

# Estimation de Volatilité en Présence de Bruit de Microstructure Endogène

Christian Yann Robert, Mathieu Rosenbaum

► **To cite this version:**

Christian Yann Robert, Mathieu Rosenbaum. Estimation de Volatilité en Présence de Bruit de Microstructure Endogène. 41èmes Journées de Statistique, SFdS, Bordeaux, 2009, Bordeaux, France, France. 2009. <inria-00386730>

**HAL Id: inria-00386730**

**<https://hal.inria.fr/inria-00386730>**

Submitted on 22 May 2009

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# ESTIMATION DE VOLATILITÉ EN PRÉSENCE DE BRUIT DE MICROSTRUCTURE ENDOGÈNE

Christian Y. Robert & Mathieu Rosenbaum

*Christian Yann Robert*  
*CREST-ENSAE Paris Tech*  
*Timbre J120, 3 Avenue Pierre Larousse,*  
*92245 Malakoff Cedex, France.*

*Mathieu Rosenbaum*  
*CMAP-École Polytechnique Paris*  
*UMR CNRS 7641, 91128 Palaiseau Cedex, France.*

*Résumé:* Ce papier considère des procédures statistiques facilement implémentables pour l'estimation de mesures de volatilité haute fréquence pour des actifs financiers. Le modèle de microstructure sous-jacent se base sur un prix efficient de type semi-martingale continue et permet de reproduire les principales caractéristiques empiriques des données ultra haute fréquence. Dans ce modèle, le bruit de microstructure est endogène mais ne dépend pas uniquement du prix efficient. En utilisant les prix de transaction observés, nous développons une nouvelle approche permettant d'approximer les valeurs du prix efficient à certains instants aléatoires. En se basant sur ces valeurs approchées, on construit un estimateur de la volatilité intégrée et on fournit sa théorie asymptotique. On donne aussi un estimateur consistant de la co-volatilité intégrée dans le cas où deux actifs (asynchrones par construction du modèle) sont observés.

*Mots clés:* Bruit de microstructure; Données ultra haute fréquence; Volatilité; Co-volatilité; Durations; Données asynchrones; Temps d'arrêt; Martingales.

*Abstract:* This paper considers practically appealing procedures for estimating intraday volatility measures of financial assets. The underlying microstructure model accommodates the inherent properties of ultra high frequency data with the assumption of continuous efficient price processes. In this model, the microstructure noise is endogenous but does not only depend on the prices. Using the (observed) last traded prices of the assets, we develop a new approach that enables to approximate the values of the efficient prices at some random times. Based on these approximated values, we build an estimator of the integrated volatility and give its asymptotic theory. We also give a consistent estimator of the integrated co-volatility when two assets (asynchronous by construction of the model) are observed.

## Bibliographie

- [1] Aït-Sahalia, Y. and Mykland, P.A. (2003). The Effects of Random and Discrete Sampling when Estimating Continuous-Time Diffusions. *Econometrica* **71** 483-549.
- [2] Aït-Sahalia, Y. and Mykland, P.A. (2004). Estimators of Diffusions with Randomly Spaced Observations: A General Theory. *The Annals of Statistics* **32** 2186-2222.
- [3] Aït-Sahalia, Y., Mykland, P.A. and Zhang, L. (2005). How Often to Sample a Continuous-Time Process in the Presence of Market Microstructure Noise. *Review of Financial Studies* **18** 351-416.
- [4] Aït-Sahalia, Y., Mykland, P.A. and Zhang, L. (2005). Ultra High Frequency Estimation with Dependent Microstructure Noise. Working paper.
- [5] Bandi, F. and Russel, J. (2005). Separating Microstructure Noise from Volatility. *Journal of Financial Economics* **79** 655-692.
- [6] Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A. and Shephard, N. (2009). Designing Realised Kernels to Measure the Ex-Post Variation of Equity Prices in the Presence of Noise. To appear in *Econometrica*.
- [7] Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A. and Shephard, N. (2009). Multivariate Realised Kernels: Consistent Positive Semi-Definite Estimators of the Covariation of Equity Prices with Noise and Non-Synchronous Trading. Working paper.
- [8] Borodin, A.N. and Salminen, P. (2002). Handbook of Brownian Motion-Facts and Formulae. Probability and Its Applications, Birkhäuser.
- [9] Delattre, S. and Jacod, J. (1997). A Central Limit Theorem for Normalized Functions of the Increments of a Diffusion Process, in the Presence of Round-Off Errors. *Bernoulli* **3** 1-28.
- [10] Engle, R. (2000). The Econometrics of Ultra High Frequency Data. *Econometrica* **68** 1-22.
- [11] Fuksawa, M. (2007). Realized Volatility Based on Tick Time Sampling. To appear in *Finance and Stochastics*.
- [12] Garman, M.B. and Klass, M.J. (1980). On the Estimation of Security Price Volatilities from Historical Data. *The Journal of Business* **53**, 67-78.
- [13] Genon-Catalot, V. and Jacod, J. (1993). On Estimating the Diffusion Coefficient for Multidimensional Processes. *Annales de l'IHP-Probabilité* **29** 119-151.
- [14] Genon-Catalot, V. and Jacod, J. (1994). Estimation of the Diffusion Coefficient for Diffusion Processes: Random Sampling. *Scandinavian Journal of Statistics* **21** 193-221.
- [15] Gloter, A. and Jacod, J. (2001). Diffusions with Measurement Errors. I - Local Asymptotic Normality. *ESAIM: Probability and Statistics* **5** 225-242.
- [16] Gloter, A. and Jacod, J. (2001). Diffusions with Measurement Errors. II - Optimal Estimators. *ESAIM: Probability and Statistics* **5** 243-260.
- [17] Gouriéroux, C. and Jasiak, J. (2001). *Financial Econometrics: Problems, Models and Methods*. Princeton University Press.
- [18] Griffin, J.E. and Oomen, R.C.A. (2008). Sampling Returns for Realized Variance Calculations: Tick Time or Transaction Time? *Econometric Reviews* **27** 230-253.

- [19] Hansen, P.R. and Lunde, A. (2006). Realized Variance and Market Microstructure Noise. *Journal of Business and Economics Statistics* **24** 127-161.
- [20] Hayashi, T. and Yoshida, N. (2005). On Covariance Estimation of Non-synchronously Observed Diffusion Processes. *Bernoulli* **11** 359-379.
- [21] Jacod, J. and Shiryaev, A.N. (2002). *Limit Theorems for Stochastic Processes*. Second Edition, Springer-Verlag.
- [22] Jacod, J., Li, Y., Mykland, P.A., Podolskij, M. and Vetter, M. (2009). Microstructure Noise in the Continuous Case: The Pre-Averaging Approach. To appear in *Stochastic Processes and their Applications*.
- [23] Kalnina, I. and Linton, O.B. (2008). Estimating Quadratic Variation Consistently in the Presence of Correlated Measurement Error. *Journal of Econometrics* **147** 47-59.
- [24] Large, J. (2007). Estimating Quadratic Variation when Quoted Prices Change by a Constant Increment. Working paper.
- [25] Li, Y. and Mykland, P.A. (2008). Rounding Errors and Volatility Estimation. Working paper.
- [26] Li, Y. and Mykland, P.A. (2007). Are Volatility Estimators Robust with Respect to Modeling Assumptions? *Bernoulli* **13** 601-622.
- [27] Phillips, P. and Yu, J. (2006). Information Loss in Volatility Measurement with Flat Price Trading. Working paper.
- [28] Revuz, D. and Yor, M. (1999). *Continuous Martingales and Brownian Motion*. Third Edition, Springer.
- [29] Robert, C.Y. and Rosenbaum, M. (2009). A New Approach for the Dynamics of Ultra High Frequency Data: the Model with Uncertainty Zones. Working paper, available at <http://www.crest.fr/pageperso/rosenbaum/rosenbaum.htm>.
- [30] Rosenbaum, M. (2007). A New Microstructure Noise Index. Working paper.
- [31] Rosenbaum, M. (2009). Integrated Volatility and Round-Off Error. To appear in *Bernoulli*.
- [32] Voev, V. and Lunde, A. (2007). Integrated Covariance Estimation using High-frequency Data in the Presence of Noise. *Journal of Financial Econometrics* **5** 68-104.
- [33] Whitt, W. (2002). *Stochastic-Process Limits*, Springer.
- [34] Zhang, L. (2006). Efficient Estimation of Stochastic Volatility Using Noisy Observations: A Multi-Scale Approach. *Bernoulli* **12** 1019-1043.
- [35] Zhang, L. (2006). Estimating Covariation: Epps Effect and Microstructure Noise. Working Paper.
- [36] Zhang, L., Mykland, P.A. and Ait-Sahalia, Y. (2005). A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-Frequency Data. *Journal of the American Statistical Association* **100** 1394-1411.