

## CRDTs: Consistency without concurrency control

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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

## *CRDTs: Consistency without concurrency control*

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*R*apport  
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## CRDTs: Consistency without concurrency control\*

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**Abstract:** A CRDT is a data type whose operations commute when they are concurrent. Replicas of a CRDT eventually converge without any complex concurrency control. As an existence proof, we exhibit a non-trivial CRDT: a shared edit buffer called Treedoc. We outline the design, implementation and performance of Treedoc. We discuss how the CRDT concept can be generalised, and its limitations.

**Key-words:** Data replication, optimistic replication, commutative operations

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## Les CRDT : Cohérence sans contrôle de concurrence

**Résumé :** Un CRDT est un type de données dont toutes les opérations concurrentes sont commutatives. Les répliques d'un CRDT convergent inéluctablement, sans nécessiter un contrôle de concurrence complexe. Comme preuve d'existence, nous montrons un CRDT non trivial: un tampon d'édition partagée appelé Treedoc. Nous en résumons la conception, la mise en œuvre et les performances. Nous discutons les limites et les possibles généralisations du concept.

**Mots-clés :** Réplication des données, réplication optimiste, opérations commutatives

## 1 Introduction

Shared read-only data is easy to scale by using well-understood replication techniques. However, sharing *mutable* data at a large scale is a difficult problem, because of the CAP impossibility result [5]. Two approaches dominate in practice. One ensures scalability by giving up consistency guarantees, for instance using the Last-Writer-Wins (LWW) approach [7]. The alternative guarantees consistency by serialising all updates, which does not scale beyond a small cluster [12]. Optimistic replication allows replicas to diverge, eventually resolving conflicts either by LWW-like methods or by serialisation [11].

In some (limited) cases, a radical simplification is possible. If concurrent updates to some datum commute, and all of its replicas execute all updates in causal order, then the replicas converge.<sup>1</sup> We call this a Commutative Replicated Data Type (CRDT). The CRDT approach ensures that there are no conflicts, hence, no need for consensus-based concurrency control. CRDTs are not a universal solution, but, perhaps surprisingly, we were able to design highly useful CRDTs. This new research direction is promising as it ensures consistency in the large scale at a low cost, at least for some applications.

A trivial example of a CRDT is a set with a single *add-element* operation. A *delete-element* operation can be emulated by adding “deleted” elements to a second set. This suffices to implement a mailbox [1]. However, this is not practical, as the data structures grow without bound. A more interesting example is WOOT, a CRDT for concurrent editing [9], pioneering but inefficient, and its successor Logoot [13].

As an existence proof of non-trivial, useful, practical and efficient CRDT, we exhibit one that implements an ordered set with insert-at-position and delete operations. It is called Treedoc, because sequence elements are identified compactly using a naming tree, and because its first use was concurrent document editing [10]. Its design presents original solutions to scalability issues, namely restructuring the tree without violating commutativity, supporting very large and variable numbers of writable replicas, and leveraging the data structure to ensure causal ordering without vector clocks.

Another non-trivial CRDT that we developed (but we do not describe here) is a high-performance shared, distributed graph structure, the *multilog* [2].

While the advantages of commutativity are well documented, we are the first (to our knowledge) to address the design of CRDTs. In future work, we plan to explore what other interesting CRDTs may exist, and what are the theoretical and practical requirements for CRDTs.

The contributions of this paper are the following: We exhibit a non-trivial, practical, efficient CRDT. We address practical issues in CRDT design such as indefinite growth, identifier size, restructuring and garbage collection. We present a novel approach side-stepping the non-scalability of consensus when dealing with dynamic, varying numbers of sites. We present some experimental data based on Wikipedia traces.

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<sup>1</sup> Technically, LWW operations commute; however they achieve this by throwing away non-winning operations. We aim instead for *genuine* commutativity that does not lose work, i.e., the output should reflect the cumulative effect of the operations.

The paper proceeds as follows. This introduction is Section 1. Section 2 presents our ordered-sequence CRDT abstraction. Section 3 examines the trace data and experimental performance of our CRDT. In Section 4 we present our solutions to some specific scalability issues. Section 5 discusses lessons learned and possible generalisations. Section 6 concludes and outlines future work.

## 2 An ordered-set CRDT

We begin by considering the requirements of a CRDT providing the abstraction of an ordered sequence of (opaque) *atoms*.

### 2.1 Model

We consider a collection of *sites* (i.e., networked computers), each carrying a *replica* of a shared ordered-set object, and connected by a reliable broadcast protocol (e.g., epidemic communication). We support a peer-to-peer, multi-master execution model: some arbitrary site *initiates* an update and executes it against its local replica; each other site eventually receives the operation and *replays* it against its own replica. All sites eventually receive and execute all operations; causally-related operations execute in order, but concurrent operations may execute in different orders at different sites.

The update operations of the ordered-set abstraction are the following:

- *insert*( $ID, newatom$ ), where  $ID$  is a fresh identifier. This operation adds atom *newatom* to the ordered-set.
- *delete*( $ID$ ), deletes the atom identified  $ID$  from the ordered-set.

Two inserts or deletes that refer to different IDs commute. Furthermore, operations are idempotent, i.e., inserting or deleting the same ID any number of times has the same effect as once. To ensure commutativity of concurrent inserts, we only need to ensure that no two IDs are equal across sites. Our ID allocation mechanism will be described next.

### 2.2 Identifiers

Atom identifiers must have the following properties: (i) Two replicas of the same atom (in different replicas of the ordered-set) have the same identifier. (ii) No two atoms have the same identifier. (iii) An atom’s identifier remains constant for the entire lifetime of the ordered-set.<sup>2</sup> (iv) There is a total order “ $<$ ” over identifiers, which defines the ordering of the atoms in the ordered-set. (v) The identifier space is *dense*.

Property (v) means that between any two identifiers  $P$  and  $F$ ,  $P < F$ , we can allocate a fresh identifier  $N$ , such that  $P < N < F$ . Thus, we are able to insert a new atom between any two existing ones.

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<sup>2</sup> Later in this paper we will weaken this property.

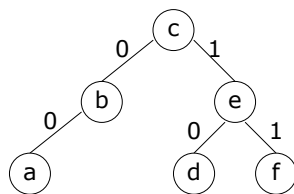


Figure 1: Example Treedoc. The TID for "b" is 0; the TID of "c" is the empty string; the TID of "d" is 10.

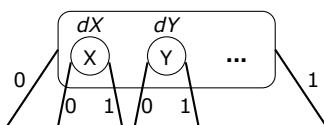


Figure 2: A treedoc major node

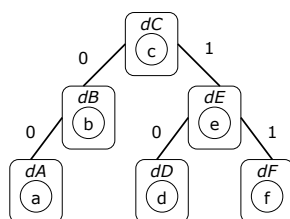


Figure 3: A treedoc node with disambiguators

The set of real numbers  $\mathbb{R}$  is dense, but cannot be used for our purpose, because, as atoms are inserted, the precision required will grow without bound. We outline a simpler solution next.

### 2.3 The Treedoc CRDT

In Treedoc, an atom identifier, henceforth called a TID, represents a path in a tree. If the tree is balanced, the average TID size is logarithmic in the number of atoms. We experimented with both binary and 256-ary trees; for lack of space we present only the binary version. The order " $<$ " is infix traversal order (i.e., left to right). Figure 1 shows a binary Treedoc that contains the text "abcdef".

In a distributed environment, different sites might concurrently allocate the same TID. To avoid this, we extend the basic tree structure, allowing a node to contain a number of internal nodes, called *mini-nodes*. A node containing mini-nodes will be called a *major node*. Figure 2 shows an example major node. Inside a major node, mini-nodes are distinguished



by a *disambiguator* that identifies the site that inserted the node. Disambiguators are unique and ordered, giving a total order between entries in the ordered-set.

Figure 3 shows a Treedoc structure with disambiguators represented at every node. Site A with disambiguator  $dA$  inserted atom  $a$ , site B inserted atom  $b$ , and so on. Mini-nodes are traversed in disambiguator order.

## 2.4 Treedoc insert and delete

We now describe the ordered-set update operations, insert and delete. We start with delete, the simpler of the two. A *delete*( $TID$ ) simply discards the atom associated with  $TID$ . We retain the corresponding tree node and mark it as a *tombstone*. (In certain cases, out of the scope of this short paper, a tombstone may be discarded immediately.)

To insert an atom, the initiator site chooses a fresh TID that positions it as desired relative to the other atoms. For instance, to insert an atom  $R$  to the right of atom  $L$ : • If  $L$  does not have a right child, the TID of  $R$  is the TID of  $L$  concatenated with 1 ( $R$  becomes the right child of  $L$ ). • Otherwise, if  $L$  has a right child  $Q$ , then allocate the TID of the leftmost position of the subtree rooted at  $Q$ .

## 2.5 Restructuring the tree

In the approach described so far, depending on the pattern of inserts and deletes, the tree may become badly unbalanced or riddled with tombstones. To alleviate this problem, the new restructuring operation *flatten* transforms a tree into a flat array, eliminating all storage overhead. As a flat array can equivalently be interpreted as a balanced tree, there is no need for the inverse operation. As the flattening operation changes the TIDs, we modify Property (iii) of Section 2.2 to allow non-ambiguous renaming.

However, flattening does not genuinely commute with update operations. We solve this using an update-wins approach: if a flatten occurs concurrently with an update, the update wins, and the flatten aborts with no effect. We use a two-phase commit protocol for this purpose (or, better, a fault-tolerant variant such as Paxos Commit [6]). The site that initiates the flatten acts as the coordinator and collects the votes of all other sites. Any site that detects an update concurrent to the flatten votes “no”, otherwise it votes “yes.” The coordinator aborts the flatten if any site voted “no” or if some site is crashed. Commitment protocols are problematic in large-scale and dynamic systems; in Section 4 we explain how we solve this issue.

## 3 Experiments

We ran a series of experiments based on cooperative editing traces.

A number of Wikipedia pages were stored as Treedocs, interpreting differences between successive versions of a page as a series of inserts and deletes. In some experiments our atoms were words; in the ones reported here an atom is a whole paragraph. We also ran

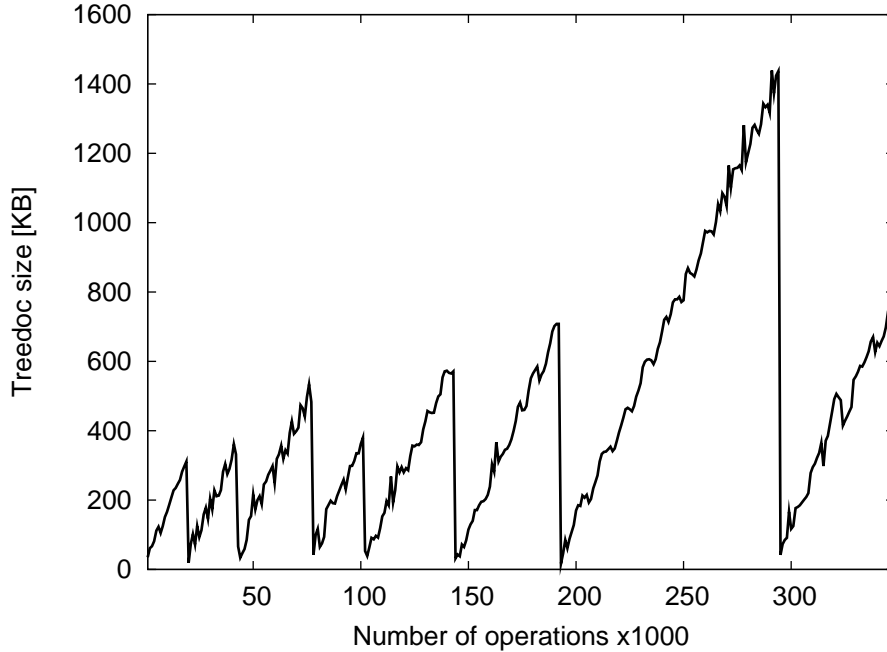


Figure 4: Treedoc size over time (GWB page)

similar experiments based on traces of SVN repositories containing LaTeX Java source code. A common observation across all experiments is that the number of deletes is surprisingly high.

We studied medium-sized Wikipedia pages such as “Distributed Computing,” reaching 20 KB of text in 800 revisions, or “PowerPC” reaching 25 KB in 400 revisions. Applying all the revisions for these pages required less than 1 second when using paragraphs as atoms, and 2 seconds using words. We also studied some frequently-edited pages, e.g., “George W. Bush” (GWB) reaching 150 KB in 40,000 revisions. Because of vandalism, the GWB page contains an even higher proportion of deletes (in the absence of flattening, 95% of nodes would be tombstones).

Hereafter we report only on the most stressful benchmark, i.e., the GWB traces, with a 256-ary tree, and full paragraphs as atoms, flattening every 1,000 revisions; 1,000 successive revisions may include up to 100,000 update operations.

Figure 4 shows the size of the GWB Treedoc structure over the first 350,000 edit operations of the GWB page. Size increases with successive edits, then falls dramatically at each periodic flattening. The decrease is attributable mostly to discarding tombstones, but also

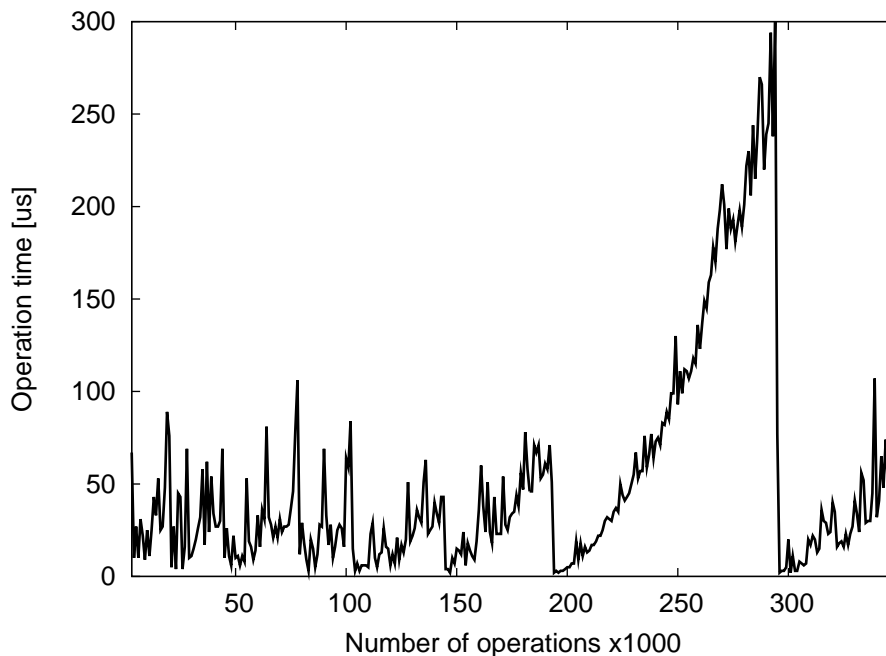


Figure 5: Execution time per operation (GWB page)

to improved balance of the tree: thus, the average TID size shrinks from 60 bytes before flattening to only 2 bytes.

Figure 5 shows execution time per operation. Again, flattening has dramatic effect. Without flattening, the per-operation time would grow up to 3ms. Periodic flattening decreases the depth of the tree to 2-3 levels, and the slowest update takes only 0.3ms.

From this we can estimate the scalability of Treedoc for concurrent updating. Assume that every site continuously initiates one update every 3 seconds. Then the system can sustain 1,000 simultaneous users without flattening, and 10,000 when flattening at 1,000-revision intervals.

## 4 Treedoc in the large scale

The CRDT approach guarantees that replicas converge. However, we saw that metadata accumulates over time, with a big impact on performance, and must be garbage-collected from time to time. The attendant commitment or consensus is a problem for scalability. In this section, we explain how Treedoc addresses this issue.

## 4.1 Supporting large and dynamic numbers of replicas

Commitment requires the unanimous agreement of a well-identified set of sites. This is problematic in a large-scale system, where sites fluctuate dynamically. In scenarios such as collaborative editing, new participants may enter at any time, leave the system definitively, or disconnect for an unpredictable period of time, while continuing to initiate updates.

To solve this problem, we partition the sites in two disjoint subsets. The *core* consists of a small group of sites that are well-known and well-connected. In the limit, the core could reduce to a single server. The sites that are not in the core are part of the *nebula*. Only core sites participate in commitment.

## 4.2 Nebula catch-up protocol

Let us call an interval between successful flattens an *epoch*. Each flatten – each change of epoch – changes the frame of reference for TIDs: TIDs from some epoch are invalid in other epochs, and sites may exchange updates only if in the same epoch. Core sites are in the same epoch by construction, but a nebula site may lag behind.

In order to communicate with the nebula, a core site executes a *catch-up* protocol, which we now describe at a high level. To simplify the description, assume that the core and the nebula sites started from the same initial state, and that the core executed a single flatten since then: If the core is in epoch  $n$  (the “new” epoch), the nebula is in epoch  $n - 1$  (“old” epoch). Updates in the old epoch use “old” TIDs, whereas those in the new epoch use “new” TIDs.

A core site maintains a buffer of update messages that it needs to send to the nebula, some in the old epoch, some in the new one. Conversely, a nebula site maintains a buffer of update messages to be sent to the core; they are all in the old epoch.

Old messages buffered in the core can be sent to the nebula site (operating in the old epoch) and replayed there. However, the converse is not true: since the core is in the new epoch, it cannot replay old updates from the nebula. The nebula must first bring them into the new epoch. To this effect, and once it has applied all old core updates, the nebula site flattens its local replica of the tree, using the tree itself to keep track of the mapping between old and new TIDs. Then it translates old TIDs in buffered messages into the corresponding new TIDs. At this point, the nebula site is in the new epoch. (It may now either join the core, or remain in the nebula.) Finally, it sends its buffered messages to the core, which can now replay them.

Since epochs are totally ordered, and since every nebula site will go through the same catch-up protocol, concurrent updates remain commutative, even if initiated in different epochs.

## 4.3 Core/nebula requirements

The requirements for managing the core and nebula follow from the above description.

Joining or leaving the core follows a membership protocol [3]. All core sites participate in flattening. Core sites may freely initiate updates and replay each others'.

Sites in the nebula are assumed to be uniquely identified (for disambiguators), but are otherwise unrestricted. The nebula may contain any number of sites, which are connected to the network or disconnected. Nebula sites may freely initiate updates, but do not participate in commitment.

Two sites may send updates to each other, and replay each others' updates, if and only if they are in the same epoch.

#### 4.4 TID translation algorithm

We now describe in more detail how a nebula site translates TIDs from the old to the new epoch. It needs to distinguish operations that were received from the core and are serialised before the flatten, from those initiated locally or received from other nebula sites, which must be serialised after the flatten. For this purpose we colour the corresponding nodes either Cyan (C for Core) or Black (Noir in French, N for Nebula).

Thus we distinguish cyan nodes, cyan tombstones, black nodes and black tombstones. A node can be both a cyan node and a black tombstone; the converse is not possible.

We will now describe the steps that a nebula site needs to take in order to execute a flatten operation. We will assume that all the operations from the core issued prior to the flatten have been executed as well as some black operations, some local and some from other nebula sites. Once the flatten is performed the site will be able to send the black operations to the core. The flatten will construct list of subtrees, each having as root a cyan node.

The first step is to go through the tree and examine only cyan nodes and tombstones. We ensure that a sentinel node  $n_b$  always exists to mark the beginning of the ordered-set and to ensure the tree is not empty. We identify the following cases:

- **cyan node** (can also be a black tombstone) - add to the list along with any black children it has
- **cyan tombstone** - add any black children to the subtree of the last node in the flattened list. We preserve the correct order by adding at the end of the subtree. If no cyan nodes have been seen so far, we add the black children to  $n_b$ .

The second step is to create the new balanced tree from the roots of the subtrees stored in the linear list. The nodes that have black children will be transformed into major nodes if both a cyan child and a black child should be placed on the same position.

The last step is to go through the new tree and generate the update operations to be sent to the core. We examine only black nodes and tombstones:

- **black node** - send *insert* operation with this TID and atom
- **black tombstone** - send *delete* operation with this TID

When a nebula site connects to the core, it sends not only black operations generated locally, but also operations received from other nebula sites. It may happen that a site receives the same update multiple times, but this causes no harm since updates are idempotent.

## 4.5 Approximate causal ordering

Vector clocks are commonly used to ensure causal ordering and to suppress duplicate messages. We observe that causal ordering is already encoded in the Treedoc structure: inserting some node always happens-before inserting some descendant of that node, and always happens-before the deletion of the same node. Operations on unrelated nodes, even if one happened-before the other, can execute in any order. Furthermore, duplicate messages are inefficient but cause no harm, since operations are idempotent. Therefore, a precise vector clock is not necessary; approximate variants that are scalable may be sufficient as long as they suppress a good proportion of duplicates.

## 5 Discussion

Massive distributed computing environments, such as Zookeeper or Dynamo [4], replicate data to achieve high availability, performance and durability. Achieving strong consistency in such environments is inherently difficult and requires a non-scalable consensus; however in the absence of consistency, application programmers are faced with overwhelming complexity. For some applications, eventual consistency is sufficient [4], but complexity and consensus are hiding under a different guise, namely of conflict detection and resolution.

In this paper we propose to use CRDTs because they ensure eventual consistency without requiring consensus. Although garbage collection is based on consensus, it remains outside the critical path of the application and hidden inside the abstraction boundary.

Not all abstractions can be converted into a CRDT: for instance a queue or a stack rely on a strong invariant (a total order) that inherently requires consensus. Treedoc on the other hand maintains a local, partial order, and the outcome of its operations need not be unique.

Even when an abstraction is not a CRDT, it is very useful to design it so that most pairs of operations commute when concurrent. Those pairs can benefit from cheap, high-performance protocols, resorting to consensus only for non-commuting pairs [8].

Generalising from Treedoc and Multilog (see Introduction) teaches us a few interesting lessons about the requirements for CRDTs. To commute, operations must have identical pre-condition; in practice, all operations should have pre-condition “true.” A central requirement is the use of unique, unchanging identifiers. To be practical, the data structure must remain compact; we ensure this by using an ever-growing tree, ensuring that metadata and identifiers remain compact (logarithmic in the size of the data).

## 6 Conclusion

The Commutative Replicated Data Type or CRDT is designed to make concurrent operations commute. This removes the need for complex concurrency control allowing operations to be executed in different orders and still have replicas converge to the same result. CRDTs enable increased performance and scalability compared to classical approaches.

Although designing a CRDT to satisfy certain requirements is not always possible, loosening invariants or precision constraints should allow the design of commutative operations.

We have proposed a CRDT called Treedoc that maintains an ordered set of atoms while providing insert and delete operations. To overcome the challenges of practicality and scalability, we explored some innovative solutions. Each atom has a unique, system-wide, compact identification that does not change between flattens. Garbage collection is a requirement in practice; it is disruptive and requires consensus, but it has lower precedence than updates, and it is not in the critical path of applications. We side-step the non-scalability of consensus by dividing sites into two categories with different roles. CRDTs require causal ordering, but since the Treedoc metadata encodes causal ordering implicitly, it does not need to be maintained strictly at the system level; this enables the use of scalable approximations of vector clocks.

Our future work includes searching for other CRDTs as well as studying the interaction between CRDTs and classical data structures.

## References

- [1] BAQUERO, C., AND MOURA, F. Using structural characteristics for autonomous operation. *Operating Systems Review* 33, 4 (1999), 90–96.
- [2] BENMOUFFOK, L., BUSCA, J.-M., MANUEL MARQUÈS, J., SHAPIRO, M., SUTRA, P., AND TSOUKALAS, G. Telex: Principled system support for write-sharing in collaborative applications. Rapport de recherche 6546, INRIA, Rocquencourt, France, May 2008.
- [3] CHOCKLER, G. V., KEIDAR, I., AND VITENBERG, R. Group communication specifications: a comprehensive study. *ACM Trans. Comput. Syst.* 33, 4 (Dec. 2001), 427–469.
- [4] DECANDIA, G., HASTORUN, D., JAMPANI, M., KAKULAPATI, G., LAKSHMAN, A., PILCHIN, A., SIVASUBRAMANIAN, S., VOSSHALL, P., AND VOGELS, W. Dynamo: Amazon’s highly available key-value store. In *21st Symp. on Op. Sys. Principles (SOSP)* (Stevenson, Washington, USA, Oct. 2007), vol. 41 of *Operating Systems Review*, ACM Sigops, ACM, pp. 205–220.
- [5] GILBERT, S., AND LYNCH, N. Brewer’s conjecture and the feasibility of consistent, available, partition-tolerant web services. *SIGACT News* 33, 2 (2002), 51–59.

- 
- [6] GRAY, J., AND LAMPORT, L. Consensus on transaction commit. *Trans. on Database Systems* 31, 1 (Mar. 2006), 133–160.
  - [7] JOHNSON, P. R., AND THOMAS, R. H. The maintenance of duplicate databases. Internet Request for Comments RFC 677, Information Sciences Institute, Jan. 1976.
  - [8] LAMPORT, L. Generalized consensus and Paxos. Tech. Rep. MSR-TR-2005-33, Microsoft Research, Mar. 2005.
  - [9] OSTER, G., URSO, P., MOLLI, P., AND IMINE, A. Data consistency for P2P collaborative editing. In *Int. Conf. on Computer-Supported Coop. Work (CSCW)* (Banff, Alberta, Canada, Nov. 2006), ACM Press, pp. 259–268.
  - [10] PREGUIÇA, N., MARQUÈS, J. M., SHAPIRO, M., AND LETÍIA, M. A commutative replicated data type for cooperative editing. In *Int. Conf. on Distributed Comp. Sys. (ICDCS)* (Montéal, Canada, 2009).
  - [11] SAITO, Y., AND SHAPIRO, M. Optimistic replication. *Computing Surveys* 37, 1 (Mar. 2005), 42–81.
  - [12] SCHIPER, N., AND PEDONE, F. Optimal atomic broadcast and multicast algorithms for wide area networks. In *Symp. on Principles of Dist. Comp. (PODC)* (Portland, OR, USA, 2007), Assoc. for Computing Machinery, pp. 384–385.
  - [13] WEISS, S., URSO, P., AND MOLLI, P. Logoot: a scalable optimistic replication algorithm for collaborative editing on P2P networks. In *Int. Conf. on Distributed Comp. Sys. (ICDCS)* (Montréal, Canada, June 2009).





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