Predicting Natural Hair Shapes by Solving the Statics of Flexible Rods

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Figure 1: *Virtual versus real hair ringlets*

Abstract

This paper presents a new physically-based method for predicting natural hairstyles in the presence of gravity and collisions. The method is based upon a mechanically accurate model for static elastic rods (Kirchhoff model), which accounts for the natural curliness of hair, as well as for hair ellipticity. The equilibrium shape is computed in a stable and easy way by energy minimization. This yields various typical hair configurations that can be observed in the real world, such as ringlets. As our results show, the method can generate different hair types with a very few input parameters, and perform virtual hairdressing operations such as wetting, cutting and drying hair.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Physically based modeling

1. Introduction and Previous Work

Due to the relevance of hair in the overall appearance of a character, many recent works have focused on the challenging issue of virtual hairstyling. However, there is currently no hairstyling method able to account for the structural properties of the different hair types existing in the world - Asian, African and Caucasoid hair - nor to predict how a given hair would look as it grows or gets wet. Such a model would prove very useful for performing virtual hairdressing operations on any given hair type.

The geometry of human hair results from complex biological and mechanical processes. Most CG hairstyling methods reproduce this result using procedural techniques and manual editing [KN02, CK05]. This gives a high controllability, but also requires some significant input and modeling skills from the user. Several attempts have been made to use physically-based modelling in the design of hairstyles [AUK92, HMT00]. In these approaches, the finest geometric hair details, such as curls or waves, are still added in a procedural way. Image-based reconstruction techniques have recently proved useful in capturing the geometry of real hair [PBS04]. However, as they only model the visible part of hair, these methods can hardly capture the geometry of curly hair because of the occlusion issue.

Our method is physically-based, and relies on the Kirchhoff equations for static Cosserat rods. This accurate mechanical model, which accounts for the curvatures and twist deformation of elastic rods, was first introduced to the CG community by Pai [Pai02]. Our specific contributions are:

1. An improvement of Pai's model, namely a new formulation of the Kirchhoff equations for an elastic rod in the static case, based on energy minimization. This allows for handling external forces such as response to collisions, while still providing efficiency and robustness.
2. The extension of Pai's model to handle elliptic strands. This is mandatory for dealing with hair from different

ethnies, since both the eccentricity[1] and the natural curliness play a great role in the overall hair shape.

3. A very simple handling of basic hairstyling operations such as wetting, cutting and drying, since only a few intuitive physical parameters need to be changed (length, mean radius and Young modulus). This opens the way for the design of future virtual prototyping systems to be used by hairdressers.

2. Static Simulation of a Hair Strand

Building an accurate hair strand model first requires a good understanding of its geometric and mechanical properties. A hair strand is a very thin and light elliptic tube that deforms in an anisotropic way. Whereas it can easily bend and sometimes twist, it strongly resists stretching and shearing. A hair strand also has *elastic* properties as it tends to recover its original shape after the stress applied on it has been removed. Lastly, a real hair strand can be naturally straight, wavy or curly.

As suggested by Audoly and Pomeau [AP05], we represent a hair strand as a Cosserat rod obeying the Kirchhoff equations. These equations describe elastic rods subject to arbitrary external forces such as gravity.

2.1. Cosserat Model for Rods

We define a hair strand as an inextensible, unshearable rod, with a boundary condition on the position and the orientation at the root (imposed by the hair clamping on the scalp). In this section, the strand is submitted to the gravity field only.

In the Cosserat model, the configuration of a rod is described by its centerline, a space curve $\mathbf{r}(s)$ (where s denotes the curvilinear abscissa of the rod), and a *material* frame $\mathscr{F}(s) = (\mathbf{n}_1(s), \mathbf{n}_2(s), \mathbf{t}(s))$ attached to each point on this curve (see Figure 1, left). Usually, vector $\mathbf{t}$ is the local tangent of the centerline $\mathbf{r}(s)$, and vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are lying in the plane of the local cross section of the rod. In our case, $\mathbf{n}_1$ and $\mathbf{n}_2$ are the principal axis of the elliptic cross section.

A Cosserat rod can locally bend in both directions $\mathbf{n}_1$ and $\mathbf{n}_2$, or twist onto itself. The amount of bending around $\mathbf{n}_1$ and $\mathbf{n}_2$, and the amount of twisting are respectively characterized by (material) curvatures κ_1 and κ_2, and torsion τ, through the following kinematics equations:

$$\frac{\partial \mathbf{u_i}}{\partial s} = \Omega \times \mathbf{u_i} \tag{1}$$

where $\Omega = \kappa_1\, \mathbf{n_1} + \kappa_2\, \mathbf{n_2} + \tau\, \mathbf{t}$ is the rotation vector of the rod and $\mathbf{u_i}$ is $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{t}$ for $i = 1, 2, 3$ respectively.

As previously mentioned, hair can be naturally curly. This intrinsic curliness can be expressed by natural curvatures and torsion κ_1^0, κ_2^0 and τ^0, which describe the state of the rod with no applied force. In the case of hair, it is reasonable to assume that natural curvatures and torsion are roughly constant along the hair shaft. As a result, the configuration of a hair strand in the absence of gravity is a helix (this is easy to show, knowing that rotation vector Ω is a constant vector).

[1] We use the mathematical definition of the eccentricity e of an ellipse. Thus, $e = 0$ for a circle and e increases as the ellipse flattens.

2.2. Potential Energy

We are looking for stable equilibrium configurations of a strand in the gravity field. One solution would consist of solving the Kirchhoff's equations for static rods [Pai02]. However, the nonlinearities in these equations and the boundary conditions that need to be imposed at both the free and the clamped ends prevent the equations from being integrated in a single pass, and one is forced to iterate the integration. Instead, our new approach is based on energy minimization. It offers the advantage of being both robust and computationally cheaper, which is essential when handling a large number of hair strands. Our method can also account for collision forces, as demonstrated in section 3.1.

Finding static configurations of a physical system is equivalent to the search for its minimal potential energy. The potential energy of a rod of length L formulates as:

$$\mathscr{E}_{hair} = \mathscr{E}_g + \mathscr{E}_e \tag{2}$$

where $\mathscr{E}_e$ is the internal elastic energy of the rod and $\mathscr{E}_g$ the energy of the rod accounting for gravity. Assuming hair to be a rod of elliptic cross section and obeying Hooke's law for elasticity, the elastic energy $\mathscr{E}_e$ can be written as:

$$\mathscr{E}_e = \int_0^L \left[\frac{EI_1}{2}(\kappa_1(s)-\kappa_1^0)^2 + \frac{EI_2}{2}(\kappa_2(s)-\kappa_2^0)^2 + \frac{\mu J}{2}(\tau(s)-\tau^0)^2\right] ds \tag{3}$$

where E is the Young modulus, μ the shearing modulus[2], I_1 (resp. I_2) the momentum of inertia of the rod's cross section with respect to $\mathbf{n}_1$ (resp. to $\mathbf{n}_2$) and J the axial momentum of inertia. Momenta of inertia depend on the principal radii of the cross section, and thus on hair's eccentricity e.

Potential gravitational energy $\mathscr{E}_g$ can be written as:

$$\mathscr{E}_g = \rho S g \int_0^L z(s)\, ds \tag{4}$$

where ρ is the volumic mass of the rod, S the area of its cross section, g the gravity field value and $z(s)$ the vertical coordinate of element ds at curvilinear abscissa s.

Note that the minimum for $\mathscr{E}_{hair}$ results from a balance of two antagonistic effects: the tendency to recover a naturally helical shape, represented by $\mathscr{E}_e$, and the downward pull of gravity, represented by $\mathscr{E}_g$.

2.3. Numerical Solving

We first divide the rod into n slices s_i of equal length ds, for the sake of simplicity. Along each (small) slice s_i, curvatures and torsion are assumed to be constant. We note C_n the vector of size $3 \times n$ composed of the n curvatures κ_1^i, κ_2^i and

[2] The shearing modulus of the rod is related to the Young modulus E and to the Poisson ratio σ by the following formula: $\mu = \frac{E}{2(1+\sigma)}$

torsions τ^i. Given the fact that the initial material frame $\mathscr{F}$ is imposed by the clamping into the scalp, vector C_n defines a unique configuration for the rod.

Our aim is first to find the vector C_n that minimizes the discrete energy $\mathscr{E}_{hair}$, and then to compute the corresponding configuration of the rod. Our algorithm is as follows:

- We first initialize energy $\mathscr{E}_{hair}$ and vector C_n;
- Then, until energy $\mathscr{E}_{hair}$ stops decreasing, we iteratively proceed the following steps:
  1. Compute the elastic energy $\mathscr{E}_e$ using Equation (3);
  2. Calculate formally the configuration $(\mathbf{r}(s), \mathscr{F}(s))$ of the rod, each slice being a helix;
  3. Compute the potential energy $\mathscr{E}_g$, which requires the integration of the kinematic relations (1), in order to determine the function $z(s)$. Indeed, $z(s) = <\mathbf{r}(s), \mathbf{z}>$ under the condition $z(0) = 0$, where $\mathbf{z}$ is the vertical normalized axis. Using the relation $\mathbf{t} = \frac{\partial \mathbf{r}}{\partial s}$, we get the following expression for $\mathscr{E}_g$:
  $$\mathscr{E}_g = \rho S g \int_0^L (L - s') < \mathbf{t}(s'), \mathbf{z} > ds' \qquad (5)$$
  which can be accurately evaluated in practice.
  4. Perform a minimization step for $\mathscr{E}_{hair} = \mathscr{E}_g + \mathscr{E}_e$, using a fast gradient descent approach [FP63].
- Knowing the vector C_n that minimizes energy $\mathscr{E}$, we compute the final geometric configuration $(\mathbf{r}, \mathscr{F})$ of the rod.

Some typical strand configurations obtained by our method are shown in Figures 1 and 2 (left). Our method is fast enough to handle several hair strands (sampled into 15 points) in real-time, and a hundred strands within a few seconds. This enables to use the method within an interactive hairstyling system.

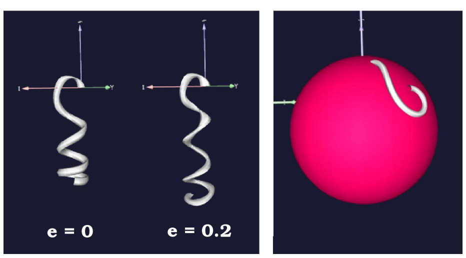

Figure 2: *Left: simulating a curly hair strand with different eccentricity values; note that increasing the eccentricity increases the regularity of the curls along the strand. Right: a curly hair strand growing on a sphere.*

3. Full Hair Modeling

Similarly to the approach of Choe *et al.* [CK05], we model hair as a set of wisps where each wisp is composed of a *master strand* and of numerous other strands that are procedurally generated within a generalized cylinder surrounding the master strand. The shape of the master strand is physically computed by energy minimization, as explained above, whereas the shape of the other strands is generated using a stochastic process similar to the one used in [CK05].

3.1. Collisions

For creating realistic hairstyles, it is necessary to account for both hair-body collisions and hair self-collisions. The latter are essential for giving an adequate volume to hair.

In our model, each wisp is modelled by a *skeleton* composed of the sample points of the master strand, and a thickness r_w^i that is computed according to several factors such as the number of hair strands within the wisp and the level of hair curliness. The body is approximated by a set of spheres for collision detection and response. We compute the collision response using elastic penalty forces F_c. The advantage is that such forces derive from a potential energy $\mathscr{E}_c$. As our static model is based on energy minimization, accounting for this kind of force simply amounts to minimizing the new energy $\mathscr{E}_{hair}$ defined by:

$$\mathscr{E}_{hair} = \mathscr{E}_g + \mathscr{E}_e + \mathscr{E}_c$$

In practice, $\mathscr{E}_c$ is computed using the amount x of penetration: $\mathscr{E}_c = \frac{1}{2} k x^2$, where k is a stiffness parameter. As illustrated in Figure 2 (right), this method properly accounts for the contacts between hair and a sphere.

Hair self-collisions are computed using the multiple layer hulls method developed by Lee and Ko [LK01]. Results show that this method is satisfying in the static case, and properly accounts for the hair volume.

3.2. Hairstyling Editing Tools

This section briefly presents the virtual hairstyling tools that are provided to the user for creating natural hairstyles. The great advantage of our approach in comparison with existing ones is that usual hairstyling operations such as curling, wetting, cutting or drying hair become straightforward.

Hair wetting has a direct impact on the hair shape, as its stiffness coefficient is roughly divided by a factor 10 [Rob02]. Moreover, as a hair strand absorbs water, its radius increases of around 13%. Ward *et al.* have proposed a hair model accounting for these properties [WGL04]. But, as this model is not characterized by adequate physical parameters, they need to control multiple structures to apply the physical changes to the hair strands. In contrast, within our accurate physically-based modelling system, the wetting of any hair strand is simply effected by modifying two relevant parameters: the Young's modulus and the mean radius. Additionally, when hair is wetted, the hair volume is procedurally reduced through the hull layers. We simply process hair drying as the inverse process of hair wetting.

Haircutting consists of removing the hair part located under a given z position. This is simply done by changing each hair length l_{prev} into $l_{prev} - l_{cut}$ where l_{cut} is the length of the removed part. As a result, hair becomes lighter after cutting, and thus its overall shape is affected, as in the real world.

4. Results and Concluding Remarks

Generated hairstyles are rendered using the accurate scattering model of Marschner *et al.* [MJC*03]. In particular, this model accounts for the visual effect due to the elliptic cross section of a hair fiber. Thus, eccentricity e represents an unified parameter for both our mechanical model and rendering. This parameter varies according to the hair type: for Asian hair, e is nearly equal to 0, whereas for Caucasoid hair, it ranges from 0 to 0.1 and for African hair, it ranges from 0 to 0.2. Other hair physical parameters E, μ and ρ are taken equal to their actual value, given by physical measurements: $E = 10\,GPa$, $\mu = 0.3$, $\rho = 1.3\,g/cm^3$ [Rob02].

Figure 1 and 3 show various hairstyles obtained with our method for different hair types and haircuts. Please also visit our website and watch our videos at http://www-evasion.imag.fr/Publications/2005/BAQLLC05/. Note that our method is able to capture realistic natural hair shapes.

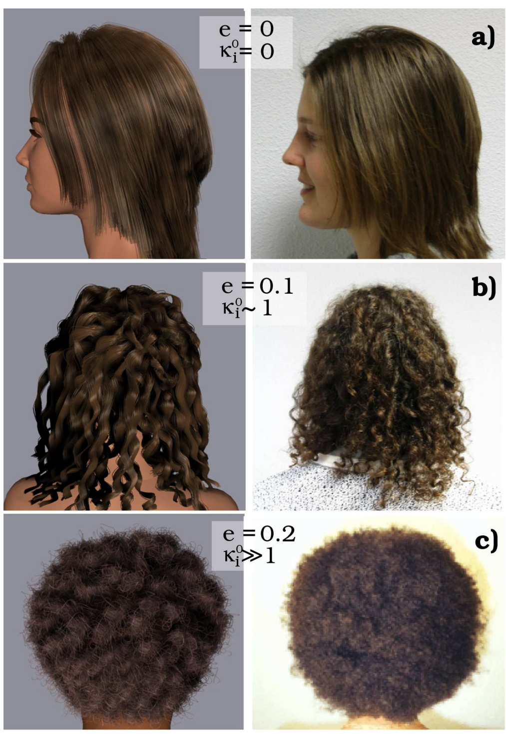

Figure 3: *Right: real pictures of (a) smooth, (b) curly and (c) African hair. Left: corresponding synthetic results, generated with adequate values for e and κ_i^0. Each synthetic hairstyle was created in less than half an hour.*

The main limitation of our method is the computational time required to calculate the equilibrium configuration of hair (6 seconds in average for the hairstyles illustrated in Figure 3). This prevents us from using more than a hundred master strands within our interactive hairstyling software. However, results are very satisfactory with this limited number of master strands, thanks to our procedural wisp model.

We are currently working on the extension of our physical model to the dynamic simulation of hair motion.

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