Distributed Coverage Control for a Multi-Robot Team in a Non-Convex Environment

Alessandro Renzaglia and Agostino Martinelli

***Abstract*— In this paper we study the problem of optimal placement for a team of mobile robots with surveillance tasks in an environment with unknown obstacles. In particular, we introduce a new approach which is based on the combination of the Voronoi partition and the potential field method. This allows obtaining distributed control which overcomes the drawbacks of these two methods, as stand-alone methods. In particular, the proposed approach is able to deal with any kind of environment. Extensive simulations are presented to show the performance of our algorithm.**

I. Introduction

In recent years it is revealed more and more the importance of using multi-robot systems for security application, otherwise impossible to be performed by a single robot. In particular by employing flying robots, many fundamental tasks are now possible. Some of these very important tasks are: surveillance of dangerous regions, like areas of chemical, biological or nuclear contamination; environmental monitoring (air quality, forest fire, ...); aiding police during surveillance missions and so on. For all these purposes an optimal deployment of the robots to well cover the environment is of primary importance. Moreover, it is fundamental to have an algorithm that takes into account the presence of obstacles for any application in a real environment. The coverage problem was introduced in literature for the first time by Gage [3]. In this work the author introduces three different kinds of coverage: blanket coverage, barrier coverage and sweep coverage. In the first one the objective is an optimal static deployment that maximizes the total detection area; the barrier coverage has the objective to minimize the probability of undetected penetration through the barrier; the last one, the sweep coverage, is a moving barrier. Poduri and Sukhatme used the potential field to obtain a coverage of a convex region with the constraint that each robot has at least K neighbors [4]. In [5] the authors have developed a method based on a local dispersion of the robots to achieve good global coverage. In [1] the Voronoi partition is used to obtain the optimal coverage of a convex region. The problem of optimal deployment is also related to the *art gallery* problem [6], well known in computational geometry. Here the aim is to find the smallest set of guards within a simple polygon such that each point of the polygon is visible by at least one guard.

Other works approach the coverage problem by a different point of view: the aim is visiting each location in a known terrain. In this case the optimization problem is to minimize the time to explore the whole region [7], [8], [9]. A survey of results for this kind of coverage problem can be find in [10].

The coverage that we want to achieve in this work is like the blanket coverage in the definitions in [3]. The main contribution of this paper is a new algorithm for the coverage of a non-convex environment. The non-convexity is due to the presence of some completely unknown obstacles and we assume the external boundary as convex (or quasi-convex) and known. Our starting point is the Voronoi partition. The limit of this method is that it can be used only when the environment to cover is convex, hence it is unusable in presence of obstacles. Our idea is to combine this method with a potential field approach based on repulsive forces between the robots and between each robot and the environment. By using only the latter method, it can be obtained a spreading out of the robots but there is a high probability that the steady state is only a local minimum usually far from the optimal solution.

The paper is organized as follows: in section II we introduce the topological concept of Voronoi tessellation and in section III we show its application to the coverage of a convex region. Then we illustrate the problem of the coverage of a non-convex region and in section V we describe the idea of combining the Voronoi partition with a potential field approach. Finally several simulation with different initial conditions and different environments are made and compared, to evaluate the performance of our algorithm.

II. Voronoi tessellation

Given an open set $\Omega \subseteq \mathbb{R}^N$, the set $\{V_i\}_{i=1}^k$ is called a tessellation (or partition) of Ω if $V_i \bigcap V_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^k V_i = \Omega$. Let $\| \cdot \|$ denote the Euclidean norm on $\mathbb{R}^N$, given a set of points $\{p_i\}_{i=1}^k$ belonging to Ω, the Voronoi region V_i corresponding to the point p_i is defined by

$$\mathcal{V}_i = \{x \in \Omega \mid \quad \|x - p_i\| \leq \|x - p_j\| \ \forall j \neq i\}\,. \quad (1)$$

The points $\{p_i\}$ are called generators.

Given a region V and a density function $\phi(q)$, defined in V, the mass, the centroid (or center of mass) and the polar moment of inertia are defined as:

$$M_V = \int_V \phi(q)dq\,, \quad C_V = \frac{1}{M_V}\int_V q\phi(q)dq\,,$$
$$J_{V,p} = \int_V \|q-p\|^2\phi(q)dq\,. \quad (2)$$

A. Renzaglia and A. Martinelli are with INRIA Rhône-Alpes, Grenoble, France `e-mail: firstname.lastname@inrialpes.fr`

Additionally, we can write the relation between $J_{V,p}$ and the polar moment of inertia about the center of mass, J_{V,C_V}:

$$J_{V,p} = J_{V,C_V} + M_V \|p - C_V\|^2 \,. \tag{3}$$

When the generators coincide with the center of mass, the tessellation is known as *centroidal tessellation*. We refer to [2], [14] for a more extended treatment on Voronoi diagrams.

III. Problem formulation

First of all, we have to define a function that measures the degree of covering of the multi-robot team. Let be p_i the position of the $i-th$ robot and $\mathcal{W}$ a tessellation of the region to cover, such that in each region W_i there is exactly one robot. Let us consider the function

$$J(\mathcal{P}, \mathcal{W}) = \sum_i \int_{W_i} f(\|q - p_i\|)\phi(q) dq \,. \tag{4}$$

The physical interpretation of the latter equation is the following: each point in W_i is weighted by

- f, which is a function of the distance between the point and the robot belonging to that region;
- ϕ, which characterizes the importance of the point.

Then, an integration over the whole region is made. The explicit dependence of f on the distance depends on the objective that we want to achieve. The final goal might be, for example, an optimal placement in order to minimize the time of intervention of at least one robot in some point of the space, or to maximize the information about the environment obtained by the robots sensors. In the first case, f will be an increasing function of the distance, in the second one it will depend on the specific characteristics of the sensor, but certainly decreasing with the distance. Our aim is to find the optimal partition $\mathcal{W}$ and optimal location P that extremize (minimize or maximize depending on the choice of $f(\|\cdot\|)$) $J(\mathcal{P}, \mathcal{W})$. It is easy to see that, at a fixed robots location, the optimal partition is, anyway, the Voronoi partition:

$$\min_{\mathcal{P},\mathcal{W}} J(\mathcal{P}, \mathcal{W}) = \min_{\mathcal{P}} J(\mathcal{P}, \mathcal{V}) \,. \tag{5}$$

Hence, we have to solve the optimization problem respect to the location only, solving the equations:

$$\nabla J_{\mathcal{V}} = [\cdots \frac{\partial J_{\mathcal{V}}}{\partial p_i} \cdots]^T = 0 \tag{6}$$

where, for simplicity sake, we have defined

$$J_{\mathcal{V}} \equiv J(\mathcal{P}, \mathcal{V}) \,. \tag{7}$$

The explicit expression of the components of (6) is:

$$\begin{aligned} \frac{\partial J_{\mathcal{V}}}{\partial p_i} = & \int_{V_i} \frac{\partial f(\|q - p_i\|)}{\partial p_i} \phi(q) \, dq \\ & + \sum_{j \in \mathcal{N}_i} \int_{\partial V_j} f(\|q - p_i\|) \, \phi(q) \frac{\partial \partial V_j}{\partial p_i} n_j \, dq \\ & + \int_{\partial V_i} f(\|q - p_i\|) \, \phi(q) \frac{\partial \partial V_i}{\partial p_i} n_i \, dq \end{aligned} \tag{8}$$

where ∂V_i denotes the boundary of the Voronoi region V_i, $n_i(q)$ denotes the outward facing normal of ∂V_i and $\mathcal{N}_i$ is the set of indices of the neighbors of p_i. It is possible to prove that the last two terms in (8) sum to zero. Indeed, only the part of ∂V_j which is shared with the boundary of the region i gives contribution in (8). Hence we can consider only these and thus we can write:

$$\bigcup_{j \in \mathcal{N}_i} \partial V_j = \partial V_i \,. \tag{9}$$

An inward normal $-n_i$ for V_i is equal to an outward normal n_j for any of its neighbors V_j, at the boundary which they share. This leads to

$$\begin{aligned} & \sum_{j \in \mathcal{N}_i} \int_{\partial V_j} f(\|q - p_i\|) \, \phi(q) \frac{\partial \partial V_j}{\partial p_i} n_j \, dq = \\ & = - \int_{\partial V_i} f(\|q - p_i\|) \, \phi(q) \frac{\partial \partial V_i}{\partial p_i} n_i \, dq \,. \end{aligned} \tag{10}$$

Hence, we can write:

$$\begin{aligned} \frac{\partial J_{\mathcal{V}}}{\partial p_i} &= \int_{V_i} \frac{\partial f(\|q - p_i\|)}{\partial p_i} \phi(q) \, dq = \\ &= - \int_{V_i} \frac{df(x)}{dx}\Big|_{\|q-p_i\|} \frac{q - p_i}{\|q - p_i\|} \phi(q) \, dq \,. \end{aligned} \tag{11}$$

Hereafter we consider only the unweighted problem, i.e. we fix $\phi(q) = 1$.

A. Possible choice of $f(\|\cdot\|)$

We write here the explicit solution of the optimization problem for two different functions $f(\|q - p_i\|)$. Let us now restrict our attention to the case in which the region to cover is 2-D.

1) $\|\cdot\|^2$: If we choose

$$f(\|q - p_i\|) = \|q - p_i\|^2 \,, \tag{12}$$

the solution of the problem is trivial and the optimal location is the centroidal one, i.e. the robots are on the centers of mass of the respective region. This statement can be proved by (11)

$$\begin{aligned} \frac{\partial J_{\mathcal{V}}}{\partial p_i} &= 2 \left[\int_{V_i} (p_i - q) \, dq \right] = \\ &= 2 M_{V_i} (p_i - C_{V_i}) \,. \end{aligned} \tag{13}$$

Because of the simplicity of its solution, this is the more frequent choice, as in [1].

2) $\|\cdot\|^{-2}$: Another possible choice is:

$$f(\|q - p_i\|) = \frac{1}{\|q - p_i\|^2} = \frac{1}{(x - p_x)^2 + (y - p_y)^2 + h^2} \tag{14}$$

where, to simplify the notation, we have indicated with p_x, p_y the components of the vector p_i, omitting the index i; h can be interpreted as the robots height of fly, assuming it is the same for each robot (here we consider the presence of this parameter to avoid the problem of the divergence in the integral, but it is easy to see that the result of the problem must be independent of its choice). This function is also of practical interest. Indeed, in many cases, the information provided by a sensor decreases quadratically with the distance. But in this case the solution is more complicated. Like in the previous example, by using (11), we have, for the y component:

$$\frac{\partial J_{\mathcal{V}}}{\partial p_y} = \iint_{V_i} \frac{2(y-p_y)}{((x-p_x)^2+(y-p_y)^2+h^2)^2}\, dx\, dy = \tag{15}$$

$$= \sum_{k=1}^{N} \int_{x_{k+1}}^{x_k} \int_0^{g_k(x)} \frac{2(y-p_y)}{((x-p_x)^2+(y-p_y)^2+h^2)^2}\, dxdy$$

where N is the number of vertices of the Voronoi polygon, (x_k, y_k) the coordinates of each of them, ordered counter-clockwise, and $x_{N+1} = x_1$. The function $g_k(x)$ is given by:

$$g_k(x) = m_k x + q_k \tag{16}$$

and the parameters m_k and q_k are:

$$m_k = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}, \quad q_k = \frac{y_k x_{k+1} - x_k y_{k+1}}{x_{k+1} - x_k}. \tag{17}$$

Considering the $k-th$ term, after the integration in y we have

$$\int_{x_k}^{x_{k+1}} \left[x^2(1+m_k^2) - 2x(p_x + m_k p_y - m_k q_k) + p_x^2 \right. \tag{18}$$

$$\left. +(p_y - q_k)^2 + h^2\right]^{-1} dx + \int_{x_k - p_x}^{x_{k+1}-p_x} \frac{dx}{x^2 + p_y^2 + h^2}.$$

When we sum over the entire boundary the second integral sum to zero, then we have to consider only the first one. It is of the type:

$$\int \frac{dx}{ax^2 - 2bx + c} = \tag{19}$$

$$= \frac{1}{\sqrt{ac-b^2}} \arctan \frac{ax-b}{\sqrt{ac-b^2}} \quad \text{if } ac - b^2 > 0$$

and in our case we have

$$ac - b^2 = (mp_x - p_y + q)^2 + h^2(1+m^2) > 0\,. \tag{20}$$

Doing the same thing for $\partial J_{\mathcal{V}}/\partial p_x$, but inverting the order of integration, we obtain a system of two equations

$$\begin{cases} g_1(p_x, p_y) = 0 \\ g_2(p_x, p_y) = 0 \end{cases} \tag{21}$$

whose solution provides the optimal position into the Voronoi region.

B. Lloyd Algorithm

Starting from an arbitrary initial robots position, one of the ways to reach the optimal configuration is provided by the Lloyd algorithm [13]. The idea is the following: calculate the Voronoi partition and the relative centers of mass, or the equivalent points (fig. 1). Hence, move each robot on its center of mass, or toward this one if its cinematical constraints do not allow it. Repeat this procedure for each time step until the convergence of the algorithm. We refer to [15] for more details and for the proof of the convergence. Later on we will refer to this algorithm with LA.

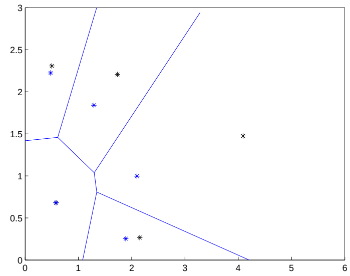


Fig. 1. Voronoi partition generated by the robots, here in blue. The black points are the center of mass of the regions.

IV. Surveillance of a non-convex region

In this section we want to extend the previous approach to a non-convex region. The importance of this problem is not only to describe a 2-D region with obstacles but also try to develop a strategy for the 3-D case, in which the non-convexity is due to terrain trasversality (buildings, hills, ...). We do not care about the visibility problem but we want to minimize the *intervention time*, i.e. the time that is necessary before at least one robot reaches every point in the space. However, because of the obstacles, it is now impossible using a cost function as in (4). A possible extension of (4) is:

$$J(\mathcal{P}) = \int_{\Omega} \min_{p_i} \tilde{d}(q, p_i)\, dq \tag{22}$$

where $\tilde{d}(q,p)$ is the distance between q and p_i, taking into account the presence of the obstacles; in other words, it is the distance that the robot p_i must cover in order to reach the point q. First of all, this distance function is strongly environment dependent. Since in our case we assume to do not know the position of the obstacles, this function is totally unknown. However, even if a map of the region is given, the optimization problem is too hard to solve. In [12] the authors, in order to achieve a similar task, have used a potential field approach. In particular they have considered only a repulsion between robot-robot and robot-obstacle but, especially when the number of robots is small respect to the dimension of the environment, it is easy to find local minima which cause not optimal solutions. By adding an attractive potential, the result can be improved, as described in the next section. The most natural choice for this attraction point is the optimal point of the Voronoi region. Let us note that to do the Voronoi tessellation the external boundary must be convex, but a non-convex boundary can be always approximated by a convex one with obstacles inside it.

V. A new approach based on the potential field method and the Voronoi partition

In this section we describe the main contribution of this paper. We propose an algorithm where the controls are determined by a potential field. The potential field method is well known in motion planning for the obstacle avoidance problem [16] and it was introduced for the first time by Khatib [11]. It has been used also in coverage problems [4],

[12]; in this last work the region to cover was unknown and not convex and the robots movement was due to a repulsive force generated by the other robots and by the obstacles, to obtain a dispersion of the robots. We will refer to this approach with RPF (Repulsive Potential Field). We propose a similar approach, where the robots and the obstacles (included the external boundary) produce a short range repulsive force but each robot is also attracted by the center of mass (or the equivalent optimal point) of its Voronoi region.
The repulsive potential used in the model is:

$$U_{rep}(\mathbf{q},\mathbf{q}_i)=\begin{cases}\frac{1}{2}k_{rep}\left(\frac{1}{\rho(\mathbf{q})}-\frac{1}{\rho_0}\right)^2 & ,\ \rho(\mathbf{q})\le\rho_0\\ 0 & ,\ \rho(\mathbf{q})>\rho_0\end{cases}\tag{23}$$

where $\mathbf{q}_i$ is the position of the robot/obstacle, $\rho(\mathbf{q})=\|\mathbf{q}-\mathbf{q}_i\|$ and ρ_0 is the range of the interaction. The artificial force induced by this potential field is $\mathbf{F}(\mathbf{q})=-\nabla U(\mathbf{q})$:

$$\mathbf{F}_{rep}(\mathbf{q},\mathbf{q}_i)=\begin{cases}k_{rep}\left(\frac{1}{\rho(\mathbf{q})}-\frac{1}{\rho_0}\right)\frac{\mathbf{q}-\mathbf{q}_i}{\rho^3(\mathbf{q})} & ,\ \rho(\mathbf{q})\le\rho_0\\ 0 & ,\ \rho(\mathbf{q})>\rho_0\end{cases}\tag{24}$$

Then, each robot feels a total repulsive force equals to:

$$\mathbf{F}_{rep}(\mathbf{q})=\sum_{i=1}^{N}\mathbf{F}_{rep}(\mathbf{q},\mathbf{q}_i)\tag{25}$$

where the sum is over the other $N-1$ robots and the closest obstacle.
The attractive potential used is:

$$U_{att}(\mathbf{q})=\frac{1}{4}k_{att}\,\rho_{goal}^4\tag{26}$$

and the relative force

$$\mathbf{F}_{att}(\mathbf{q})=k_{att}(\mathbf{q}_{goal}-\mathbf{q})\,\rho_{goal}^2\tag{27}$$

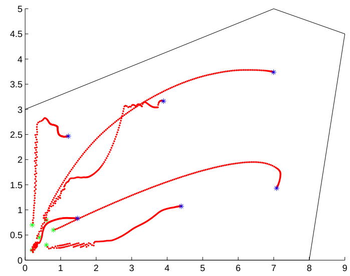
where $\rho_{goal}=\|\mathbf{q}-\mathbf{q}_{goal}\|$ and $\mathbf{q}_{goal}$ is, in our case, the center of mass. Added to these forces, we also consider a viscous term, νv, in order to have more regular trajectories. The equation of motion is:

$$\mathbf{F}_{tot}=\mathbf{F}_{rep}+\mathbf{F}_{att}=m\,\ddot{\mathbf{q}}-\nu\,\dot{\mathbf{q}}\tag{28}$$

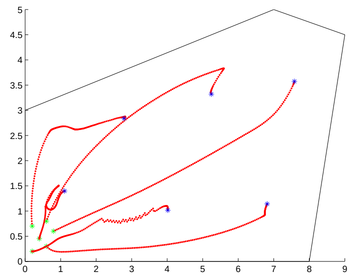
where m is the virtual robot mass which, without any loss of generality, we assume unitary. We will refer to our approach with RAPF (Repulsive and Attractive Potential Field).

VI. Simulation Results

We propose some numerical simulations using several kinds of environments and we compare our algorithm with the others existing in literature: in particular, in the convex environment we compare our RAPF with the LA; in the case of non-convex environment the comparison will be made with the RPF. In all the simulations we have used the same values for the parameters of the potentials and the same cinematical constraints for the robots. To have a quantitative result we have computed the cost function (22) discretizating the space to obtain the distance between each site of the lattice and the closest sensor. We have limited



(a)



(b)

Fig. 2. Convex environment without obstacles. In fig (a) the coverage is obtained by the RAPF, in fig (b) with the LA. The initial configurations are in green, in blue the final ones, in red the trajectories to reach them.

our study to homogeneous teams, i.e. all the robots have the same cinematical constraints (velocity and acceleration) and limited range communication with other robots and obstacles detection. Furthermore, we assume that the robots know exactly their position in a common frame of reference (e.g. by using GPS). In this simulation we have considered only the center of mass of the Voronoi regions like the optimal point to reach, i.e. making the choice (12), because it is the most suitable for our intent. In the shown simulations, the team is composed by six robots with a maximum speed of 1 m/s.

A. Convex environment

Even if the main goal of this paper is to show the applications of our approach to a non-convex environment, first of all we apply it to a convex region to prove that the result is the same as using the LA. In particular, if the region to cover is not regular and/or the robots initial positions are very close to one another, our algorithm is quite better. This can be explained in the following way: although the centroidal Voronoi partition is the optimal solution for the coverage problem, it is not trivial which is the best way to reach such configuration. In particular in the initial time steps a better spreading out of the robots can be obtained by means of repulsive potential.

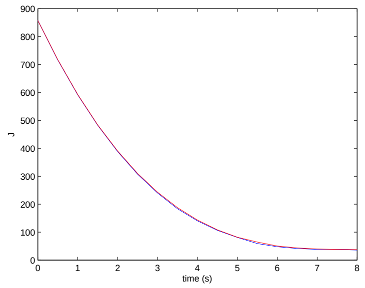


Fig. 3. Cost functions for the convex environment.

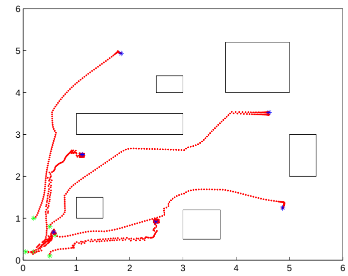
In our simulation it can be seen that, even if the trajectories are quite different (fig. 2), the two cost functions are almost identical (fig. 3). We conclude that the improvement of RAPF is negligible in the case of convex environments.

B. Non-convex environment

We show the results of the simulations for two different environments. We have remarked that the best choice of the potential parameters is environment dependent if we do not consider the Voronoi attraction. However, by using the RAPF, it is almost independent: it is a fundamental aspect if we do not have any information about the obstacles. By comparing the cost functions (fig. 5 and 7) it is possible to see that our algorithm, not only improves the performance of the steady state, but also the performance during the robots motion. In figure 8 is shown the simulation made with a different initial robots configuration in the same environment than in figure 6. Let us note that, by our algorithm, the final placement does not vary a lot with the initial conditions, while by the repulsive potential field it is strongly dependent. Due to space limitation we describe here only few results, but this behavior has been verified in many other configurations.

VII. Conclusions and Future Work

We have presented a new algorithm for the deployment of a robots team to achieve an optimal coverage of a non-convex environment with unknown obstacles. In particular, we want to minimize the distance between each point of the accessible space and the closest robot. The idea is to use a potential field method, based on the repulsion between the robots and between each robot and its closest obstacle, with the adjoint of the attraction towards the centers of mass of the Voronoi region. We have shown several results to evaluate the performance of the algorithm. Regarding a convex environment, the improvement is negligible. However, regarding the more realistic non-convex case, our algorithm significantly outperforms other existing approaches.
Future works will focus on considering more general scenarios obtained by including a weight function which gives more importance to some regions respect to others. Another interesting development is the extension of these results for a heterogeneous team, for example considering different velocity constraints for each robot.



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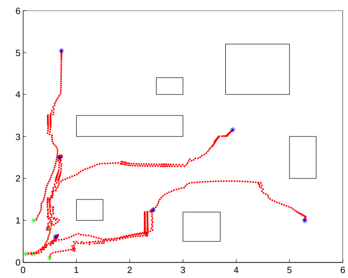

(b)

Fig. 4. Coverage obtained with our algorithm (fig. (a)) and with the only repulsive potential field (fig. (b)). In green the initial configurations, in blue the final ones, in red the trajectories to reach them.

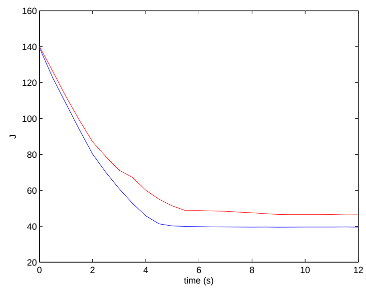

Fig. 5. Cost functions for the environment shown in fig. 4. In blue the cost function obtained with our algorithm, in red with the RPF.

VIII. Acknowledgments

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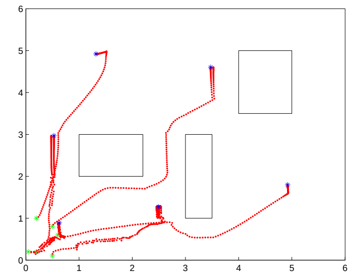

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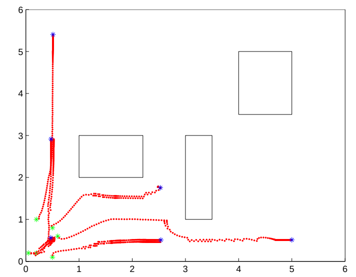

(b)

Fig. 6. Coverage obtained by the RAPF (fig. (a)) and by the RPF (fig. (b)). In green the initial configurations, in blue the final ones, in red the trajectories to reach them. The initial position of the robots is the same as in fig. 4, but the environment is different.

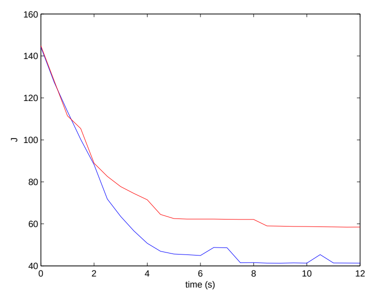

Fig. 7. Cost function for the environment shown in fig. 6. In blue the cost function obtained with our algorithm, in red with the RPF.

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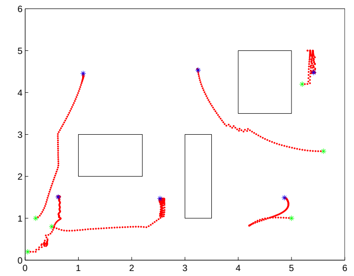

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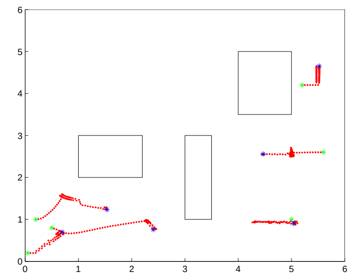

(b)

Fig. 8. Coverage obtained by the RAPF (fig. (a)) and by the RPF (fig. (b)). The environment is the same as in fig. 6, but the initial position of the robots is different.

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