission of the packet. These CSMA/CA protocols form the basis of the IEEE 802.11 standard, which, however, adds an additional MAC acknowledgment sent just after the end of the received packet. The relay self selection technique protocol proposed in this article can operate with the CSMA/CA technique of the IEEE 802.11 standard with this acknowledgment modified (replaced by) the active signaling phase described in the previous section.

3) MACs with RTS/CTS: Our relay self selection technique could also be adapted to other protocols such as MACAW [17], MACA/PR [18], DBTMA [19] etc., which use a Request To Send/Clear To Send (RTS/CTS) exchange before the actual transmission of the data. In these protocols, the RTS packet should be sent to all neighbors and the CTS packet should encompass the active signaling phase. (Thus, in this case, the relay self selection will take place *before* the reception of the packet in question.)

4) Aloha: Our relay self selection can also work with non-slotted Aloha. However, it would be more beneficial in the case of a slotted Aloha protocol in which, at each time slot, each node with packets to be sent tosses a coin with a bias p (for heads) and accesses the channel when getting heads. In fact, the use of a slotted structure allows the throughput of the Aloha protocol to be improved by a factor of 2 (see Chapter 4 of [20]). In addition, the slotted structure also improves the efficiency of the relay self selection technique. The combination of the slotted Aloha with the relay self selection protocol has been analyzed in [12], [14], [1] and many very interesting properties have been shown, especially concerning network scaling.

IV. Performance evaluation model

We now describe our model for the performance evaluation of the opportunistic routing. This model will be used for simulations as well as for the mathematical analysis.

A. Network architecture

We consider networks formed of nodes randomly distributed on the plane. Specifically, nodes are assumed to be sampled according to some homogeneous Poisson point process with intensity λ. In our simulations, we use a finite planar network on the square $[0, 1000]$ m $\times[0, 1000]$ m. The locations of the nodes do not change with time slots, but mobility is taken into account in the radio channel model (see model M3 in Section IV-C below). In our simulations, the default option is $\lambda = 10^{-3}$ nodes /m^2.

B. O-D pairs and background traffic

In the simulations O-D pairs are selected on opposite parts of the network, as shown in Figure 5, with a distance of about 1130 m from each other. This represents a moderate distance (approx. 9 hops away for a transmission range of 140 m.).

For a fixed O-D pair, and for a given set of network nodes sampled according to a Poisson point process, a basic simulation experiment allows one to get a sample of the end-to-end transmission of one packet of the tagged O-D pair flow, assuming some given physical (radio), MAC and routing model that will be described below (cf Sections IV-C–IV-F). In this end-to-end transmission we track the route selected for this packet, the transmission attempts at each relay node and the end-to-end delay. For the sake of simplicity, in the simulations:

- the tagged packets of the O-D pair are treated as higher priority packets at each node. We should of course add a queueing delay to account for the competition with cross traffic, but under natural homogeneous traffic and stability assumptions, this would amount to adding a delay with the same law at each node, and should hence not change the main conclusions of the comparison study.
- all nodes are assumed to always have packets to transmit, and they always transmit whenever authorized by the MAC; these transmissions allow us to take the background traffic into account through the interference they create at each time slot, and in turn, determine which nodes capture the tagged packet transmission.

We repeat a large number of such basic experiments to evaluate means. We consider both packets sent from O to D for the same and for different network samples.

Note that even if we track only the packets of the tagged O-D pair, the cross-traffic is taken into account via the interference experienced by the tagged packets due to other transmitters in each time slot. We still assume that each node always has a packet to transmit and that interference plays an important role in determining a successful reception.

C. Radio channel models

The power used by all the transmitters is assumed to be equal to some constant $S = 1$. We use the following simplified power attenuation function $l(r) = (Ar)^{-\beta}$ for some constants $A > 0$ and $\beta > 2$, which gives the fraction of the emitted power that is received at distance r from the transmitter. Even if this function has a pole at the origin, it is reasonable and commonly used if the density λ of points is not too large or, equivalently, the points are not too close to each other. In the simulations, we take a path-loss exponent of $\beta = 3$.

In certain models, in addition to the above attenuation function, we assume that the received powers are multiplicatively modified by some location and possibly time dependent random path-loss factors. The following 3 scenarios will be considered.

(M1): *Path loss factors are constant* equal to 1. This assumption might correspond to a very slow channel fading and/or coding which allows for empirical averaging over fading effects during packet transmission (e.g. based on symbol interleaving).

(M2): *Path loss factors are position dependent*; they are sampled independently for each transmitter-receiver pair and stay constant for all time slots of the simulation. This corresponds to a slow fading or shadowing effect.

(M3): *Path loss factors are position and time dependent*; they are sampled independently for each time slot and each transmitter-receiver pair. This might correspond to user mobility and will be the default option in the simulations below.

For models M2 and M3, we assume a *Rayleigh fading*, where path-loss factors are exponential random variables with parameter 1 (see e.g. [21, p. 50 and 501]). The model also includes a thermal noise independent of everything else with power denoted by W. In the simulations, the default option is $W = 0$.

D. Capture model

Let us suppose that some station transmits during a given time slot. We assume that it can successfully transmit to a given receiver of this time slot if the SINR ratio at this receiver is not less than some fixed threshold T. By the SINR we mean the ratio between the power received from the given transmitter (attenuated and modified by the path-loss factor) and the sum of powers received from all other transmitters of the given time slot, including the power W of the thermal noise; see (6.1) for the corresponding formula. In the simulations, the default value is $T = 10$.

E. MAC models

We will only consider slotted MAC scenarios; i.e. the time is divided into equal slots. In each slot, the first part is dedicated to the transmission of the data sent by the initial source or repeated by the intermediate nodes, and the second part of the slot is dedicated to the acknowledgment packet which is sent by potential relays to elect the best relay and to acknowledge the reception of the packet; cf. Figure 3. However, only the reception of the data part in each time slot will be simulated and the nodes having successfully received the data will be identified (cf. Section IV-D). In this study we assume that the self-selection procedure perfectly designates the best (according to a given criterion) relay node among them.

1) Aloha: The first simulations presented in this paper assume a slotted Aloha. In this model, at each time slot each node tosses a coin independently of everything else. The nodes tossing heads are the transmitters of this time slot; the other nodes are the receivers. This model will also be the basis of the mathematical analysis based on stochastic geometry. The main parameter of this model is the probability of tossing heads, denoted by p, which is referred to as the medium access probability. Nodes which are not authorized to transmit at a given time slot are considered as potential receivers at this time slot.

2) CSMA/CA: Next, we present simulations using a CMSA/CA protocol. To simplify the simulation, we simulate a slotted CSMA/CA system meaning that at each time slot, transmitting nodes are selected among the nodes with a pending packet according to the CSMA/CA rule. More precisely, in each time slot the nodes with a pending packet try to access the channel in a random order and succeed only if they satisfy the CSMA/CA rule; i.e. if the detected signal is below the carrier sense threshold [4]. This carrier sense threshold is thus the key parameter of the CSMA/CA simulation. Nodes which are not authorized to transmit at a given time slot are considered as potential receivers at this time slot.

In this simple model we also neglect the collision window in which transmitters can start their transmissions without sensing each other.

[4] In [22] it is shown that this model is a good approximation of a real CSMA when the packets are of the same length and if we also consider the overhead induced by the backoff algorithm.

F. Routing

We now describe two routing strategies: conventional shortest-path and opportunistic routing where the latter aims at minimizing the remaining distance to the destination at each hop.

1) Shortest path routing: By this we understand routing along the routes with the least number of hops as found by Dijkstra's algorithm [4]. For each given network, this amounts to finding paths of minimal weight between a given origin O and destination D (cf. Section IV-B) in a graph with edges between all pairs of nodes and where the weight of the edge between nodes x and y is 1 if $|x - y| \leq R$ and ∞ otherwise, where R is the maximum transmission range and is considered as a parameter of this routing protocol. This shortest path is used to route all packets of this O-D pair. A given MAC scheme (Aloha or CSMA/CA) is then used to let the tagged packets progress from O to D along this path.

2) Opportunistic radial routing: It should be recalled that in opportunistic routing, the next hop on the route to the destination is not known a priori and the routing algorithm should be described together with the MAC. Consider a tagged packet of the O-D pair flow located at some current node A.

`Until` A `is the destination D do:`

1. `Until` A `is selected by the MAC to transmit, end-to-end delay++;`
2. `When` A `is selected by the MAC to transmit do:`
    - 2.1. `All the nodes which are selected by the MAC to transmit are transmitters, the remaining nodes are receivers;`
    - 2.2. `The set of transmitters together with the fading variables at that time slot determine the interference everywhere at this time slot;`
    - 2.3. `The set of receivers` $\mathcal{S}$ `which satisfy the SINR capture condition at this time slot receive the tagged packet successfully;`
    - 2.4. `Among the nodes of` $\mathcal{S} \cup \{A\}$`, the nearest to the destination, say` B`, is the next relay;`
    - 2.5. `The other nodes of` $\mathcal{S}$ `discard the tagged packet;`
    - 2.6. `end-to-end delay++;`
    - 2.7. `if` $A \neq B$ `then number-of-hops++;`
3. $A := B$.

A more formal description of this routing protocol, as well as a proof of its convergence (the fact that it delivers the packet to the destination in a finite number of time slots) is presented in Section VI under the Aloha MAC assumption. Figure 5 gives three examples of radial paths obtained by simulation for different radio channel models. The path that is the closest to the segment joining the origin to the destination node is obtained with a shortest path routing algorithm. The second path moving farther away from this segment corresponds to the time-space opportunistic radial routing strategy under the M1 model. The third path, which allows one to search for relays very far from the transmitter corresponds to time-space opportunistic radial routing in the presence of fading (here under the M3 assumptions).

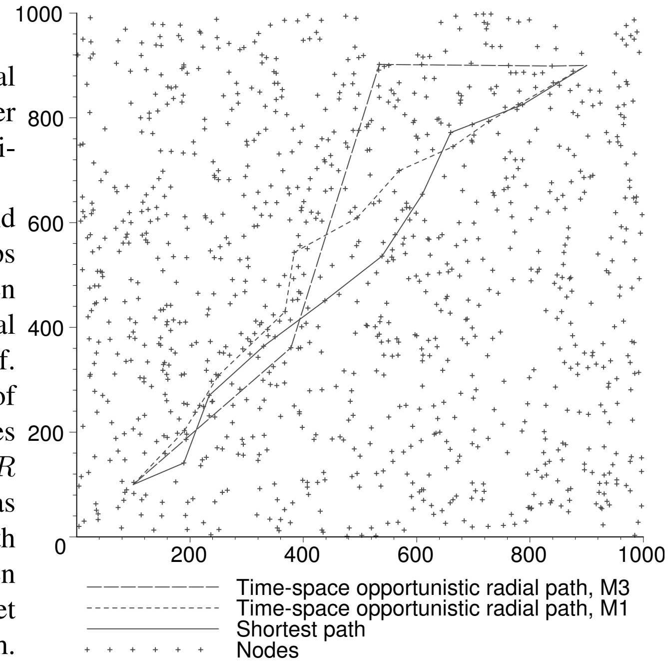

Fig. 5. Samples of routing paths with opportunistic radial routing (with and without fading) and with a shortest path algorithm (for Aloha MAC).

G. Nodes positioning

1) Perfect positioning: Note that opportunistic routing requires that the nodes know their geographical positions. In our simulations we will assume first that all the nodes have perfect knowledge of their positions. The nodes can acquire such knowledge using, e.g. GPS. Under this assumption we will compare opportunistic routing with optimized self selection of relays to conventional routing based on the shortest path algorithm.

2) A simple localization algorithm: As the assumption that each network node knows its exact position may be considered too demanding in practice, we will also study the performance of our relay self selection algorithm with a weaker assumption, namely, that only a fraction of the network nodes have perfect knowledge

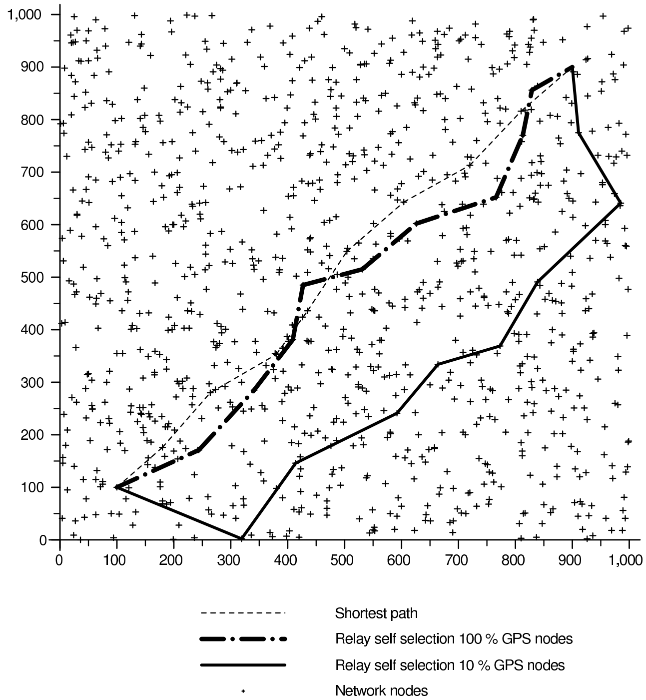

Fig. 6. Samples of routing paths with shortest path and opportunistic radial routing obtained under different assumptions for node positioning assumptions (for slotted Aloha MAC).

of their positions (e.g. are equipped with GPS). The other nodes will use these so-called 'auto-localized' nodes as anchors to estimate their own positions. A simple localization algorithm can approximate the position of a node as the barycenter of its auto-localized neighbors (see [23]).

Three examples of paths obtained by simulation are given in Figure 6. The path which is the closest to the segment joining the origin to the destination node is obtained with a shortest path routing algorithm combined with the slotted Aloha as the access scheme. The second path moving farther away from this segment corresponds to the relay self selection mechanism using the slotted Aloha. In this path, all the network nodes have perfect knowledge of their positions. The third path, which is a path on the right of the direct line between the source node and the destination node, corresponds to the relay self selection mechanism using the slotted Aloha but with only 10% of the network nodes having perfect knowledge of their position. The other nodes compute their positions using the simple localization algorithm; see [23]. In this case, at first glance one might assume that the relay self selection would not perform as well as shortest path routing. However, as we will see, this is not the case. We have to bear in mind that the important metric is the end-to-end delay between O and D and not the number of hops.

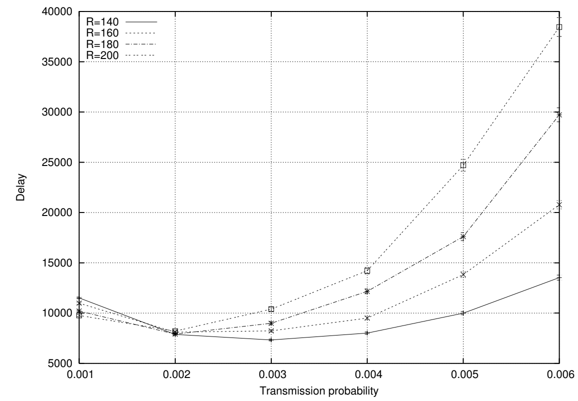

Fig. 7. Shortest path routing algorithm: end-to-end delay versus p for various transmission ranges.

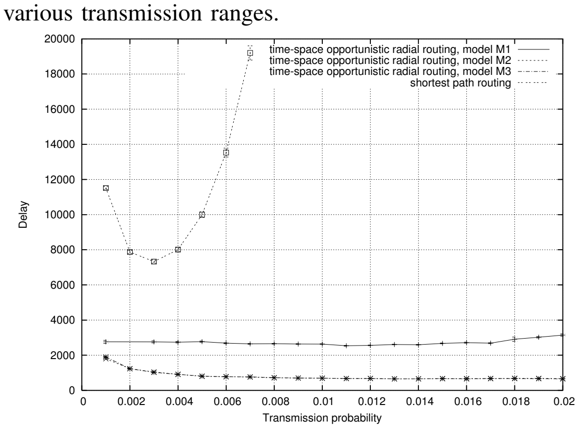

Fig. 8. Comparison between routing strategies: end-to-end delay versus transmission probability p.

H. Performance metrics

For a given tagged packet of the O-D pair and a given network of nodes we consider:

- the *end-to-end delay*, defined as the number of time slots it takes for this packet to go from O to D,
- the *number of hops* made by this packet from O to D,
- the average *local delay* (delay per hop) defined as the ratio end-to-end-delay/number of hops.

I. Averaging and Confidence Intervals

In order to calculate the means of the above performance characteristics, we average over 80 different networks connecting a given O-D pair and for each network we average over 5 packets for the O-D pair. The results are always presented with confidence intervals corresponding to a confidence level of 95%. Note that some of these confidence intervals are small and can only be seen when zooming in on the corresponding plots.

V. Simulation Results

A. Aloha

1) Mean End-to-End Delay: For shortest path routing, the maximum transmission range parameter R (recall from Section IV-F1 that this is a parameter of Dijkstra's algorithm) has first been optimized in order to make the comparison fair. The end-to-end delays for various values of R and of the transmission probability p are presented in Figure 7. We see that the best delay is obtained with $p = 0.003$ and with $R = 140$ m. This value, which is our default value for shortest path routing in what follows, is actually the smallest value of the transmission range which connects the network with high probability in this case.

In Figure 8, we compare the shortest path algorithm and our time-space opportunistic routing. In this figure we give the mean end-to-end delay as a function of the transmission probability p under different fading scenarios. Here is the main observation of the paper.

Observation 5.1: The algorithm based on time-space diversity significantly outperforms the conventional shortest path routing strategy: the average delay of a packet is at least two and a half times smaller for this strategy than for Dijkstra's algorithm.

We also see that the discrepancy between the conventional shortest path routing strategy and time-space opportunistic routing becomes much larger for a large p. Moreover, the performance of opportunistic routing is much less sensitive to a suboptimal choice of the parameter p.

Figure 9, which refines Figure 8 for opportunistic routing strategies, shows that:

Observation 5.2: Letting time-space opportunistic routing take advantage of the varying fading (e.g. due to mobility) is beneficial in terms of mean end-to-end delays.

The analysis of the simulation results shows that opportunistic routing in the presence of fading (M2 and M3) performs roughly four times better in terms of end-to-end delay than opportunistic routing in the absence of fading (M1), see Figure 9. Opportunistic routing with slow fading (M2) or with fast fading (M3) offers similar performance. Only very long simulations (not presented here) show that opportunistic routing in M3 leads to slightly shorter delays than in M2.

Here is the second most important observation of this paper. Figure 10, which plots the mean end-to-end delay for the M3 time-space opportunistic routing, shows that:

Observation 5.3: There is an optimal value of p that minimizes the mean end-to-end delay of the time-space opportunistic routing algorithm, and that this optimal value p^* seems to be the same for all values of the node density λ.

Similar observations (not presented here) hold for the M1–M2 models described in Section IV-C.

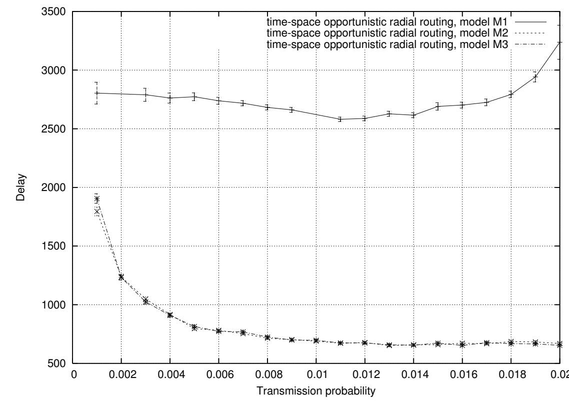

Fig. 9. Effect of fading on time-space opportunistic radial routing: end-to-end delay versus transmission probability p.

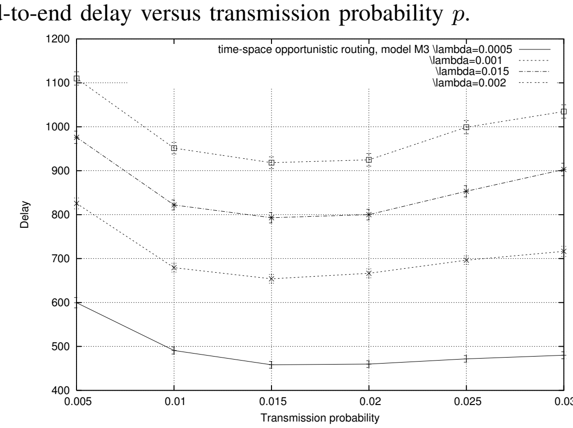

Fig. 10. Time-space opportunistic radial routing; end-to-end delay versus p for various values of the node density.

In Figure 11 we see that :

Observation 5.4: The mean end-to-end delay of the time-space opportunistic routing algorithm is of the order of $\sqrt{\lambda}$ where λ is the node density.

The matching is excellent for opportunistic routing in M3 (and in M2 although it is not shown in Figure 11), for opportunistic routing in M1 there is rough matching. The discrepancy seen for small λ may be caused by side effects.

2) Mean Number of Hops, Mean Local Delays: Figure 12 gives the average number of hops to reach the destination for the two routing strategies with p varying from 0.001 to 0.02.

Observation 5.5: In the case without fading M1, for small values of p, the time-space opportunistic path is shorter (has a smaller mean number of hops) than the Dijkstra shortest path, whereas it is longer for large values of p. In the presence of fading, time-space opportunistic routing offers shorter paths than Dijkstra type routing for $p \leq 0.014$ and slightly larger paths than Dijkstra type routing for $p > 0.014$.

We also observe that for time-space opportunistic routing, the mean number of hops to reach the destination increases with p. This can be easily understood since when p increases, the time-space diversity decreases and

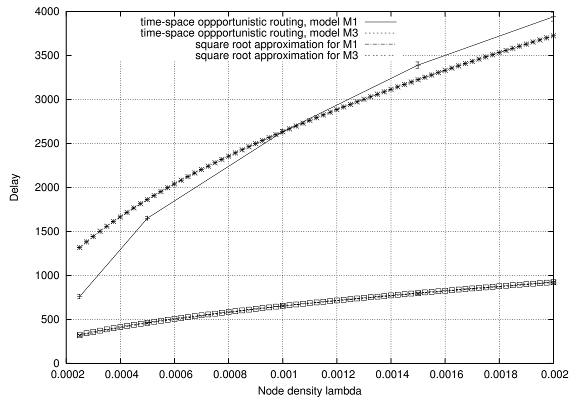

Fig. 11. End to end delay of time-space opportunistic radial routing algorithms (with and without fading) versus node density λ, comparison with $\sqrt{\lambda}$.

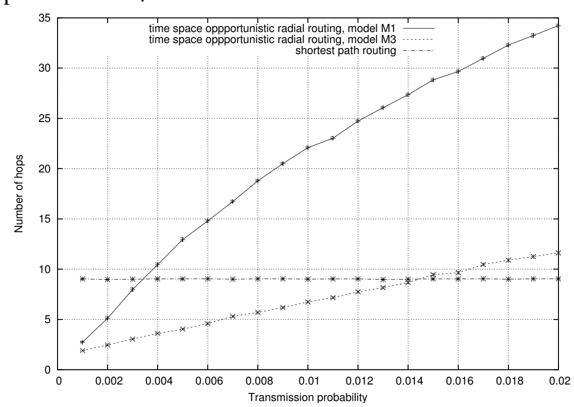

Fig. 12. Mean number of hops from origin to destination with time-space opportunistic radial routing (with and without fading) and with a shortest path routing algorithm.

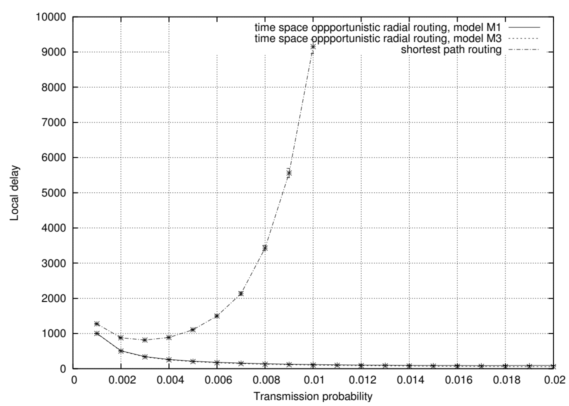

Fig. 13. Mean local delay for time-space opportunistic radial routing (with and without fading) and with a shortest path routing algorithm.

thus the number of hops to reach the destination tends to increase.

Figure 13 studies the mean local delay for the same three scenarios as above.

Observation 5.6: In time-space opportunistic routing, for each p, the mean delay per hop is much smaller than the delay per hop for Dijkstra's algorithm.

This explains why the average delay is smaller for time-space opportunistic routing than for Dijkstra's algorithm even if the number of hops may be larger.

% GPS	20%	10%	7%	5%	4%	3%	2%	1%
% Delay exc	7.7%	6.1%	5.8%	4.3%	4.8%	7.7%	13.8%	39%

Fig. 14. Percentage of packets with delay exceeded versus percentage of GPS nodes (for slotted Aloha)

3) Impact of imperfect node positioning: Figure 8 provides the mean end-to-end delay of the relay self selection algorithm when only 10% of the network nodes have GPS information. Figure 8 actually shows that the performance of the optimized relay self selection is not significantly affected by the lack of precision induced by the simple localization algorithm when p is less than 0.004. For higher values, using the localization algorithm leads to an increase in the end-to-end delay and to significant packet loss due to an excessive delay (5 % of packets with a delay exceeding 10.000). However, if we use the localization algorithm and if we maintain a small value of p, the end-to-end delay will be approximately 3000 whereas the mean delay is 2500 for the relay self selection with $p = 0.012$. Even with 10% of GPS nodes, the relay self selection mechanism significantly outperforms the shortest path routing scheme in terms of the end-to-end delay. Last but not least, we can also observe in Figure 8 that the tuning of the Aloha parameter p is much easier with relay self selection than with the shortest path algorithm.

In Figure 14, when $p = 0.01$, we present the percentage of packets end-to-end delay of which exceeds 10.000 slots for various percentages of nodes having GPS information. We see that when more than 3% of the nodes have GPS information, less than 10% of the packets have delays exceeding 10.000. The performance of the relay self selection mechanism seems to be quite insensitive to the percentage of nodes having GPS information and thus to a lack of precision in knowledge of the node positions.

4) Impact of the path-loss exponent: Figure 15 shows the gain in terms of mean end-to-end delay of the relay self selection algorithm over conventional shortest path routing, as a function of the path-loss exponent. In order to perform a fair comparison of both techniques, we chose the p which optimizes the mean end-to-end delay. We observe that this gain varies from 2.8 for $\beta = 3$ to nearly 4 for $\beta = 5$. In this study we have not considered the effect of fading. However it is easy to take such an effect into account with simple models which are not presented in this paper for reasons of space. In these fading-aware models another important additional gain (up to 4) can be obtained. Actually, the fading effect adds spatial diversity which is used by the relay self selection mechanism to improve its performance.

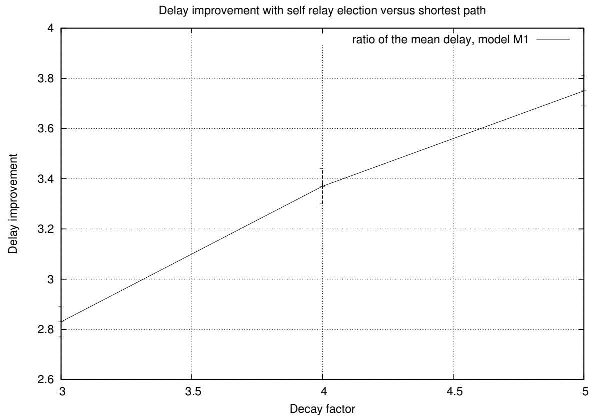

Fig. 15. Gain in average delay with relay self selection versus shortest path versus path loss exponent β (for slotted Aloha).

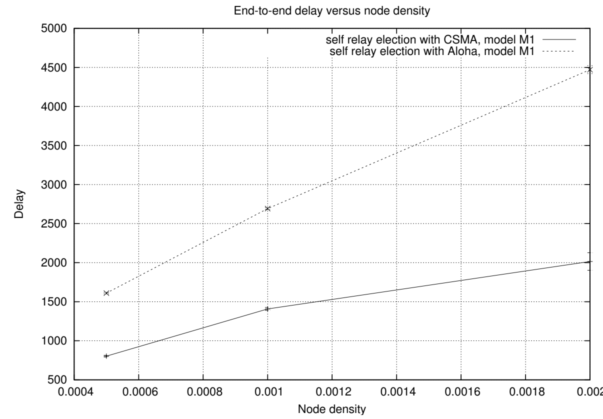

Fig. 17. Comparison of the mean end-to-end delay versus node density of the self selection mechanism used with a CSMA/CA protocol.

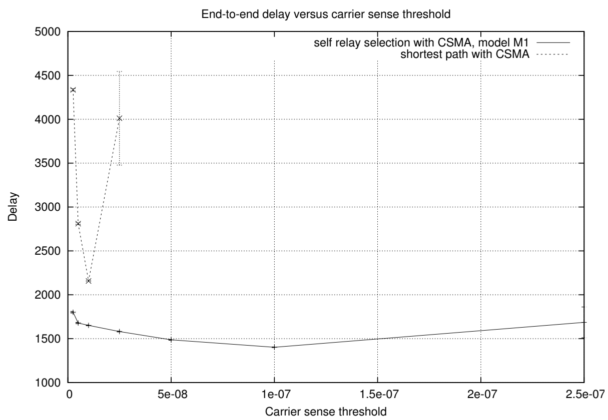

Fig. 16. Mean end-to-end delay versus the defer threshold of the CSMA access protocol used as access scheme, comparison between the relay self selection and the shortest path routing algorithms.

B. CSMA/CA

We compare the traditional routing strategy based on a shortest path Dijkstra algorithm and the routing with self relay selection described in Section III. Both routing schemes use the slotted CSMA protocol as the access scheme. We consider the end-to-end delay of these two schemes, and the results of these simulations are presented in Figure 16. It can be seen that, here again, the relay self selection mechanism significantly outperforms the shortest path algorithm. In terms of end-to-end access delay, the obtained gain is around 1.5.

As the Aloha case, we can observe that the tuning of the carrier sense threshold is much easier with the relay self selection than with the shortest path algorithm.

In Figure 17 we compare the end-to-end delay of the relay self selection mechanism used with Aloha and CSMA/CA for various values of the node density. Both protocols are optimized w.r.t. the transmission probability p for Aloha and w.r.t. the carrier sense threshold for CSMA. We note that CSMA/CA offers much smaller end-to-end delays, the gain is around a factor 0.5, nearly 0.45 for a node density of 0.002. The gain in performance of CSMA over Aloha is confirmed in Figure 18 where we can see that the actual mean number of hops to reach the destination is smaller for CSMA than for Aloha. Since CSMA offers a better exclusion area around the transmitter the packet can go farther towards the destination.

Nonetheless the improvement in performance of CSMA shown in Figures 17 and 18 comes at the price of an optimization w.r.t. the carrier sense threshold which is not independent of the node density. This is shown in Figure 19. The optimizations for $\lambda = 0.0005$, $\lambda = 0.001$ and $\lambda = 0.002$ lead to very different values of the carrier sense threshold. This is in contrast to the Observation 5.3, which says that the optimal Aloha MAP p does not depend on the density of nodes.

The last remark is in line with a remark made in [14] where we showed that CSMA with a fixed carrier sense threshold offers a maximum throughput $0(1)$. This throughput does not scale with λ whereas for Aloha scheme it scales as $0(\sqrt{\lambda})$; i.e. according to Gupta and Kumar's well known law [24].

VI. Mathematical Analysis

In this section we will study opportunistic routing under the Aloha MAC assumption using the theory of point processes. In particular we will show how to optimally tune the MAC parameters so as to minimize the average number of time slots required to carry a typical packet from origin to destination on long paths. We show that this optimization is independent of the network density.

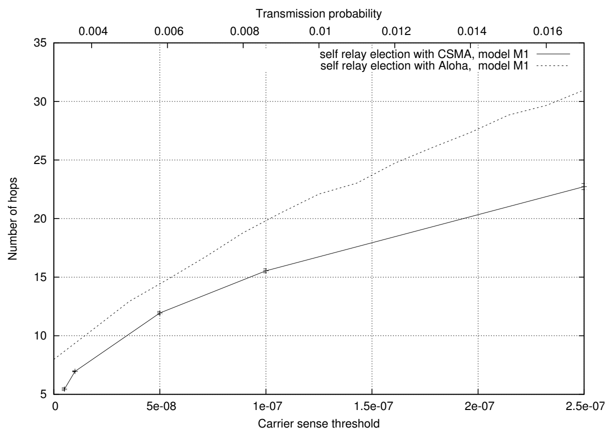

Fig. 18. Mean number of hops the self selection mechanism used with CSMA or Aloha versus carrier sense threshold (CSMA) or transmission probability (Aloha).

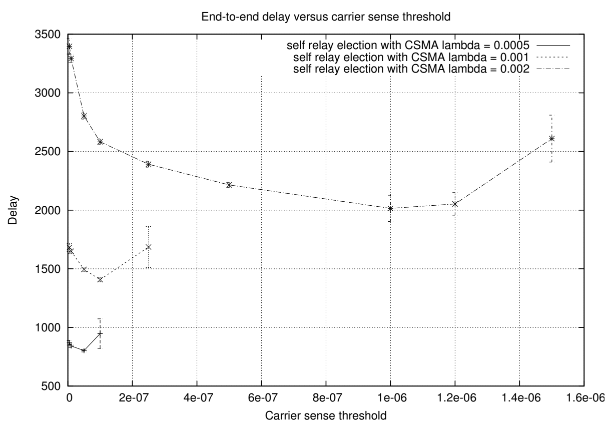

Fig. 19. Mean end-to-end delay of the self selection mechanism used with a CSMA/CA protocol versus the carrier sense threshold

A. *Minimizing End-to-end Delay for Long Paths*

The aim of this section is to explain Observation 5.3, namely that (and in what sense) there is an optimal value p^* that minimizes the mean end-to-end delay of the opportunistic routing for all values of the node density λ. In our explanation we will use some "fictitious" *directional routing* model which consists in a non-terminating do loop of the same nature as in opportunistic radial routing (see Section IV-F2) but with 2.4 replaced by:

2.4' `Among the nodes of` $\mathcal{S} \cup \{A\}$`, the one with the largest abscissa in the direction of the vector (O,D), is the next relay;`

Note that directional routing as defined above does not aim at delivering a packet to a particular destination. It corresponds rather to a situation where the destination is at infinity, in the same direction as from O to D[5] However, it is a suitable mathematical model for opportunistic radial packet forwarding between O-D pairs that are separated by a large distance. Indeed, when the remaining distance to the destination is large, then the optimal receiver for the radial path and that for the directional path (see steps `2.4` and `2.4'`) tend to coincide (cf. Figure 5 right).

A path can be seen as a sequence of *progress segments* ξ_k for the tagged packet, where the k-th segment is that connecting the location of the packet at the k-th time slot to its location at the $k+1$-st slot. The k-th segment is degenerate and reduces to a point if the node harboring this packet at slot k does not transmit at this time slot or transmits without capture by a node closer to the destination.

Let us define the *progress* P_k of a given progress segment ξ_k as the length of the projections of ξ_k on the O-D pair direction. Observation 5.3 could be explained by the following two properties.

1) *Radial and directional routing coincide far from the destination*: (cf. Figure 5 right).
2) For all n, the maximization w.r.t. p of the mean progress of directional routing in n time steps is invariant with respect to the intensity of the underlying Poisson point process.

In what follows we will present a mathematical framework in which these two properties can be formalized.

B. *The detailed model*

Let $\Phi = \{X_i\}_i$ be a homogeneous Poisson point process, with points $X_i \in \mathbb{R}^2$ representing the locations of nodes on the Euclidean plane. We denote by λ the intensity of Φ. We will consider two independent sequences $\{\mathbf{e}_i\}_i, \{\mathbf{F}_i\}_i$ of independent marks of the points of Φ.

For all i, let $\mathbf{e}_i = \{e_i^n\}_n$ be a sequence of independent and identically distributed (i.i.d.) Bernoulli random variables, where e_i^n represents the transmission indicator of node i at time n, or equivalently the fact that this node tosses heads at time n. We assume that $\mathbf{P}\{\, e = 1 \,\} = 1 - \mathbf{P}\{\, e = 0 \,\} = p$, where e is the generic transmission indicator.

[5] If one wants to consider the directional routing that delivers the packet to the real destination D, then the direction in step `2.4'` should be modified at each location of the packet in such a way that it always points towards D. Such a routing algorithm may however lead to some oscillations when the packet is close to the destination (cf. Figure 1), and seems to offer no advantage with respect to the radial algorithm. Thus we prefer to consider it only as a mathematical model with D at infinity.

For all i, let $\mathbf{F}_i = \{F_{i,j}^n\}_{j,n}$ be a family of random vectors, where $F_{i,j}^n$ represents the fading between nodes i and j at time n. We assume that for each n, the variables $\{F_{i,j}^n\}_{i,j}$ are i.i.d. with the same law as a generic variable denoted by F a general non-negative distribution with mean 1. For different n the variables $F_{i,j}^n$ need not be independent. We will consider the following 3 scenarios:

(M1) $F_{i,j}^n \equiv 1$ for all i, j, n,
(M2) $F_{i,j}^n = F_{i,j}^0$ for all i, j, n,
(M3) $F_{i,j}^n$ are i.i.d. for all i, j, n.

Let $W > 0$ be a given random variable representing the thermal noise which is assumed to be independent of Φ. Denote by $l(r) = (Ar)^{-\beta}$ the attenuation at distance r. Let S denote the (fixed) power used by transmitters.

Let $\Phi_1^n = \{X_i : e_i^n = 1\}$ denote the point process of transmitters at slot n and $\Phi_0^n = \{X_i : e_i^n = 0\}$ that of receivers. Suppose that a transmitter is located at $X_i \in \Phi_1^n$. Consider a receiver located at $X_j \in \Phi_0^n$. Transmitter X_i can establish a successful channel to receiver X_j if and only if

$$\frac{SF_{i,j}^n l(|X_i - X_j|)}{W + I_{\Phi_1^n \setminus \{X_i\}}(X_j)} \geq T\,, \tag{6.1}$$

where $I_{\Phi_1^n \setminus \{X_i\}}$ is the shot-noise process of $\Phi_1^n \setminus \{X_i\}$:

$$I_{\Phi_1^n \setminus \{X_i\}}(X_j) = \sum_{X_k \in \Phi_1^n \setminus \{X_i\}} SF_{k,j}^n l(|X_k - X_j|).$$

Let $\delta(X_i, X_j, n)$ be the indicator that the event (6.1) holds.

When restricted to a bounded window, with $W \equiv 0$ and exponential F in M2 and M3, the above model corresponds to the simulation scenarios described in Section IV.

C. Opportunistic Neighborhood

Let us define the set of *neighbors* of $X_i \in \Phi$ at time n as $\{X_i\}$ plus the set of receivers which capture the packet sent by X_i at time n provided X_i transmits:

$$\begin{aligned} &V(X_i, n) \\ &\quad = \{X_i\} \cup \begin{cases} \{\Phi_0^n \ni X_j : \delta(X_i, X_j, n) = 1\} & \text{if } X_i \in \Phi_1^n \\ \emptyset & \text{otherwise.} \end{cases} \end{aligned}$$

Remark: The above SINR-based notion of neighborhood is quite different from that based on the maximum transmission range (used for the shortest path routing in Section V). Besides the fact that the neighborhood of a given node is different in different time slots (even in the M1 scenario), using (6.1) one can prove (cf [25]) that *no receiver at a given time slot can be a neighbour of more than* $(1+T)/T$ *transmitters*. In particular, if $T > 1$ then different transmitters, have *disjoint sets of neighbors*, even if they are very close to each other.

We will now show that the opportunistic neighbourhood is always finite (which is important, both from the practical and theoretical points of view) and then calculate the mean number of nodes in some particular case. Note that the assumptions of the following result are satisfied for all the scenarios considered (M1–M3).

Proposition 6.1: Assume that $\mathbf{E}[F^{2/\beta}] < \infty$ *and that either* $\mathbf{E}[F^{-2/\beta}] < \infty$ *or* $\mathbf{E}[W^{-2/\beta}] < \infty$. *Then for any of the models M1–M3* $\mathbf{P}\{\,\#V(X_i, n) = \infty \text{ for some } i, n\,\} = 0$.

Proof: By Campbell's formula [26, page 119], it is enough to prove that $\mathbf{E}^0[\#V(0, n)] < \infty$ for fixed n, where the expectation $\mathbf{E}^0$ is taken with respect to the *Palm probability* $\mathbf{P}^0$. In the case of our independently marked Poisson point process it corresponds to the addition of a node at the origin $X_0 = 0$ endowed with an independent MAC sequence $\mathbf{e}_0$ and fading sequence $\mathbf{F}_0$ (see Slivnyak's theorem, [26, page 41]). In what follows we consider time $n = 0$ and omit it in the notation. Using Campbell's theorem for the second time and the fact that Φ_1 and Φ_0 are independent Poisson point processes with respective intensities λp and $\lambda(1-p)$, we have

$$\begin{aligned} &\mathbf{E}^0[\#V(0)] \\ &\quad = 1 + \mathbf{E}^0\Big[\mathbb{I}(e_0 = 1) \sum_{i \neq 0} \mathbb{I}(X_i \in V(0))\Big] \\ &\quad = 1 + p\lambda(1-p) \int_{\mathbb{R}^2} \mathbf{E}^{0,x}[\delta(0,x) | e_0 = 1, e_x = 0]\,\mathrm{d}x\,, \end{aligned}$$

where the expectation $\mathbf{E}^{0,x}$ is taken with respect to the *two-fold Palm probability* $\mathbf{P}^{0,x}$, which in our case corresponds to the addition of two nodes, at $X_0 = 0$ and at $X = x$, endowed with independent MAC and fading sequences $\mathbf{e}_0, \mathbf{e}_x$, $\mathbf{F}_0, \mathbf{F}_x$.

Noting that under $\mathbf{P}^{0,x}$ given $e_0 = 1, e_x = 0$ the variable $I_{\Phi_1 \setminus \{0\}}(x)$ has the same distribution as $I_{\Phi_1}(0)$ under the stationary distribution $\mathbf{P}$, and passing to polar coordinates, we get from Fubini's theorem that

$$\begin{aligned} &\int_{\mathbb{R}^2} \mathbf{E}^{0,x}[\delta(0,x) | e_0 = 1, e_x = 0]\,\mathrm{d}x \\ &\quad = \mathbf{E}\Big[\int_{\mathbb{R}^2} \mathbb{I}\Big(A|x| \leq T \frac{W + I_{\Phi_1}(x)}{SF_{0,x}}\Big)^{-1/\beta} \mathrm{d}x\Big] \\ &\quad = \frac{2\pi}{A} \mathbf{E}\Big[\int_0^\infty r \mathbb{I}\Big(r \leq T \frac{W + I_{\Phi_1}(0)}{SF}\Big)^{-1/\beta} \mathrm{d}r\Big]\,, \end{aligned}$$

where F is independent of W and $I_{\Phi_1}(0)$. Consequently, we obtain

$$\mathbf{E}^0[\#V(0)] = 1 + \frac{p(1-p)\lambda\pi}{AT^{2/\beta}}\mathbf{E}[(SF)^{2/\beta}] \times\mathbf{E}[(W + I_{\Phi_1}(0))^{-2/\beta}] \,.$$

Since we assume $\mathbf{E}[F^{2/\beta}] < \infty$, the right hand side in the last formula is finite provided $\mathbf{E}[W^{-2/\beta}] < \infty$. If this latter condition does not hold, in particular if $W = 0$ with some positive probability, we need to prove the finiteness of the same negative moment of the shot-noise. For this we can proceed as follows:

$$\mathbf{E}[(I_{\Phi_1}(0))^{-2/\beta}] \le \mathbf{E}[(SF)^{-2/\beta}]\mathbf{E}[(\max_{X_i\in\Phi_1} l(|X_i|))^{-2/\beta}] \,.$$

We assume $\mathbf{E}[F^{-2/\beta}] < \infty$ and for our particular attenuation function,

$$\begin{aligned}\mathbf{E}[(\max_{X_i\in\Phi_1} l(|X_i|))^{-2/\beta}] &= \int_0^\infty \mathbf{P}\{\max_{X_i\in\Phi_1} l(|X_i|) \le r^{-\beta/2}\}\, dr \\ &= \int_0^\infty \mathbf{P}\{\min_{X_i\in\Phi_1} |X_i| \ge \sqrt{r}/A\}\, dr \\ &= \int_0^\infty e^{-\pi\lambda p r/A^2}\, dr = \frac{A^2}{\pi\lambda p} < \infty .\end{aligned}$$

which completes the proof. ∎

Remark: For exponential F in models M2–M3, the expected number of neighbors $\mathbf{E}^0[\#V(0)]$ can be calculated explicitly. Indeed, the expectation $\mathbf{E}^{0,x}[\dots]$ in (6.2) is equal to

$$\begin{aligned}&\mathbf{E}^{0,x}[\delta(0,x)|e_0=1,e_x=0] \\ &= \mathbf{E}\left[\exp\left(-\frac{TW}{Sl(|x|)}\right)\right]\mathbf{E}\left[\exp\left(-\frac{TI_{\Phi_1}(x)}{Sl(|x|)}\right)\right] \\ &= \psi_W(T/Sl(|x|))\,\psi_{I_{\Phi_1}}(T/Sl(|x|))\,,\end{aligned}$$

where $\psi_W, \psi_{I_\Phi}(\cdot)$ are, respectively, the Laplace transforms of W and $I_\Phi = I_\Phi(0)$. This last function is known in closed from. In particular, for $W \equiv 0$ we obtain the following formula:

$$\mathbf{E}^0[\#V(0)] = 1 + \frac{(1-p)\beta}{4\pi T^{2/\beta}\Gamma(2/\beta)\Gamma(1-2/\beta)}\,,$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,\mathrm{d}t$ is the Gamma function, as is easily shown by calculations similar to those in [12], Section III.

D. Opportunistic Routing

A *point map* is a mapping which, for a given realization of the point process Φ, maps each of its points $X_i \in \Phi$ to some (possibly the same) point of Φ.

1) Next Relay in Radial Routing: Define the following family of point maps: for $n \ge 0$

$$\mathcal{A}^n(X_i) = \mathcal{A}^n(X_i,\Phi) = \arg\min\{|X_j| : X_j \in V(X_i,n)\}\,.$$

The above point maps are almost surely well defined due to the well known fact that the probability of finding two or more points of the homogeneous Poisson point process equidistant to the origin is equal to 0. They represent the motion of a packet from X_i at time n to $\mathcal{A}^n(X_i)$ in the *time-space opportunistic radial routing towards the final destination at the origin 0 of the plane*.

In order to describe the route a packet makes from a given point $X \in \mathbb{R}^2$ of the plane to the origin 0, let us add these points to the stationary configuration of nodes and let us use the notation $\Phi^{0,X} = \Phi\cup\{0,X\}$. Recall that $\Phi^{0,X}$ represents the distribution of nodes under the two-fold Palm distribution $\mathbf{P}^{0,X}$. Denote by $\mathbf{e}_0, \mathbf{e}_X, \mathbf{F}_0, \mathbf{F}_X$ the MAC and fading marks of nodes at 0 and X under $\Phi^{0,X}$. In the case of independently marked Poisson point process they are independent of everything else and have the respective generic distributions.

The *radial (time-space opportunistic) path* of a packet from the source node X at time 0 towards the destination node at the origin 0 of the plane is the sequence of visited nodes $\{Y_n\}_n$ defined by:

$$Y_0 = X, \quad Y_{n+1} = \mathcal{A}^n(Y_n,\Phi^{X,0}) \text{ for } n \ge 0\,.$$

2) Convergence of the Radial Routing: We will say that the time-space opportunistic radial routing algorithm *converges* if its path is such that $Y_n \equiv 0$ after some finite n. The following result indicates that in the presence of the external noise, the (varying) fading is beneficial for the convergence.

Proposition 6.2: Assume that either (i) $W \equiv 0$ or (ii) M3 holds with F having unbounded support (i.e. $\overline{B}(s) = \Pr\{F > s\} > 0$ for all s). Then the time-space opportunistic radial routing algorithm converges almost surely under $\mathbf{P}^{0,X}$.

Proof: The convention that $X_i \in V(X_i,n)$ implies that no node of norm larger than $|Y_n|$ will ever be selected as the next relay. Hence, for all n, $|Y_{n+1}| \le |Y_n|$. In order to prove convergence, it is hence enough to show that the probability that $Y_{n+k} = Y_n$ for all $k \ge 1$ and for some n is 0 when $Y_n \ne 0$. Assume M3 holds. Denote by $\mathcal{G}$ the σ-algebra generated by Φ. Conditionally on $\mathcal{G}$ and on the event $Y_n = X_i \ne 0$ for a given $X_i \in \Phi\cup\{X\}$, we have

$$\mathbf{P}^{0,X}\Big\{\, Y_n = Y_{n+1} = \ldots = Y_{n+k} \,\Big|\, \mathcal{G}, Y_n = X_i \neq 0 \,\Big\}$$
$$= \prod_{i=0}^{k-1} \mathbf{P}^{0,X}\{\, Y_{n+i} = Y_{n+i+1} \,|\, \mathcal{G}, Y_{n+i} = X_i \neq 0 \,\}$$
$$= \Big(\mathbf{P}^{0,X}\{\, Y_n = Y_{n+1} \,|\, \mathcal{G}, Y_n = X_i \neq 0 \,\}\Big)^k$$

so it is enough to prove that

$$\mathbf{P}^{0,X}\{\, Y_{n+1} = Y_n \,|\, \mathcal{G}, Y_n = X_i \neq 0 \,\} < 1$$

to conclude the proof. But we have

$$\begin{aligned}
&\mathbf{P}^{0,X}\{\, |Y_{n+1}| < |Y_n| \quad | \quad \mathcal{G}, Y_n = X_i \neq 0 \,\} \\
&\quad \geq \mathbf{P}^{0,X}\{\, \mathcal{A}^n(X_i, \Phi) = 0 \quad | \quad \mathcal{G} \,\} \\
&\quad = p(1-p)\mathbf{P}^{0,X}\Big\{\, \frac{SF^n_{i,0}l(|X_i|)}{W + I_{\Phi^n_1 \setminus \{X_i\}}(0)} \geq T \\
&\qquad\qquad \Big|\, \mathcal{G}, e^n_i = 1, e^n_0 = 0 \,\Big\}.
\end{aligned}$$

Since $F^n_{i,0}$ is independent of $I_{\Phi^n_1 \setminus \{X_i\}}(0)$ and W, the probability $\mathbf{P}^{0,X}\{\ldots\}$ in the last formula can be expressed as

$$\mathbf{E}^{0,X}\left[\overline{B}\bigg(\frac{T(W + I_{\Phi^n_1 \setminus \{X_i\}}(0))}{Sl(|X_i|)}\bigg) \,|\, \mathcal{G}, e^n_i = 1, e^n_0 = 0\right]$$

and is positive by the assumption $\overline{B}(s) > 0$ and the fact that $I_{\Phi^n_1 \setminus \{X_i\}}(0) < \infty$ a.s. as a Poisson shot-noise. This concludes the proof of case (ii).

Consider now the case (i) $W \equiv 0$. Let $\mathcal{H}$ denote the σ-algebra generated by Φ and the fading variables (under M1 or M2, these variables do not vary over time). Using the same argument as before it is enough to prove that

$$\mathbf{P}^{0,X}\{\, Y_{n+1} = Y_n \,|\, \mathcal{H}, Y_n = X_i \neq 0 \,\} < 1\,.$$

And we have

$$\begin{aligned}
&\mathbf{P}^{0,X}\{\, |Y_{n+1}| < |Y_n| \quad | \quad \mathcal{H}, Y_n = X_i \neq 0 \,\} \\
&\quad \geq \mathbf{P}^{0,X}\{\, \mathcal{A}^n(X_i, \Phi) = 0 \quad | \quad \mathcal{H} \,\} \\
&\quad = \mathbf{P}^{0,X}\left\{\, \frac{SF_{i,0}l(|X_i|)}{I_{\Phi^n_1 \setminus \{X_i\}}(0)} \geq T,\, e^n_i = 1, e^n_0 = 0 \,\Big|\, \mathcal{H} \right\} \\
&\quad = p(1-p)\mathbf{P}^{0,X}\Big\{\, I_{\Phi^n_1 \setminus \{X_i\}}(0) \leq \frac{SF_{i,0}l(|X_i|)}{T} \\
&\qquad\qquad \Big|\, \mathcal{H},\, e^n_i = 1, e^n_0 = 0 \,\Big\}.
\end{aligned}$$

The proof then follows from the fact that the $\mathcal{H}$-conditional law of the Poisson shot-noise process $I_{\Phi^n_1 \setminus \{X_i\}}(0)$ puts a positive mass on the interval $[0, z]$ for all positive z. ■

Remark: Note that result of Proposition 6.2 *cannot* be immediately concluded from the fact that at any time and current location of the packet there is a positive probability of delivering it directly to the destination. In fact, our routing protocol is *not allowed* to wait for such an event. We also remark, that under M1 and M2 with $W > 0$, there is a non-negative probability that the packet is trapped forever at some isolated node.

3) Directional Path: We now define the directional point map. Denote by $\langle x, y\rangle$ the scalar product in $\mathbb{R}^2$ and for a given unit vector (think of a "direction") $\mathrm{d} \in \mathbb{R}^2$, $|\mathrm{d}| = 1$, define

$$\mathcal{A}^n_{\mathrm{d}}(X_i) = \arg\max\{\langle X_j, \mathrm{d}\rangle : X_j \in V(X_i, n)\}\,.$$

It is well known that the probability of finding two or more points of a homogeneous Poisson point process on a line with a given direction is equal to 0. Moreover, under the assumptions of Proposition 6.1, the point maps $\mathcal{A}^n_{\mathrm{d}}$ are well defined.

Consider $\Phi^X = \Phi \cup \{X\}$ and let the node at X be marked by an independent MAC sequence $\mathbf{e}_X$ and fading sequence $\mathbf{F}_X$. The d-*directional path* followed by a packet routed from X in the direction d by the time-space opportunistic *directional routing* algorithm is the sequence $\{Z_n = Z_n(X)\}_{n\geq 0}$ defined by

$$Z_0 = X, \quad Z_{n+1} = \mathcal{A}^n_{\mathrm{d}}(Z_n, \Phi^X), \quad \text{for } n \geq 0\,.$$

Property (1) can be formalized as the following conjecture: *The finite-dimensional distributions of the sequence $\{Y_n(X) - X\}_n$ under $\mathbf{P}^{0,X}$ converge weakly to those of $\{Z_n(0)\}$ under $\mathbf{P}^0$ when $|X| \to \infty$ such that* $-X/|X| = \mathrm{d}$. Roughly speaking this result is due to the fact that the optimal choices in "arg min" in $\mathcal{A}$ and "arg max" in $\mathcal{A}_{\mathrm{d}}$ coincide with high probability when the packet is far from the destination. A formal proof could follow the lines of [27, Lemma 1,Theorem 1].

E. Scaling Properties of the Directional Paths

The directional routing reveals interesting scaling properties which allow one to understand why Property (2) holds true and hence to substantiate Observation 5.4. We will prove the following result where $\mathbf{P}^0 = \mathbf{P}^0_\lambda$ denotes the probability measure under which the underlying Poisson point process Φ has intensity λ.

Proposition 6.3: Assume that $W \equiv 0$ and $\mathbf{E}[F^{2/\beta}] < \infty$, $\mathbf{E}[F^{-2/\beta}] < \infty$ so that the directional path $\{Z_n\}$ is well defined. Then for any of the models M1–M3, the law of the sequence $\{Z_n = Z_n(0)\}_n$ under $\mathbf{P}^0_\lambda$ is the same as that of $\{Z_n/\sqrt{\lambda}\}_n$ under $\mathbf{P}^0_1$.

Proof: Note that the distribution of the underlying Poisson point process $\Phi = \{X_i\}_i$ under $\mathbf{P}^0_\lambda$ is the same as the distribution of $\Phi(\lambda) = \{(X_i/\sqrt{\lambda})\}_i$ under $\mathbf{P}^0_1$. Moreover, under our assumptions on l and

$W = 0$, the SINR (in fact SIR) is invariant with respect to the scaling $\Phi(\lambda)$ of the point process. Indeed, $l(|X_i/\sqrt{\lambda} - X_j/\sqrt{\lambda}|) = \lambda^{\beta/2} l(|X_i - X_j|)$ and $I_{\Phi_1^n(\lambda)\setminus\{X_i/\sqrt{\lambda}\}}(X_j/\sqrt{\lambda}) = \lambda^{\beta/2} I_{\Phi_1^n\setminus\{X_i\}}(X_j)$. Moreover, the dilation (our scaling) is a conformal mapping (preserves angles). Consequently, the directional point map $\mathcal{A}_{\mathrm{d}}^n(X_i/\sqrt{\lambda})$ acting on $\Phi^0(\lambda)$ is equal to $1/\sqrt{\lambda}\mathcal{A}_{\mathrm{d}}^n(X_i)$ acting on Φ_1^0. This completes the proof. ■

For fixed n, consider now the optimization of the mean progress of the directional path in n hops with respect to the transmission probability p:

$$p^*(n, \lambda) = \arg\max_{0 \le p \le 1} \mathbf{E}_\lambda^0[\langle Z_n, \mathrm{d}\rangle].$$

The following corollary is a simple consequence of Proposition 6.3 (cf. Observation 5.3).

Corollary 6.4: Under the assumptions of Proposition 6.3 the optimal transmission probability $p^*(n, \lambda) = p^*(n)$ does not depend on λ.

F. Other Point Maps and their Optimal Transmission Probabilities

Several other point maps can be used or have already been used for routing. Rephrased in the terminology of the present paper, the authors of [12] used the following directional point map:

$$\tilde{\mathcal{A}}_{\mathrm{d}(X_i)} = \arg\max_{X_j \in \Phi_0} \{\langle X_j - X_i, \mathrm{d}\rangle p_{|X_j - X_i|}\},$$

where $p_{|x|} = \mathbf{E}^{0,x}[\delta(0, x) | e_x = 0, e_0 = 1]$ is the probability of successful transmission from 0 to x. Note that this point map is *less adaptive (more parametric)* than $\mathcal{A}_{\mathrm{d}}$ as it does not take advantage of the actual state of the SINR conditions at the receivers but only of their distance to the emitter; the indicator of successful reception is replaced there by the reception probability at a given location.

The distribution function of the associated progress in one hop: $\tilde{P}_1 = \max_{X_j \in \Phi_0}\{\langle X_j, \mathrm{d}\rangle p_{|X_j|}\}$ was calculated under $\mathbf{P}^0$ under conditions which can be rephrased as:

(M4) $F_{i,j}^n = F_i^n$ for all i, j, n and F_i^n are i.i.d. exponential with mean 1.

Under these conditions it was shown that the mean progress $\mathbf{E}^0[P_1]$ offered in one time slot by the directional routing $\mathcal{A}_{\mathrm{d}}$ is not smaller than $\mathbf{E}^0[\tilde{P}_1]$ offered by $\tilde{\mathcal{A}}_{\mathrm{d}}$. Moreover, $\mathbf{E}^0[\tilde{P}_1]$ was shown to scale like $1/\sqrt{\lambda}$ and to be maximized by a value of the transmission probability $\tilde{p}^* \approx 0.05$. It was argued in [12] that such a one-hop optimization of $\tilde{A}_{\mathrm{d}}$ is sufficient if the locations of nodes are independently re-sampled in each time slot. This last scenario is similar in spirit to the *Poisson Weighted Infinite Tree* model of [28], and was argued to be reasonable if the nodes are highly mobile; this model is easier to analyze as the successive hops of the routing become i.i.d. In consequence, in this "highly mobile" scenario the optimization of the mean progress $\mathbf{E}^0[\tilde{P}_1]$ in one slot minimizes the mean end-to-end delay over a long multi-hop radial path.

For the setting of Section V, the optimal transmission probability is $p^* \approx 0.014$ for M1 and $p^* \approx 0.018$ for M2 and M3. These optimal values differ slightly from the optimal transmission probability $\tilde{p}^* \approx 0.05$ obtained in [12] for M4 with the same parameter setting ($\beta = 3, W = 0, T = 10$). The discrepancy can be explained by the differences alluded to above: a less adaptive point map $\tilde{\mathcal{A}}_{\mathrm{d}}$ and different radio channel assumptions.

VII. Conclusion

In this paper we have proposed time-space opportunistic schemes using an optimized relay self selection technique. This technique uses signalling bursts to select the best relay towards the destination. We have shown that this optimized relay self selection technique can work with various access schemes such as Aloha, CSMA or even TDMA.

We have used simulations to show that time-space oppor- tunistic routing schemes signicantly improve the performance of multi-hop networks compared to conventional shortest path routing algorithms. The gain in terms of average delay incurred by a packet traveling from a source to a distant destination node is at least 2.8 depending on the actual network parameters. We have also shown that time-space opportunistic routing schemes still performs well even if only a small percentage of the nodes know their geographical position exactly, the other nodes estimating their position using a very simple localization algorithm.

We have compared our optimized relay self selection technique when we use Aloha or CSMA as the access scheme. This comparison has shown that CSMA actually outperforms Aloha in terms of end-to-end delay when the carrier sense threshold of CSMA is tuned for the node density of the network. In contrast, we have shown that for Aloha the optimization of the transmission p does not depend on the node density of the network.

We have also proposed a new mathematical framework based on stochastic geometry to prove some of the observations made in the simulations. In particular, this framework allowed us to prove that these routing algorithms can be optimized so as to minimize the average end-to-end delay incurred by a packet over long paths and that the optimum transmission probability does not depend on the node density in the random homogeneous case. The potential of this mathematical framework is well illustrated by the fact that the scaling property of Proposition 6.3 and the invariance property of Corollary 6.4 remain true for more general scenarios, e.g. when the routing is defined by more general classes of point maps. Various challenging problems remain open. On the practical side, we would quote in particular the evaluation of the overhead associated with such schemes. The simulation study also leads to several conjectures.

REFERENCES

[1] Baccelli, F., Błaszczyszyn, B., and Mühlethaler, P. (2008) On the performance of time-space opportunistic routing in multihop mobile ad hoc networks. *Proc. of WiOpt*, Berlin.

[2] Baccelli, F., Błaszczyszyn, B., Ermel, E., and Mühlethaler, P. (2008) An optimized relay self selection technique for opportunistic routing in mobile ad hoc networks. *Proc. of European Wireless Conference*, Prague.

[3] Adjih, C., Clausen, T., Jacquet, P., Laouiti, A., Minet, P., Muhlethaler, P., Qayyum, A., and Viennot, L. (2003). Optimized link-state routing protocol. RFC 3626.

[4] Dijkstra, E. W. (1959) A note on two problems in connexion with graphs. *Numerische Mathematik*, **1**, 269–271.

[5] Perkins, C., Belding-Royer, E., and Das, S. (2003). Ad hoc On-demand Distance Vector (AODV) routing. RFC 3561, Experimental.

[6] Karp, B. (2000) Geographic routing for wireless networks. Phd thesis. Harvard University.

[7] Finn, G. G. (1987) Routing and addressing problems in large metropolitan-scale internetworks. Technical Report ISI/RR-87-180. Information Sciences Institute.

[8] Stojmenovic, I. and Lin, X. (2001) Loop-free hybrid single-path/flooding routing algorithms with guaranteed delivery for wireless networks. *IEEE Transaction on Parallel and Distributed Systems*, **12**, 1023–1032.

[9] Stojmenovic, I. and Lin, X. (2001) Power-aware localized routing in wireless networks. *IEEE Transaction on Parallel and Distributed Systems*, **12**, 1122–1133.

[10] Karp, B. and Kung, H. T. (2000) Gpsr: Greedy perimeter stateless routing for wireless networks. *6th Annual International Conference on Mobile Computing and Networking, MobiCom 2000, August 6.-11., 2000, Boston, Massachusetts, USA*, August, pp. 243–254. ACM / IEEE.

[11] Blum, B., He, T., Son, S., and Stankovic, J. (2003) IGF: A state-free robust communication protocol for wireless sensor networks. Technical Report CS-2003-11. Department of Computer Science, University of Virginia, USA.

[12] Baccelli, F., Blaszczyszyn, B., and Mühlethaler, P. (2003) An Aloha Protocol for Multihop Mobile Wireless Networks. *Proceedings of the Allerton Conference, University of Illinois, Urbana Champaign*, November.

[13] Biswas, S. and Morris, R. Exor: opportunistic multi-hop routing for wireless networks. *SIGCOMM '05*, New York, NY, USA, pp. 133–144. ACM Press.

[14] Baccelli, F., Blaszczyszyn, B., and Muhlethaler, P. (2006) An aloha protocol for multihop mobile wireless networks. *IEEE Transactions on Information Theory*, **52**, 421–436.

[15] Jacquet, P., Minet, P., Mühlethaler, P., and Rivierre, N. (1997) Priority and collision detection with active signalling: The channel access mechanism of hiperlan. *Wireless Personnal Communication.*, January, pp. 11–25.

[16] (1996). ETSI *HIPERLAN functional specifications*, ETS 300-654.

[17] Bharghavan, V., Demers, A., Shenker, S., and Zhang, L. (1994) Macaw: a media access protocol for wireless lan's. *SIGCOMM '94: Proceedings of the conference on Communications architectures, protocols and applications*, pp. 212–225.

[18] Lin, C. and Gerla, M. (2002) Maca/pr: An asynchronous multimedia multihop wireless network. *Proceedings of IEEE INFOCOM '97*, pp. 975 – 985.

[19] Haas, Z. and Deng, J. (2002) Dual busy tone multiple access (dbtma)-a multiple access controlscheme for ad hoc networks. *IEEE Transactions on Communications*, pp. 975 – 985.

[20] Bertsekas, D. and Gallager, R. (1988) *Data Networks*. Prentice Hall.

[21] Tse, D. and Viswanath, P. (2005) *Foundamentals of Wireless Communication*. Cambridge University Press.

[22] Muhlethaler, P. and Najid, A. (2003) Optimization of CSMA multihop adhoc network. RR 4928. INRIA. Also published in European Wireless 2003.

[23] Ermel, E., Fladenmuller, A., Pujolle, G., and Cotton, A. (2004) Improved position estimation in wireless heterogeneous networks. *Networking 2004*, May.

[24] Gupta, P. and Kumar, P. R. (2000) The capacity of wireless networks. *IEEE Transactions on Information Theory*, **46**, 388–404.

[25] Baccelli, F. and Blaszczyszyn, B. (2001) On a coverage process ranging from the boolean model to the poisson voronoi tessellation, with applications to wireless communications. *Adv. Appl. Prob. SGSA*, **33**, 293–323.

[26] Stoyan, D., Kendall, W., and Mecke, J. (1995) *Stochastic Geometry and its Applications*. Wiley, Chichester.

[27] Bordenave, C. (2006) Navigation on a poisson point process. Technical Report RR-5790. INRIA.

[28] Aldous, D. and Steel, J. (2002). The objective method : Probabilistic combinatorial optimization and local weak convergence. in preparation.