Cycle Covering

Jean-Claude Bermond, Lilian Chacon, David Coudert, Francois Tillerot

Cycle Covering

J-C. BERMOND
CNRS-I3S-INRIA, France

L. CHACON
France Telecom R&D, France

D. COUDERT
CNRS-I3S-INRIA, France

F. TILLEROT
France Telecom R&D, France

Abstract

This paper considers the design of a survivable WDM network based on covering the initial network with sub-networks, which are protected independently from each other. We focus on the case where the optical WDM network is a ring, there are requests between any pair of vertices and the covering is done with small cycles. This problem can be modelled as follows: Find a covering of the edges of a logical graph I (here the complete graph K_n) by subgraphs I_k of a certain kind (here cycles C_k of length k), such that, for each I_k, there exists in the physical graph G (here C_n) a disjoint routing of the edges of I_k. The aim is to minimize the number of subgraphs I_k in the covering. We give optimal solutions for that problem.

Keywords

cycle covering, WDM, graph, survivability

1 Introduction

With the growth in data traffic and the surging demand for new services, higher capacity core networks are needed. Wavelength Division Multiplexing (WDM) technology is expected to provide solutions to this challenge. Early deployment of WDM technology has been point-to-point to increase link bandwidth by carrying several high bit-rate signals on different wavelengths in a same fiber (optical channels). Beyond this fiber capacity expansion, recent advances in WDM equipment focus on optical networking, with the ability to add, drop and construct wavelength routed networks. Routing and resource allocation problems in non survivable WDM networks have been studied for several years [1, 2, 8, 14]. Network survivability [17] (i.e. the ability to recover traffic affected by failures) is becoming a key issue in the design of ultra-high capacity networks based on WDM technology. This study considers the design of a survivable WDM network based on covering the initial network with sub-networks, which are protected independently from each other.

The planning of an optical layer can be divided into two sub-problems: computing the routing of the wavelength demands on the physical layer (routing problem) and allocating the resources to the routed demands (resource allocation problem) [1, 8, 14]. The survivability against equipment or link failure consists in computing new routes for the demands affected by a failure; thus the optical layer must be over-dimensioned. Two survivability schemes can be implemented : protection or restoration. Protection can be done by using a pre-assigned capacity between nodes in order to replace the failed or degraded transport entities. On the other hand, restoration can be realized by using any capacity available between nodes in order to find a transport entity that can replace the failed one. Furthermore, restoration is based on re-routing algorithms to find a new path to recover failed network entities, at the time the failure occurs. Dividing the network into independent sub-networks provides an intermediate solution for survivability. Indeed it allows resource sharing within the limits of a given sub-network, and uses of fast automatic protection in case of failure [16].

This paper focuses on the design of an optical layer based on survivable sub-networks. The objective is to cover the initial set of demands (logical graph) by a set of cycles. The ring topology is chosen as sub-network since it minimizes the complexity of the routing problem with full survivability for any *single failure*. Indeed we used on the cycle half of the capacity for the demands and in case of failure we reroute the traffic going through the failed link via the remaining part of the cycle using the other half of capacity. It will be interesting to get very small cycles as subnetworks as they are more easy to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one). Furthermore this cycle should satisfy the disjoint routing cycle (DRC) property, implying that it is embedded in an elementary cycle of the physical graph.

Our final aim is to minimize the cost function of the network. This cost function depends on many parameters and is difficult to study in full generality ; in first approximation, we can reduce it to minimize the number of cycles of the covering.

Finally, we will restrict ourselves to a particular case of the general problem which is interesting for practical networks and shows already the difficulty of the problem. Mainly, we will suppose that the physical network is a ring and the set of requests is the All-to-All one.

Compared to previous studies based on a similar approach [10, 6], we provide an analytical model and optimal solutions for the ring physical topology.

A similar problem is considered in [5] and [7]. They use the same ring survivability conditions but their aim is to minimize the sum of the number of vertices of the rings.

In the next section, we precise our model and hypotheses. In section 3, we recall related results from design theory, and in section 4, we give the solution for a covering with the minimum number of cycles showing that such a solution is attained with cycles of length at most 4 which is very good for our original problem.

2 Modelling

We model the optical telecommunications network by a graph, called the physical graph and denoted by G. The vertices of the graph represent the optical switches and the edges the fiber-optics links. In fact, G is an oriented symmetric multigraph; indeed each time there is a fiber optic from a node x to a node y there is also the opposite one and if there are k such fibers we should consider in the model k arcs from x to y, and k arcs from y to x. For simplicity we consider simple graphs and, for reasons stated after, we can consider instead of the symmetric digraph the underlying undirected graph obtained by replacing each pair of symmetric arcs by an edge.

Usually the physical graph has no regularity properties, just enough connectivity in order to insure a good routing for the demands even in case of failures. However many designers of an optical network builds it with loops interconnected between themselves. Thus, the first case to consider is when the physical graph is a loop (or ring), which means that the graph G is a cycle (ring). So, in the following, we will suppose that *the graph G is an undirected cycle of length n,* denoted C_n.

The family of requests (or demands), called *instance of communications*, is modelled by a graph, called logical (or virtual) graph and denoted by I. The vertices represent the nodes of the physical graph and the edges correspond to the requests between these nodes. In general, it is a digraph but in telecommunication networks used for applications like telephone the requests are symmetric (i.e. for each communication request from node A to node B there is a similar request from node B to node A). Thus, the logical graph will be undirected. Furthermore, we will suppose that the routing of symmetric requests is done by a symmetric routing. There is no constraint forcing that, but it is the way requests are actually managed. So the symmetry of the routing explains why we consider undirected graphs for the physical one. Finally the instance of communication called *total exchange* (or All-to-All), where each node wants to communicate with all the others simultaneously, is important. In such a case, the logical graph I will be the complete graph. In the following sections, we will suppose that *the logical graph I is the complete graph on n vertices, K_n.*

Routing an instance consists in finding a set of paths such that to each (symmetric) request (edge of I) is associated a path in the physical graph G between the pairs of nodes communicating in the request. Here we do not consider the allocation of wavelengths to the request (that is done later in the last phase of the network design). The securization problem considered in the introduction can be modelled by finding a covering of the edges of the logical graph I by subgraphs I_k. In general, we wish the I_k to have a simple structure and a small number of vertices. Therefore the interesting case is when I_k is a small cycle. Indeed, on the cycle we use half of the capacity for the demands, and in case of failure we reroute the traffic through the failed link via the remaining part of the cycle using the other half of the capacity. It will be interesting to get very small cycles as subnetworks as they are easier to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one). The problem of finding a covering of the edges of a graph (in particular K_n) by small cycles has been studied in the literature (see next section).

But here there is another constraint due to the fact that the requests have to be routed on the physical network G, which can be modelled as follows (one can think that to each subnetwork, here cycle, we associate a wavelength): for each subgraph I_k of the covering there should exist *edge disjoint routing in G* i.e. the paths associated in G to any pair of requests of I_k should be edge disjoint. We call this property *the disjoint routing constraint* (DRC). As an illustration, let G be C_4 and I be K_4 (See Figure 1). A first covering is given by the two C_4's $(1,2,3,4,1)$ and $(1,3,4,2,1)$ (See Figure 1.(c)), but there does not exist an edge disjoint routing for the cycle $(1,3,4,2,1)$, as it is impossible to associate the requests $(1,3)$ and $(2,4)$ to edge disjoint paths in G. In counterpart, the covering given in Figure 1.(d) by the C_4 $(1,2,3,4,1)$ and the two C_3's $(1,2,4,1)$ and $(1,3,4,1)$ satisfy the edge disjoint routing property.

Note that if a covering of I by triangles C_3 $(= K_3)$ is wanted, the DRC is satisfied as soon as G is Hamiltonian or 3-connected. However 2-connectivity is not enough as shown by the example of Figure 2.(a) where it is impossible to satisfy DRC for the triangle ABC. Furthermore, a covering by C_k satisfies DRC if G is k-connected; indeed in a k-connected graph there is always a cycle containing k given vertices.

When G is the cycle C_n, where the vertices are labelled with integers modulo n, represented by the set $\{0,1,...,n-1\}$, a C_k satisfies DRC if and only if its vertices can be ordered cyclically modulo n, that is if the vertices can be written $(a_1,a_2,\ldots,a_k)$ with $0 \leq a_1 \leq a_2 \leq \cdots \leq a_k \leq n-1$. As an example, in Figure 2.(b), the cycle $(0,2,3,6,0)$ satisfies DRC, but the cycle $(0,4,3,6,0)$ does not satisfy it.

The general problem can be summarized as follows: *Find a covering of the edges of a logical graph I by subgraphs I_k, such that, for each I_k, there exists in the physical graph G a disjoint routing of the edges of I_k and such that the cost of*

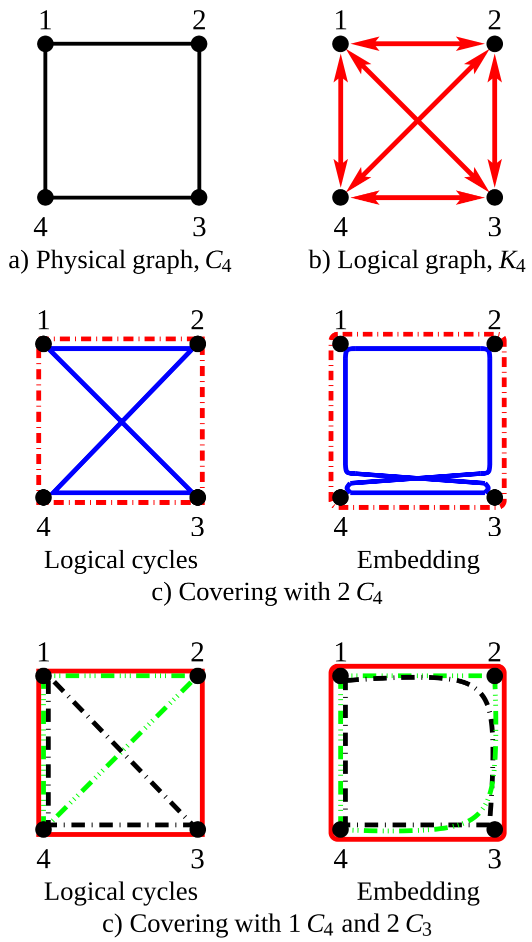
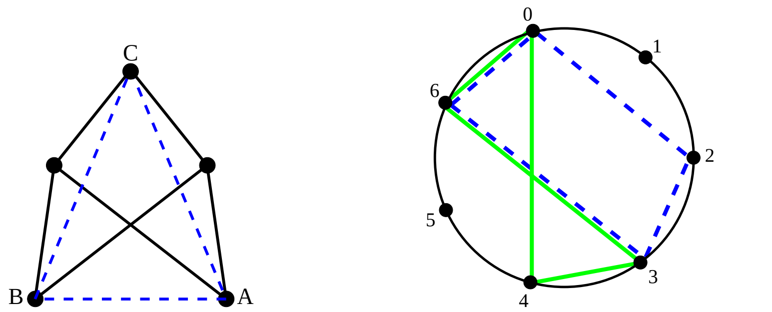


Figure 1: Cycle covering example.



a) 2-connectivity is not enough b) vertices must be cyclically ordered

Figure 2: Disjoint Routing Constraint

the network is minimized.

The aim is to minimize the cost of the network; that is a very complex function depending of the size of the ADM (Add and Drop Multiplexer) put in each node, the number of wavelengths (associated to the subnetworks) in transit in each optical node and a cost of regeneration and amplification of the signal. When the physical graph is a ring that corresponds *to minimize the number of subgraphs* I_k *in the covering* (as there is a unique physical path associated to a request). Furthermore it reduce the complexity (in terms of number of sub-networks) of the whole network. Moreover, a ring network is not able to support more than a single failure as it is only 2-connected.

Here we solve the problem when the physical graph G is the cycle of length n, C_n, I is the complete graph K_n and the I_k are cycles C_k of length k.

3 Results without disjoint routing constraint

Our problem has not yet been studied in the literature; however, without the disjoint routing constraint, it is equivalent to find a covering of the edges of a graph I by subgraphs I_k. When $I = K_n$ and $I_k = K_k$, this problem is known as the *covering design problem* (See the surveys of Stinson [15] and Mills and Mullin [12]). Moreover, the problem of finding a perfect covering of the edges is related to the existence of an $(n,k,1)$-design. In particular, for the covering by 3-cycles, we have the following results (See [13, 12]):

Theorem 1 *[13, 12] The minimum number of 3-cycles required to cover the edges of* K_n *is* $\left\lceil \frac{n}{3} \left\lceil \frac{n-1}{2} \right\rceil \right\rceil$.

When $n \equiv 1 \text{ or } 3 \mod 6$, the edges of K_n can be partitioned into $\frac{n(n-1)}{6}$ 3-cycles, also called Steiner triples.

When I_k is isomorphic to C_k, the partition problem of the edges of K_n in C_k is known as *cycle designs* (See [11] for a survey). Also, the covering of the edges of K_n by C_k, $k > 3$, was considered in [3], where one can found the following result:

Theorem 2 *[3] The minimum number of 4-cycles required to cover the edges of K_n is*

$$\left\lceil \frac{n}{4} \left\lceil \frac{n-1}{2} \right\rceil \right\rceil + \varepsilon(n) \quad \textit{with } \varepsilon(n) = \begin{cases} 1 & \textit{if } n \equiv 3 \mod 8 \\ 0 & \textit{otherwise.} \end{cases}$$

Finally, the partitioning problem of the edges of K_n by graphs isomorphic to a given graph H is considered in [4, 9].

4 Our results

Here, we consider covering of the edges of K_n by cycles satisfying the *disjoint routing constraint*. Such a covering will be called a *DRC-covering* of K_n.

We will denote by $\rho(n)$ the minimum number of cycles needed in a DRC-covering of K_n.

4.1 Lower bounds

Theorem 3 $\rho(2p+1) \geq \frac{p(p+1)}{2}$ *with* $p \geq 1$*, and* $\rho(2p) \geq \frac{p^2+1}{2}$*,* $p \geq 2$*.*

Proof. Let C_k^j be a k-cycle of DRC-covering of K_n. The disjoint routing property implies that its vertices are cyclically ordered modulo n. Thus C_k^j can be written $(a_1^j, a_2^j, \ldots, a_k^j, a_1^j)$, with $0 \leq a_1^j \leq a_2^j \leq \cdots \leq a_k^j \leq n-1$.

Let $\delta_i^j = a_{i+1}^j - a_i^j$, $1 \leq i \leq k-1$, and $\delta_k^j = n + a_1^j - a_k^j$. The disjoint routing property implies $\sum_i \delta_i^j = n$.

For an edge (x,y) of K_n with $x < y$, we call difference of the edge the value $y - x$ if $y - x \leq n/2$ or $x + n - y$ otherwise (it corresponds to the distance between x and y on a cycle of length n).

In the odd case, $n = 2p+1$, the covering must contain the n edges of difference d for every d, $1 \leq d \leq p$. Each difference correspond to an δ_i^j, with $\delta_i^j = d$ or $n - d$. Thus, $\sum_{i,j} \delta_i^j \geq \sum_{d=1}^{p} nd = n\frac{p(p+1)}{2}$. Remind that $\sum_i \delta_i^j = n$. Consequently, if the covering contains $\rho(n)$ cycles, we have $n\rho(n) \geq n\frac{p(p+1)}{2}$ and finally, $\rho(n) \geq \frac{p(p+1)}{2}$.

In the even case, $n = 2p$, the covering must contain n edges of difference d, $1 \leq d \leq p-1$ and $\frac{n}{2} = p$ edges of difference p. Furthermore, since the degree of the nodes in K_n is odd (equal to $n-1$) and the degree of the nodes of a cycle is even (equal to 2), the covering must contain extra edges (i.e. in each vertex, there is an edge covered at least twice). Thus, there are at least $\frac{n}{2}$ extra edges of difference at least 1 in the covering (corresponding to a perfect matching). Consequently, $\sum_{i,j} \delta_i^j \geq \left(\sum_{d=1}^{p-1} nd\right) + pp + p = p(p^2+1)$ and if the covering contains $\rho(n)$ cycles, we obtain $n\rho(n) = 2p\rho(n) \geq p(p^2+1)$ and finally $\rho(n) \geq \frac{p^2+1}{2}$. $\square$

Note that for both odd and even cases, the length of the cycles involved in the DRC-covering of K_n has no influence on the lower bound of $\rho(n)$. Thus, as it will be confirm in the following, small cycles are sufficient to reach the lower bound.

4.2 Minimum DRC covering

Theorem 4 *When $n = 2p+1$, $\rho(n) = \frac{p(p+1)}{2}$. Furthermore, the DRC-covering of K_{2p+1} consists of p C_3 and $\frac{p(p-1)}{2}$ C_4.*

Proof. (By induction on p) K_3 is covered using one C_3. Thus, the theorem is true when $p = 1$.

Suppose now that the theorem is true for K_{2p+1}. We will show that it is also true for $n = 2p+3$. For that, let us arrange the vertices of K_{2p+3} in the following order: $A, 0, 1, \ldots, p-1, B, p, \ldots, 2p$.

We build a DRC-covering of K_{2p+3} from a DRC-covering of K_{2p+1} as follows. The cycles of the DRC-covering of K_{2p+3} will be

- the $p(p+1)/2$ cycles of a DRC-covering of the K_{2p+1} on vertices $0, 1, \ldots, p-1, p, \ldots, 2p$,
- the p C_4 of a DRC-decomposition of the $K_{2p,2}$ constructed between vertices $0, \ldots, p-1, p+1, \ldots, 2p$ on one side and vertices A and B on the other side, namely the C_4 $(A, i, B, p+1+i, A)$, $0 \leq i \leq p-1$,
- the C_3 (A, B, p, A).

One can check that each edge of K_{2p+3} is covered by exactly one of these cycles and altogether we have $p(p+1)/2 + p + 1 = (p+1)(p+2)/2$ cycles. Furthermore, there are exactly $p+1$ C_3 and $p(p+1)/2$ C_4. $\square$

In Figure 3, we show a covering of K_5 obtained in that way. Let us called the vertices of K_5 $A, 0, B, 1, 2$ in that order. The DRC-covering of K_5 consists of the unique cycle $(0, 1, 2, 0)$ of the covering of K_3, plus the C_4 $(A, 0, B, 2, A)$ of a DRC-decomposition of the $K_{2,2}$ constructed between vertices A and B and vertices 0 and 2, plus the C_3 $(A, B, 1, A)$.

Theorem 5 *When $n = 2p$, $p \geq 3$, $\rho(n) = \left\lceil \frac{p^2+1}{2} \right\rceil$. Furthermore, when $n = 4q$, the DRC-covering of K_{4q} consists of 4 C_3 and $2q^2 - 3$ C_4, and when $n = 4q+2$ the DRC-covering of K_{4q+2} consists of 2 C_3 and $2q^2 + 2q - 1$ C_4.*

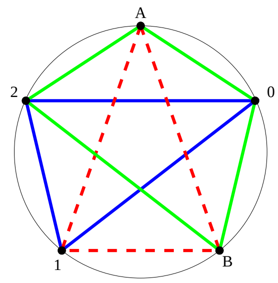


Figure 3: Covering of K_5 obtained from the covering of K_3.

In order to prove this theorem, we first need to prove some lemmas.

Lemma 1 *K_6 can be covered by 2 C_3 and 3 C_4.*

Proof. The covering is given by the two C_3: $(0,1,3,0)$ and $(0,1,4,0)$, plus three C_4: $(0,2,4,5,0)$, $(1,2,3,5,1)$ and $(2,3,4,5,2)$, as shown in Figure 4. Furthermore, there are three edges, $(0,1)$, $(2,3)$ and $(4,5)$, covered exactly twice (they form a perfect matching). □

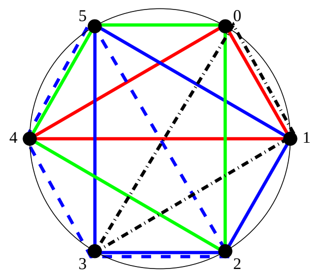


Figure 4: K_6

Lemma 2 *If there exists a DRC-covering of K_{4q+2} with $\rho(4q+2) = 2q^2+2q+1$ cycles, then there exists a DRC-covering of K_{4q+4} with $\rho(4q+4) = 2q^2+4q+3$ cycles.*

Proof. Let us call and range the vertices of K_{4q+4} in the following order: $A, 0, 1, \ldots, 2q, B, 2q+1, \ldots, 4q+1$.

We build a DRC-covering of K_{4q+4} from a DRC-covering of K_{4q+2} as follows. The cycles of the DRC-covering of K_{4q+4} will be

- the $2q^2 + 2q + 1$ cycles of a DRC-covering of the K_{4q+2} on vertices $0, 1, \ldots, 2q, 2q+1, \ldots, 4q+1$,
- the $2q$ C_4 of a DRC-decomposition of the $K_{4q,2}$ constructed on vertices $1, \ldots, 2q, 2q+1, \ldots, 4q$ on one side, and vertices A and B on the other side, namely, the C_4 $(A, i, B, 2q+i, A)$, $1 \leq i \leq 2q$,
- the 2 triangles $(A, 0, B, A)$ and $(A, B, 4q+1, A)$.

One can check that every edge of K_{4q+4} is covered by one of these cycles and that altogether we have $2q^2+2q+1+2q+2 = 2q^2+4q+3 = \left\lceil \frac{(2q+2)^2+1}{2} \right\rceil$ cycles. Furthermore, there are exactly 4 C_3 in the covering (2 from the DRC-covering of K_{4q+2} and the 2 extra C_3). □

To illustrate this proof, we indicate in Figure 5 the cycles involved in the covering of K_8.

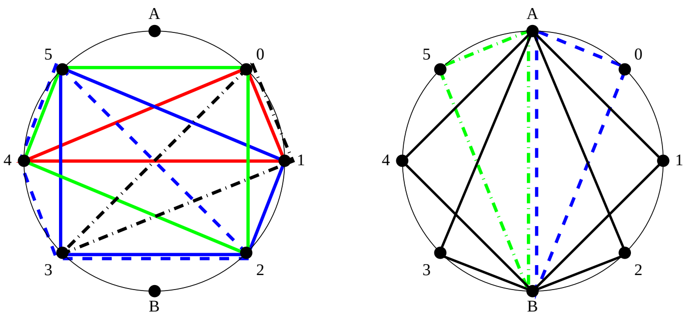


Figure 5: Cycles involved in the covering of K_8.

Lemma 3 *If there exists a DRC-covering of K_{4q+2} with $\rho(4q+2) = 2q^2+2q+1$ cycles, then there exists a DRC-covering of K_{4q+6} with $\rho(4q+6) = 2q^2+6q+5$ cycles.*

Proof.

We will prove a little stronger theorem, imposing some extra properties in the decomposition which will be kept in the construction.

Let us suppose that there exists a DRC-covering of K_{4q+2}, where the nodes $0, 1, \ldots, 4q+1$ are cyclically ordered, with the following properties:

- the edges (of the perfect matching) $(0,1), (2,3), \ldots, (4q, 4q+1)$ are covered exactly twice, while other edges are covered exactly once.

- the edge $(0,1)$ belongs to the 3-cycle $(0,1,x,0)$, for some x different from $0,1$.

We will show that there exists a DRC-covering of K_{4q+6} with the same properties. Note that these properties are satisfied by the covering of K_6 of Lemma 1 (x being either 3 or 4).
Let us call and range the vertices of K_{4q+6} in the following order: $0,A,B,1,\ldots,2q+1,C,D,2q+2,\ldots,4q+1$. The cycles of the DRC-covering of K_{4q+6} will be

- the $2q^2+2q$ cycles of the covering of K_{4q+2} except the 3-cycle $(0,1,x,0)$,
- the $2q$ 4-cycles $(A,i,C,f(i),A)$, with $2 \leq i \leq 2q+1$ and where f is a bijection from $\{2,3,\ldots,2q+1\}$ to $\{2q+2,\ldots,4q+1\}$,
- the $2q+1$ 4-cycles $(B,j,D,g(j),B)$, $1 \leq j \leq 2q+1$, and where g is a bijection from $\{1,2,\ldots 2q+1\}$ to $\{2q+2,\ldots,4q+1,0\}$.
- the 3 C_4 (A,B,C,D,A), $(0,A,1,x,0)$, $(B,1,C,D,B)$ and the C_3 $(0,A,C,0)$,

One can check that each edge of K_{4q+6} is covered by one of these cycles and that altogether, we have $2q^2+2q+2q+2q+1+3+1 = 2q^2+6q+5 = \left\lceil \frac{(2q+2)^2+1}{2} \right\rceil$ cycles. Furthermore, there are still exactly 2 C_3 in the covering. Also, the edges $(0,A),(B,1),(2,3),\ldots,(2q,2q+1),\ldots,(C,D),(2q+2,2q+3),\ldots,(4q,4q+1)$ (corresponding to a perfect matching) are covered twice, while other edges are covered only once. Moreover, the edge $(0,A)$, which is covered twice, appears in the 3-cycle $(0,A,C,0)$. □

Now, we are able to prove Theorem 5.

Proof. of Theorem 5 (By induction)
The theorem is true for $n=6$ as shown by Lemma 1. Note that the covering of K_6 satisfies the two extra properties needed in the proof of Lemma 3. So using Lemma 3, one can build by induction the DRC-covering of K_{4q+2}, $q \geq 1$, by $\rho(4q+2) = 2q^2+2q+1$ cycles. Then, using Lemma 2, one can build the DRC-covering of K_{4q+4}, $q \geq 1$, by $\rho(4q+4) = 2q^2+4q+3$ cycles.
So Theorem 5 is proved. □

5 Conclusion

The problem of the design of a survivable WDM network was considered as an extension of the classical edge covering problem by addition of the disjoint routing constraint. In particular, we have studied the case of a physical ring network with the all-to-all (K_n) communication instance. For this design problem, we give a solution with the optimal number of sub-network (cycles). As an extension of

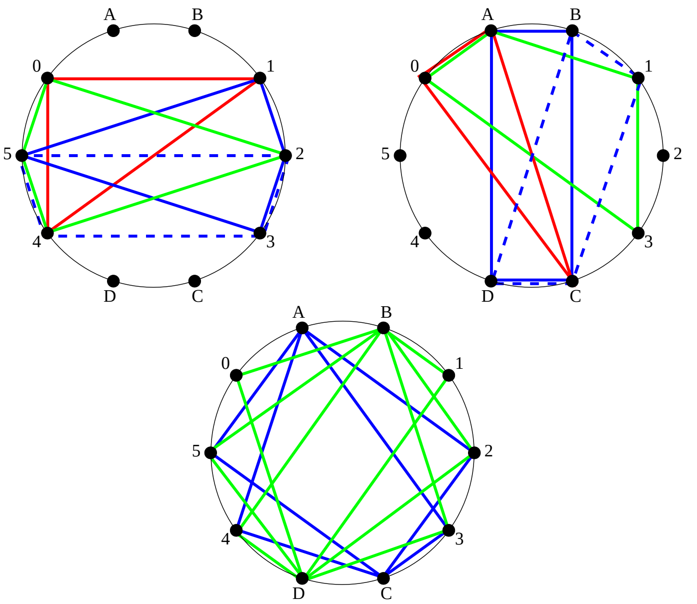


Figure 6: Cycles involved in the covering of K_{10}.

this problem, we are now investigating cases with other communication instances such as λK_n (or more general logical graphs). We also think to consider other network topologies for example trees of rings, grids or tori. Moreover it will be interesting to consider in these extensions the real cost function.

References

[1] B. Beauquier. *Communications dans les réseaux optiques par multiplexage en longueur d'onde*. PhD thesis, Université de Nice Sophia-Antipolis, Janvier 2000.

[2] B. Beauquier, J-C. Bermond, L. Gargano, P. Hell, S. Perennes, and U. Vaccaro. Graph problems arising from Wavelength-Routing in All-Optical Networks. In *IEEE Workshop on Optics and Computer Science*, 1997.

[3] J-C. Bermond. *Cycles dans les graphes et G-configurations.* Thèse d'Etat, 1975, Université de Paris Sud, Orsay.

[4] J-C. Bermond and D. Sotteau. Graph decomposition and G-designs. In *5th British Combinatorial Conference*, Congressus Numerantium 15, pages 53–72, Aberdeen, 1975. Utilitas Math. Pub.

[5] T. Eilam, S. Moran, and S. Zaks. Approximation algorithms for survivable optical networks. In *The 14th international Symposium on Distributed Computing (DISC)*, pages 104–118, 2000.

[6] G. Ellinas, A. Hailemariam, and T. Stern. Protection cycles in mesh WDM networks. *IEEE Journal on Selected Areas in Communications*, 18(10):1924–1937, 2000.

[7] O. Gerstel, P. Lin, and G. Sasaki. Wavelength assignement in a WDM ring to minimize system cost instead of number of wavelengths. In *IEEE INFOCOM*, volume 1, pages 94–101, 1998.

[8] P.E. Green. *Fiber optic networks*. Prentice Hall, 1993.

[9] K. Heinrich. Graph decompositions and designs. In J.H. Dinitz and D.R. Stinson, editors, *Contemporary designs a collection of surveys*. Wiley, 1992.

[10] E. Limal, B. Mikkelsen, and K.E. Stubkjr. Building WDM wide area resilient network from 4-node semi-mesh and mesh sub-networks. In *IEEE International Conference on Communication (ICC), paper 36.4*, Montreal, Canada, June 1997.

[11] C.C. Lindner and C.A. Rodger. Decomposition into Cycles II : cycle systems. In J.H. Dinitz and D.R. Stinson, editors, *Contemporary designs a collection of surveys*. Wiley, 1992.

[12] W.H. Mills and R.C. Mullin. Coverings and packings. In J.H. Dinitz and D.R. Stinson, editors, *Contemporary designs a collection of surveys*. Wiley, 1992.

[13] R.G. Stanton and M.J. Rogers. Packings and covering by triples. *Ars combinatoria*, 13:61–69, 1982.

[14] T. Stern and K. Bala. *Multiwavelength optical networks, a layered approach*. Addison Wesley, 1999.

[15] D.R. Stinson. *Coverings*, chapter 8 of the CRC handbook of Combinatorial designs by C.J. Colbourn and J.H. Dinitz. CRC Press, 1996.

[16] F. Tillerot, E. Didelet, A. Daviaud, and G. Claveau. Efficient network upgrade based on a WDM optical layer with automatic protection switching. In *OSA Optical Fiber Communications Conference (OFC)*, San Jose (CA), USA, Feb. 22-27 1998.

[17] Tsong-Ho Wu. Emerging Technologies for Fiber Network Survivability. *IEEE Communications Magazine*, 33(2):58–74, 1995.

Jean-Claude Bermond is with the MASCOTTE project, CNRS-I3S-INRIA, 2004, route des Lucioles, B.P. 93, F-06902 Sophia-Antipolis Cedex.
E-mail: jean-claude.bermond@sophia.inria.fr

Lilian Chacon is with France Telecom R&D, Technopole Anticipa, 2, avenue Pierre Marzin, F-22307 Lannion Cedex.
E-mail: lilian.chacon@rd.francetelecom.fr

David Coudert is with the MASCOTTE project, CNRS-I3S-INRIA, 2004, route des Lucioles, B.P. 93, F-06902 Sophia-Antipolis Cedex.
E-mail: david.coudert@sophia.inria.fr

François Tillerot is with France Telecom R&D, Technopole Anticipa, 2, avenue Pierre Marzin, F-22307 Lannion Cedex.
E-mail: francois.tillerot@rd.francetelecom.fr