Component-trees and multivalued images: A comparative study

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Abstract. In this article, we discuss the way to derive connected operators based on the component-tree concept and devoted to multi-value images. In order to do so, we first extend the grey-level definition of the component-tree to the multi-value case. Then, we compare some possible strategies for colour image processing based on component-trees in two application fields: colour image filtering and colour document binarisation.

Key words: Component-trees, multi-value images, connected operators.

1 Introduction

Connected operators can be defined from various ways (for instance by region-adjacency graph merging [9], levelings [14], geodesic reconstruction, etc.). One possibility is to consider an image *via* its component-tree structure. Component-trees [19] (also known under different denominations [20, 7, 13]) have been devoted to several image processing tasks (segmentation, filtering, coding, etc.) and have, until now, exclusively involved grey-level images. Since there is an increasing need to process colour - and more generally multi-value - images, we propose to explore some solutions to use component-tree-based operators with such images. These solutions are experimentally assessed in the context of colour image filtering and colour document binarisation.

In Sec. 2, we briefly recall previous work and usual concepts related to multi-value image processing and component-trees. In Sec. 3, we propose an extension of the "grey-level" definition of the component-tree structure in order to use it with multi-value images. In Sec. 4, processing strategies allowing to use component-trees with multi-value images are described. In Sec. 5, we present two examples using the component-trees with colour images. A discussion and perspectives are given in Sec. 6.

2 Related work

2.1 Multi-value image processing

The extension of mathematical morphology to the case of colour/multi-value images is an important task, which has potential applications in multiple areas. For decades, a significant amount of work has been devoted to this specific purpose (see *e.g.* [3] for a recent survey).

Several attempts have been made to extend connected operators to colour images. Some of them are based on the contraction of a region adjacency graph structure [21]. Some others consider colour extrema using specific vectorial orderings [10, 8]. However until now, no attempts have been made to use the component-tree data structure in combination with multi-value images.

In the mathematical morphology framework, two main ways are usually proposed to perform colour image processing. The first one, called *marginal processing*, consists in processing separately the different channels of a multi-value image, thus reducing the problem to the processing of mono-value images and their fusion to recover a multi-value result. This approach is straightforward, unfortunately it may also induce several drawbacks such as the generation of false colours, for instance.

The second one, called *vectorial processing*, consists in defining a total order (or preorder) relation on the set of multi-value components. To this end, various vector-based orderings have been proposed [4]: conditional ordering (C-ordering, including lexicographic ordering), reduced ordering (R-ordering, which implies to reduce a vector value to a scalar one) which has been extensively studied in [11], "partial ordering" (P-ordering, where vectors are gathered into equivalence classes as in [22]). Recently, usual morphological operators for colour images have been derived from a total ordering based on a reduced ordering (leading to a preorder) completed by a lexicographic ordering to obtain a total order [2].

2.2 Component-trees

The component-tree structure provides a rich, scale-invariant, description for grey-level images [20, 19]. It has been involved, in particular, in the development of attribute filtering [6, 20], object identification [12, 16, 18], and image retrieval [15, 1]. In the context of segmentation or recognition tasks, it enables to perform object detection without having to precompute a specific threshold (which is usually an error-prone task).

Another advantage of this structure lies in its low algorithmic cost: efficient algorithms have been designed to compute it [20, 19, 5]. Moreover, the component-tree computation can be done offline, therefore leading to very fast (real-time) and interactive processing [24].

Until now, component-tree-based processing has always involved binary or grey-level images. We now propose to investigate the multi-value case.

3 Component-trees and multi-value images

3.1 Multi-value images

Let $n \in \mathbb{N}^*$. Let $\{(T_i, \leq_i)\}_{i=1}^n$ be a family of (finite) totally-ordered sets (namely the sets of *values*). For any $i \in [1..n]$, the infimum of T_i is denoted by $\perp_i$. Let $\mathbf{T}$ be the set defined by $\mathbf{T} = \prod_{i=1}^n T_i = T_1 \times T_2 \times \ldots \times T_n$. A value $\mathbf{t} \in \mathbf{T}$ is then a vector composed of n scalar values: $\mathbf{t} \in \mathbf{T} \Leftrightarrow \mathbf{t} = (t_i)_{i=1}^n = (t_1, t_2, \ldots, t_n)$ with $t_i \in T_i$ for any $i \in [1..n]$. Let

$\leq$ be the binary relation on $\mathbf{T}$ defined by: $\forall \mathbf{t}, \mathbf{u} \in \mathbf{T}, (\mathbf{t} \leq \mathbf{u} \Leftrightarrow \forall i \in [1..n], t_i \leq_i u_i)$. Then $(\mathbf{T}, \leq)$ is a complete lattice, the infimum $\perp$ of which is defined by $\perp = (\perp_i)_{i=1}^n$.

Let $d \in \mathbb{N}^*$. A (discrete) multi-value image is defined as a function $\mathbf{F} : \mathbb{Z}^d \to \mathbf{T}$. For all $i \in [1..n]$, the mappings $F_i : \mathbb{Z}^d \to T_i$ defined such that $\forall x \in \mathbb{Z}^d, \mathbf{F}(x) = (F_i(x))_{i=1}^n$ are called the *channels* (or the *bands*) of the multi-value image F. The support of $\mathbf{F}$ is defined by $\text{supp}(\mathbf{F}) = \{x \in \mathbb{Z}^d \mid \mathbf{F}(x) \neq \perp\}$ and we note $\text{supp}(\mathbf{F}) = E$. In the sequel, we will assume that for any image $\mathbf{F}$, $\text{supp}(\mathbf{F})$ is finite. We will then assimilate an image $\mathbf{F}$ to its (finite) restriction $\mathbf{F}_{|E} : E \to \mathbf{T}$.

3.2 Component-trees

Let $X \subseteq E$. The connected components of X are the subsets of X of maximal extent. The set of all the connected components of X is noted $\mathcal{C}[X]$.

Let $\mathcal{R}$ be a total preorder on $\mathbf{T}$, *i.e.* a binary relation verifying (*i*) reflexivity ($\forall \mathbf{t} \in \mathbf{T}, \mathbf{t} \mathrel{\mathcal{R}} \mathbf{t}$), (*ii*) transitivity ($\forall \mathbf{t}, \mathbf{u}, \mathbf{v} \in \mathbf{T}, (\mathbf{t} \mathrel{\mathcal{R}} \mathbf{u}) \wedge (\mathbf{u} \mathrel{\mathcal{R}} \mathbf{v}) \Rightarrow (\mathbf{t} \mathrel{\mathcal{R}} \mathbf{v})$) and (*iii*) totality ($\forall \mathbf{t}, \mathbf{u} \in \mathbf{T}, (\mathbf{t} \mathrel{\mathcal{R}} \mathbf{u}) \vee (\mathbf{u} \mathrel{\mathcal{R}} \mathbf{t})$).

We set $\mathcal{P}(E) = \{X \mid X \subseteq E\}$. For all $\mathbf{t} \in \mathbf{T}$, let $X_{\mathbf{t}} : \mathbf{T}^E \to \mathcal{P}(E)$ be the thresholding function defined by $X_{\mathbf{t}}(\mathbf{F}) = \{x \in E \mid \mathbf{t} \mathrel{\mathcal{R}} \mathbf{F}(x)\}$, for all $\mathbf{F} : E \to \mathbf{T}$. Since $\mathcal{R}$ is transitive, we have $\forall \mathbf{F} : E \to \mathbf{T}, \forall \mathbf{t}, \mathbf{t}' \in \mathbf{T}, \mathbf{t} \mathrel{\mathcal{R}} \mathbf{t}' \Leftrightarrow X_{\mathbf{t}'}(\mathbf{F}) \subseteq X_{\mathbf{t}}(\mathbf{F})$.

Let $\mathbf{F} : E \to \mathbf{T}$ be a multi-value image. Let $\mathcal{K} = \bigcup_{\mathbf{t} \in \mathbf{T}} \mathcal{C}[X_{\mathbf{t}}(\mathbf{F})]$. Then the relation $\subseteq$ is a partial order on $\mathcal{K}$, and the Hasse diagram $(\mathcal{K}, L)$ of the partially-ordered set $(\mathcal{K}, \subseteq)$ is a tree (*i.e.* a connected acyclic graph), the root of which is the supremum $R = \sup(\mathcal{K}, \subseteq) = E$. This rooted tree $(\mathcal{K}, L, R)$ is called the *component-tree of* $\mathbf{F}$. The elements $\mathcal{K}$, R and L are the set of the *nodes*, the *root* and the set of the *edges* of the tree, respectively.

Note that if $\mathbf{F} : E \to \mathbf{T}$ is monovalued (*i.e.* if $n = 1$ or, equivalently, $\mathbf{T} = (T_1)$) and equipped with a total order relation, then $\mathbf{F}$ can be assimilated to a function taking its values in a totally-ordered set $([0..|T_1| - 1], \leq_{\mathbb{Z}})$, and we actually retrieve the "usual" component-tree definition for grey-level images.

3.3 Tree pruning

The nodes $\mathcal{K}$ of a component-tree store information, also called *attributes*, on the associated connected components of an input image $\mathbf{F}$. Practically, to each node $N \in \mathcal{K}$, we then associate an attribute $\sigma(N) \in K$ (where K is a set of knowledge).

Let $Q : K \to \mathbb{B}$ be a criterion (*i.e.* a predicate on K). By setting $\mathcal{K}_Q \subseteq \mathcal{K}$ as $\mathcal{K}_Q = \{X \in \mathcal{K} \mid Q(X)\}$, we generate the subset of the nodes satisfying the criterion Q. Such a selection process enables to perform pruning on the component-tree of an image $\mathbf{F}$ according to a given criterion, leading to a filtering process which generates an image reconstructed from $\mathbf{F}$ with respect to $\mathcal{K}_Q$.

As it will be illustrated in the next section, for any given criterion Q, it is generally possible to define a connected operator $\Psi : \mathbf{T}^E \to \mathbf{T}^E$ which associates to a multi-value image $\mathbf{F}$ the image $\Psi(\mathbf{F})$ generated from $\mathcal{K}_Q$. Similarly, a segmentation operator $\Gamma : \mathbf{T}^E \to \mathcal{P}(E)$ can be derived by setting $\Gamma(\mathbf{F}) = \bigcup_{X \in \mathcal{K}_Q} X$.

4 Processing strategies for multi-value images

In this section, we explore some original processing strategies allowing to define connected operators for multi-value images based on the component-tree structure. These different approaches are illustrated in Fig. 1 on a synthetic image, considering an example of area opening.

4.1 Marginal processing

Based on the marginal approach, an operator Ψ can be used to handle multi-value images by processing separately each one of the n component-trees associated to each channel of the image.

For all $i \in [1..n]$, let $\mathcal{K}^i$ be the set of nodes of the individual channel F_i, according to the total order relation $\leq_i$. Then, an operator Ψ can be defined by

$$\Psi(\mathbf{F}) = \bigvee_{i \in [1..n]} \bigvee_{X \in \mathcal{K}^i_Q} C_{X,\mathbf{v}_i(X)} , \qquad (1)$$

where, $C_{X,\mathbf{v}_i(X)} : E \to \mathbf{T}$ is the cylinder function defined by $C_{X,\mathbf{v}_i(X)}(x) = \mathbf{v}_i(X)$ if $x \in X$ and $\perp$ otherwise, $\mathbf{v}_i(X) = (\perp_1, \ldots, \perp_{i-1}, m_i(X), \perp_{i+1}, \ldots, \perp_n)$ and $m_i(X)$ is the "value" of the component X in the channel F_i: $m_i(X) = \min\{F_i(x) \mid x \in X\}$.

A well-known drawback of marginal processing lies in the possible appearance of "false" values, *i.e.* values that are not present in the original image. Note however that Ψ is a connected operator, ensuring that no false contours are introduced.

4.2 Vectorial processing

By contrast to the marginal case, considering a vectorial approach for processing multi-value images can ensure that no false values will be introduced in the result image. However, the nature of the relation $\mathcal{R}$ influences the way $\Psi(\mathbf{F})$ is reconstructed from the filtered set of nodes $\mathcal{K}_C \subseteq \mathcal{K}$. In particular, the image reconstruction is different whether $\mathcal{R}$ is a total *preorder* or a total *order* relation.

Total order If $\mathcal{R}$ is a total order on $\mathbf{T}$ (*i.e.* if $\mathcal{R}$ is anti-symmetric: $\forall \mathbf{t}, \mathbf{u} \in \mathbf{T}, (\mathbf{t} \mathcal{R} \mathbf{u}) \wedge (\mathbf{u} \mathcal{R} \mathbf{t}) \Rightarrow (\mathbf{t} = \mathbf{u})$), an operator Ψ can be defined by

$$\Psi(\mathbf{F}) = \bigvee^{\mathcal{R}}_{X \in \mathcal{K}_Q} C_{X,\mathbf{v}(X)} , \qquad (2)$$

where $\mathbf{v}(X) = \min_{\mathcal{R}}\{\mathbf{F}(x) \mid x \in X\}$.

This is the case, for instance, when considering the lexicographic ordering, a case of C-ordering, or a total ordering based on the distance w.r.t. a reference vector and completed by a lexicographic ordering, as described in [2].

Total preorder If $\mathcal{R}$ is a total preorder, there may possibly exist some $\mathbf{t}, \mathbf{u} \in \mathbf{T}$ such that $\mathbf{t} \neq \mathbf{u}$ while $(\mathbf{t}\, \mathcal{R}\, \mathbf{u}) \wedge (\mathbf{u}\, \mathcal{R}\, \mathbf{t})$. For any $\mathbf{U} \subseteq \mathbf{T}$, let $\mathrm{Min}_{\mathcal{R}}\mathbf{U} = \{\mathbf{u} \in \mathbf{U} \mid \forall \mathbf{u}' \in \mathbf{U}, \mathbf{u}\, \mathcal{R}\, \mathbf{u}'\}$. Then we can derive an operator Ψ by

$$\Psi(\mathbf{F}) = \bigvee^{\mathcal{R}}_{X \in \mathcal{K}_Q} C_{X,\mathbf{v}(X)} \,, \tag{3}$$

where $\mathbf{v}(X)$ is a value computed from the set $\mathbf{V}(X) = \mathrm{Min}_{\mathcal{R}}\{\mathbf{F}(x) \mid x \in X\}$ (note that $\mathrm{Min}_{\mathcal{R}}\{\mathbf{F}(x) \mid x \in X\} = \{\min_{\mathcal{R}}\{\mathbf{F}(x) \mid x \in X\}\}$ whenever $\mathcal{R}$ is a total order). The value $\mathbf{v}(X)$ may belong to the set $\mathbf{V}(X)$ (for example by ranking the vectors using a total ordering and taking a representant value, as the median vector), or may be computed from the values of $\mathbf{V}(X)$ (by taking, for example, the mean vector of $\mathbf{V}(X)$).

Using component-trees based on a total preorder allow us, in particular, to merge in a same component several flat-zones taking distinct values. In particular, it can lead to highlight structures of interest according to some predetermined knowledge. In order to illustrate this assertion, let us consider the reduced ordering $\leq_r$ defined w.r.t. a function $r : \mathbf{T} \to \mathbb{R}$ by $\mathbf{t} \leq_r \mathbf{u} \Leftrightarrow r(\mathbf{t}) \leq r(\mathbf{u})$ for all $\mathbf{t}, \mathbf{u} \in \mathbf{T}$. If r is non-injective, $\leq_r$ induces a preorder relation on $\mathbf{T}$. For instance, we can consider the distance to a reference value $\mathbf{r} \in \mathbf{T}$, *i.e.* $r(\mathbf{t}) = d(\mathbf{r}, \mathbf{t})$. By choosing $\mathbf{r}$ close to the "colour" of some given structures of interest (*i.e.* to be removed, to be segmented, etc.), such structures may probably appear as single nodes close to the leaves of the tree. More generally, one could define a set of reference values $S = \{\mathbf{r}_i\}_{i=1}^{k}$ $(k \in \mathbb{N}^*)$. A reduced ordering could then be induced by the function $\phi_S : \mathbf{T} \to \mathbb{R}$ defined by $\phi_S(\mathbf{t}) = \min_{i \in [1..k]}\{d(\mathbf{r}_i, \mathbf{t})\}$ for all $\mathbf{t} \in \mathbf{T}$.

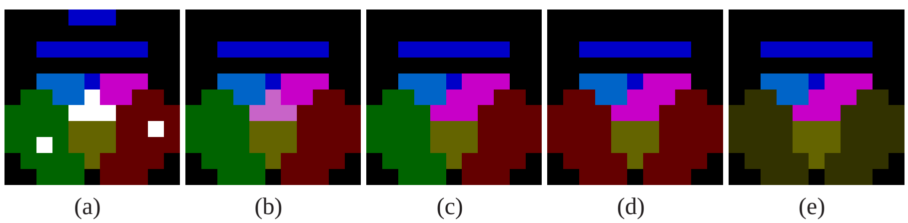


Fig. 1. Area opening of size 6 on a synthetic RGB image $\mathbf{F} : [0, 10]^2 \to [0..255]^3$. (a) Original image, (b) marginal processing, (c) vectorial processing based on a total lexicographic order (L), (d, e) vectorial processings relying on a distance-based reduced ordering using $\mathbf{r} = (255, 255, 255)$ as reference, with median reconstruction (P_{median}) (d) and mean reconstruction (P_{mean}) (e).

5 Experiments

In this section we illustrate the proposed approaches in the context of two application fields. Let $\mathbf{F} : E \to \mathbf{T}$ be a colour image defined in the RGB space $\mathbf{T} = T_r \times T_g \times T_b = [0..255]^3$, with $E \subseteq \mathbb{Z}^2$.

The following methods have been considered. The first one (M) is based on marginal processing. The other ones (L, dL, P_{mean}, P_{median}) are based on vectorial processing. Two of them rely on a total ordering of the image values: lexicographic ordering (L)

$$\mathbf{t} \leq_L \mathbf{t}' \Leftrightarrow (t_r < t'_r) \vee ((t_r = t'_r) \wedge (t_g < t'_g)) \vee ((t_r = t'_r) \wedge (t_g = t'_g) \wedge (t_b \leq t'_b)) , \quad (4)$$

and a distance-based reduced ordering followed by a lexicographic ordering (dL)

$$\mathbf{t} \leq_{dL} \mathbf{t}' \Leftrightarrow (r(\mathbf{t}) < r(\mathbf{t}')) \vee ((r(\mathbf{t}) = r(\mathbf{t}')) \wedge (\mathbf{t} \leq_L \mathbf{t}')) . \quad (5)$$

The two other ones are based on a total distance-based preordering

$$\mathbf{t} \leq_P \mathbf{t}' \Leftrightarrow r(\mathbf{t}) \leq r(\mathbf{t}') , \quad (6)$$

with $r(\mathbf{t}) = ||\mathbf{t} - \mathbf{r}||$ ($||\,.\,||$ being the Euclidean norm). One (P_{mean}) uses the *mean* vector to reconstruct the filtered image (see Sec. 4.2), the other one (P_{median}) uses the *median* vector based on a total lexicographic ordering $\leq_L$.

5.1 Colour image filtering

The proposed strategies have been evaluated from their performance in the context of image filtering. For this purpose, an image processing scheme based on area opening [23] using a component-tree implementation has been assessed. A filtering operator Ψ was defined by: $\Psi = \complement \circ \gamma_\lambda \circ \complement \circ \gamma_\lambda$, where $\complement$ denotes the colour image complement defined for all $p \in E$ by: $\complement\mathbf{F}(p) = (255 - F_r(p), 255 - F_g(p), 255 - F_b(p))$ and γ_λ is the area opening of parameter λ.

The experiments have been carried out by processing colour images corrupted by random noise and Gaussian noise.
The methods dL, P_{mean}, P_{median} used $\mathbf{r} = (255, 255, 255)$ as reference vector. From a qualitative point of view, the operator based on marginal processing (M) outperforms the other ones, since it is the one that visually preserves at best the image (see Fig. 2). The filtering operator based on preordering using a mean reconstruction (P_{mean}) preserves correctly image details, however it tends to reduce the quantisation of the image values, therefore reducing the overall image saturation. Note that all the other methods introduce undesired coloured artifacts.

As in [3], quantitative denoising comparisons based on the Normalised Mean Square Error (NMSE) measure have been performed. Given a reference image $\mathbf{F}$, the NMSE measure of the denoised image $\mathbf{F}'$ is defined by

$$\text{NMSE} = \frac{\sum_{p \in E} ||\mathbf{F}(p) - \mathbf{F}'(p)||^2}{\sum_{p \in E} ||\mathbf{F}(p)||^2} . \quad (7)$$

For the sake of comparison, the filtering operator Ψ has been compared with the OCCO filter [3] (using a marginal processing) defined by

$$\text{OCCO}_B = \frac{1}{2}\gamma_B[\phi_B(\mathbf{F})] + \frac{1}{2}\phi_B[\gamma_B(\mathbf{F})] , \quad (8)$$

Fig. 2. Comparison of filtering operators Ψ based on area opening using different processing strategies. (a) Lenna image corrupted by 15% random noise. (b) Marginal processing (M). (c) Lexicographic ordering (L). (d) Distance-based total ordering (dL). (e) Preordering (P_{mean}). (f) Preordering (P_{median}).

	M	L	dL	P_{mean}	P_{median}	OCCO
Random noise (15%)	**0.33**	0.68	1.39	0.99	1.27	2.32
Gaussian noise ($\mu = 0, \sigma = 32$)	1.13	4.07	4.08	1.34	3.49	**0.84**

Table 1. NMSE results of the proposed denoising operators based on different strategies (Lenna image, see Fig. 2).

where γ_B and ϕ_B denote the opening and closing operators, respectively. In these experiments B is the elementary ball. The results are summarised in Table 1.

In the case of random noise, the proposed filtering approach based on marginal processing leads to the lowest NMSE, emphasising the efficiency of connected based operators in this particular context.

In the case of Gaussian noise, the marginal strategy still performs better than the other ones based on the filtering Ψ operator. We notice however that the OCCO operator achieves the lowest NMSE in this context. Indeed, in the case of Gaussian noise, the original image values are not necessarily present in the corrupted version. Therefore, a denoising operator should enable to choose values that are not present in the image in order to restore the original ones. This is the case of the M, P_{mean} and OCCO methods, that are the best ones in this context. However, in a denoising context, the P_{mean} strategy suffers from the quantisation of the image values that results from the non-injective distance-based reduced ordering.

5.2 Colour document binarisation

Connected operators can be efficiently involved in object detection tasks. Based on the proposed strategies, an object extraction scheme relying on connected operators was experimented and applied to the case of colour document binarisation. This binarisation method was initially designed for grey-level document images and is fully described in [17]. We summarise it hereafter.

The core of the method is based on the concept of the component-tree *branch*. Let $(\mathcal{K}, L, R)$ be the component-tree of a monovalued (grey-level) image. The set of regional maxima (*i.e.* the set of tree *leaves*) is defined by $\mathcal{M} = \{X \in \mathcal{K} \mid \forall Y \in \mathcal{K}, Y \not\subset X\}$. The branch of the tree starting from the leaf $M \in \mathcal{M}$ is defined by the (unique) sequence of nodes $\mathcal{B}_{\mathcal{K}}(M) = (X_k)_{k=1}^{t} \in \mathcal{K}$, such that $X_1 = M$, $X_t = R$, $\forall k \in [1, t-1], X_k \subset X_{k+1} \wedge \forall Y \in \mathcal{K}, X_k \subseteq Y \subset X_{k+1} \Rightarrow Y = X_k$.

The method is based on the assumption that, for each branch of the tree, there exists a node corresponding to an object of interest. In the considered application, such a node is the one that maximises a *contrast* criterion based on the Fisher discriminant

$$J_\lambda(X) = (\mu_1 - \mu_2)^2/(\sigma_1^2 + \sigma_2^2)\ , \tag{9}$$

where μ_1 and σ_1 (resp. μ_2 and σ_2) denote the mean and standard deviation of the original values of the node X (resp. of the neighbourhood of X) and the parameter λ defines the size of the neighbourhood of X. Therefore, for each branch, a unique node maximising the criterion is preserved which allows to filter the component-tree without the use of any threshold parameter. However, using this procedure, each regional maximum can possibly create a component. Therefore, prior to the maximisation procedure, a rough binarisation based on the image grey-levels is first applied to discard irrelevant regional maxima.

Finally, a maximisation procedure on the tree branch aiming at finding the most plausible components based on the bounding-box size is performed. The proposed approach is then composed of three steps, each devoted to preserve relevant components according to a chosen criterion.

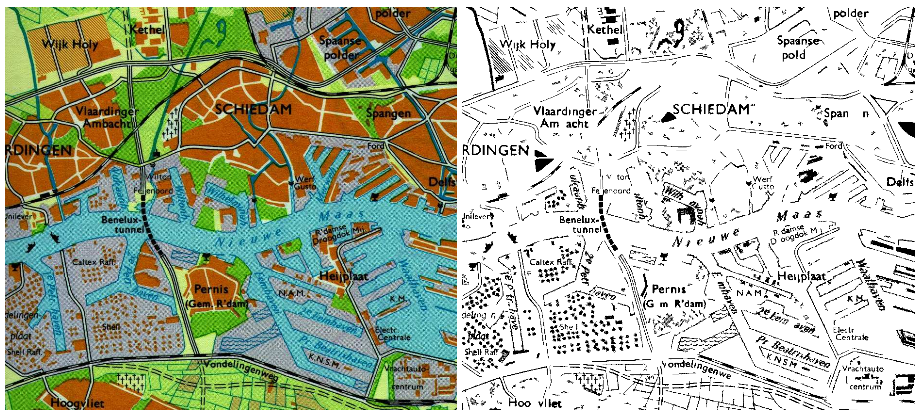
1. Rough binarisation based on a K-Means classifier applied to the pixel values.
2. For each branch of the tree, selection of the node X maximising the contrast measure $J_\lambda(X)$ (X marked as *active*).
3. For each branch of the tree, preservation of the active node maximising a size criterion (related to the bounding box of the component).

This method was applied on a set of colour documents from the MediaTeam Oulu Document Database[1]. It was implemented following the proposed strategies. Note that

[1] `http://www.mediateam.oulu.fi/downloads/MTDB`

in order to extract the relevant components, prior knowledge related to the values of the objects of interest is necessary. As a consequence, marginal processing and total orderings (L and dL) - logically - led to unsatisfactory results.

The reduced ordering based on multiple reference vector provided interesting results, since it enabled to highlight the objects of interest w.r.t. the background. However it was not appropriate in this application: for example, in Fig. 3(a), the letter "G" in black in the image upper left became connected to the river in blue, therefore preventing the correct extraction of this component. The best result was obtained by using multiple distance-based reduced ordering, each aiming at extracting characters of a given colour and by taking the supremum of the results (see Fig. 3).



(a) (b)

Fig. 3. Binarisation method of colour documents. (a) Original colour document, (b) Binarisation method based on a processing strategy involving two distance-based reduced orderings (dL) aiming at extracting black and dark blue characters.

6 Conclusion

In this article we have proposed an extension of the definition of the component-tree to the case of multi-value images equipped (at least) with a total preorder. Some solutions have also been explored to define connected operators based on the component-tree structure in the case of such images. The interest of component-tree-based operators in combination with colour images has, in particular, been illustrated in the context of denoising and binarisation.

In the case of denoising applications, the marginal processing approach remains the most efficient one, as pointed out in other works [3]. As far as object detection is considered, prior knowledge related to the object values is necessary, and therefore distance-based reduced ordering become more suited to such issues. We also believe that the usefulness of colour connected operators based on component-tree could be greatly increased by considering other - more perceptual - colour spaces. This possibility has been considered in other works [3, 2] and has not been developed here for the sake of generality.

In terms of computational efficiency, marginal processing and strategies based on reduced ordering are the fastest in the case of colour images, since the number of different values remains limited (less than 255 for 24 bits colour images in marginal processing). Approaches based on total colour ordering lead to the construction of component-trees having a large depth (around 100 000 different values in the case of the Lenna image) and huge number of nodes, therefore implying longer processing times (although specific algorithms have been designed for this case [5]).

In this paper we have not explored the case in which the thresholding function $X_{\mathbf{t}}$ is defined w.r.t. the partial order $\leq$ of $\mathbf{T}$. In this case the Hasse diagram $(\mathcal{K}, L)$ obtained from the set $\mathcal{K} = \bigcup_{\mathbf{t} \in \mathbf{T}} C[X_{\mathbf{t}}(F)]$ is not a tree anymore. This leads to a, more general, graph structure, which will be investigated in future works.

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