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# Cycle Covering

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## Abstract

This paper considers the design of a survivable WDM network based on covering the initial network with sub-networks, which are protected independently from each other. We focus on the case where the optical WDM network is a ring, there are requests between any pair of vertices and the covering is done with small cycles. This problem can be modelled as follows: Find a covering of the edges of a logical graph  $I$  (here the complete graph  $K_n$ ) by subgraphs  $I_k$  of a certain kind (here cycles  $C_k$  of length  $k$ ), such that, for each  $I_k$ , there exists in the physical graph  $G$  (here  $C_n$ ) a disjoint routing of the edges of  $I_k$ . The aim is to minimize the number of subgraphs  $I_k$  in the covering. We give optimal solutions for that problem.

## Keywords

cycle covering, WDM, graph, survivability

## 1 Introduction

With the growth in data traffic and the surging demand for new services, higher capacity core networks are needed. Wavelength Division Multiplexing (WDM) technology is expected to provide solutions to this challenge. Early deployment of WDM technology has been point-to-point to increase link bandwidth by carrying several high bit-rate signals on different wavelengths in a same fiber (optical channels). Beyond this fiber capacity expansion, recent advances in WDM equipment focus on optical networking, with the ability to add, drop and construct wavelength routed networks. Routing and resource allocation problems in

non survivable WDM networks have been studied for several years [1, 2, 8, 14]. Network survivability [17] (i.e. the ability to recover traffic affected by failures) is becoming a key issue in the design of ultra-high capacity networks based on WDM technology. This study considers the design of a survivable WDM network based on covering the initial network with sub-networks, which are protected independently from each other.

The planning of an optical layer can be divided into two sub-problems: computing the routing of the wavelength demands on the physical layer (routing problem) and allocating the resources to the routed demands (resource allocation problem) [1, 8, 14]. The survivability against equipment or link failure consists in computing new routes for the demands affected by a failure; thus the optical layer must be over-dimensioned. Two survivability schemes can be implemented : protection or restoration. Protection can be done by using a pre-assigned capacity between nodes in order to replace the failed or degraded transport entities. On the other hand, restoration can be realized by using any capacity available between nodes in order to find a transport entity that can replace the failed one. Furthermore, restoration is based on re-routing algorithms to find a new path to recover failed network entities, at the time the failure occurs. Dividing the network into independent sub-networks provides an intermediate solution for survivability. Indeed it allows resource sharing within the limits of a given sub-network, and uses of fast automatic protection in case of failure [16].

This paper focuses on the design of an optical layer based on survivable sub-networks. The objective is to cover the initial set of demands (logical graph) by a set of cycles. The ring topology is chosen as sub-network since it minimizes the complexity of the routing problem with full survivability for any *single failure*. Indeed we used on the cycle half of the capacity for the demands and in case of failure we reroute the traffic going through the failed link via the remaining part of the cycle using the other half of capacity. It will be interesting to get very small cycles as subnetworks as they are more easy to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one). Furthermore this cycle should satisfy the disjoint routing cycle (DRC) property, implying that it is embedded in an elementary cycle of the physical graph.

Our final aim is to minimize the cost function of the network. This cost function depends on many parameters and is difficult to study in full generality ; in first approximation, we can reduce it to minimize the number of cycles of the covering.

Finally, we will restrict ourselves to a particular case of the general problem which is interesting for practical networks and shows already the difficulty of the problem. Mainly, we will suppose that the physical network is a ring and the set of requests is the All-to-All one.

Compared to previous studies based on a similar approach [10, 6], we provide an analytical model and optimal solutions for the ring physical topology.

A similar problem is considered in [5] and [7]. They use the same ring survivability conditions but their aim is to minimize the sum of the number of vertices of the rings.

In the next section, we precise our model and hypotheses. In section 3, we recall related results from design theory, and in section 4, we give the solution for a covering with the minimum number of cycles showing that such a solution is attained with cycles of length at most 4 which is very good for our original problem.

## 2 Modelling

We model the optical telecommunications network by a graph, called the physical graph and denoted by  $G$ . The vertices of the graph represent the optical switches and the edges the fiber-optics links. In fact,  $G$  is an oriented symmetric multi-graph; indeed each time there is a fiber optic from a node  $x$  to a node  $y$  there is also the opposite one and if there are  $k$  such fibers we should consider in the model  $k$  arcs from  $x$  to  $y$ , and  $k$  arcs from  $y$  to  $x$ . For simplicity we consider simple graphs and, for reasons stated after, we can consider instead of the symmetric digraph the underlying undirected graph obtained by replacing each pair of symmetric arcs by an edge.

Usually the physical graph has no regularity properties, just enough connectivity in order to insure a good routing for the demands even in case of failures. However many designers of an optical network builds it with loops interconnected between themselves. Thus, the first case to consider is when the physical graph is a loop (or ring), which means that the graph  $G$  is a cycle (ring). So, in the following, we will suppose that *the graph  $G$  is an undirected cycle of length  $n$* , denoted  $C_n$ .

The family of requests (or demands), called *instance of communications*, is modelled by a graph, called logical (or virtual) graph and denoted by  $I$ . The vertices represent the nodes of the physical graph and the edges correspond to the requests between these nodes. In general, it is a digraph but in telecommunication networks used for applications like telephone the requests are symmetric (i.e. for each communication request from node  $A$  to node  $B$  there is a similar request from node  $B$  to node  $A$ ). Thus, the logical graph will be undirected. Furthermore, we will suppose that the routing of symmetric requests is done by a symmetric routing. There is no constraint forcing that, but it is the way requests are actually managed. So the symmetry of the routing explains why we consider undirected graphs for the physical one. Finally the instance of communication called *total exchange* (or All-to-All), where each node wants to communicate with all the others simultaneously, is important. In such a case, the logical graph  $I$  will be the complete graph. In the following sections, we will suppose that *the logical graph  $I$  is the complete graph on  $n$  vertices,  $K_n$* .

Routing an instance consists in finding a set of paths such that to each (symmetric) request (edge of  $I$ ) is associated a path in the physical graph  $G$  between the pairs of nodes communicating in the request. Here we do not consider the allocation of wavelengths to the request (that is done later in the last phase of the network design). The securization problem considered in the introduction can be modelled by finding a covering of the edges of the logical graph  $I$  by subgraphs  $I_k$ . In general, we wish the  $I_k$  to have a simple structure and a small number of vertices. Therefore the interesting case is when  $I_k$  is a small cycle. Indeed, on the cycle we use half of the capacity for the demands, and in case of failure we reroute the traffic through the failed link via the remaining part of the cycle using the other half of the capacity. It will be interesting to get very small cycles as subnetworks as they are easier to manage and less costly to reroute. Also, we will associate a wavelength to each cycle (in fact two: one for the normal traffic and one for the spare one). The problem of finding a covering of the edges of a graph (in particular  $K_n$ ) by small cycles has been studied in the literature (see next section).

But here there is another constraint due to the fact that the requests have to be routed on the physical network  $G$ , which can be modelled as follows (one can think that to each subnetwork, here cycle, we associate a wavelength): for each subgraph  $I_k$  of the covering there should exist *edge disjoint routing in  $G$*  i.e. the paths associated in  $G$  to any pair of requests of  $I_k$  should be edge disjoint. We call this property *the disjoint routing constraint (DRC)*. As an illustration, let  $G$  be  $C_4$  and  $I$  be  $K_4$  (See Figure 1). A first covering is given by the two  $C_4$ 's  $(1, 2, 3, 4, 1)$  and  $(1, 3, 4, 2, 1)$  (See Figure 1.(c)), but there does not exist an edge disjoint routing for the cycle  $(1, 3, 4, 2, 1)$ , as it is impossible to associate the requests  $(1, 3)$  and  $(2, 4)$  to edge disjoint paths in  $G$ . In counterpart, the covering given in Figure 1.(d) by the  $C_4$   $(1, 2, 3, 4, 1)$  and the two  $C_3$ 's  $(1, 2, 4, 1)$  and  $(1, 3, 4, 1)$  satisfy the edge disjoint routing property.

Note that if a covering of  $I$  by triangles  $C_3$  ( $= K_3$ ) is wanted, the DRC is satisfied as soon as  $G$  is Hamiltonian or 3-connected. However 2-connectivity is not enough as shown by the example of Figure 2.(a) where it is impossible to satisfy DRC for the triangle  $ABC$ . Furthermore, a covering by  $C_k$  satisfies DRC if  $G$  is  $k$ -connected; indeed in a  $k$ -connected graph there is always a cycle containing  $k$  given vertices.

When  $G$  is the cycle  $C_n$ , where the vertices are labelled with integers modulo  $n$ , represented by the set  $\{0, 1, \dots, n-1\}$ , a  $C_k$  satisfies DRC if and only if its vertices can be ordered cyclically modulo  $n$ , that is if the vertices can be written  $(a_1, a_2, \dots, a_k)$  with  $0 \leq a_1 \leq a_2 \leq \dots \leq a_k \leq n-1$ . As an example, in Figure 2.(b), the cycle  $(0, 2, 3, 6, 0)$  satisfies DRC, but the cycle  $(0, 4, 3, 6, 0)$  does not satisfy it.

The general problem can be summarized as follows: *Find a covering of the edges of a logical graph  $I$  by subgraphs  $I_k$ , such that, for each  $I_k$ , there exists in the physical graph  $G$  a disjoint routing of the edges of  $I_k$  and such that the cost of*

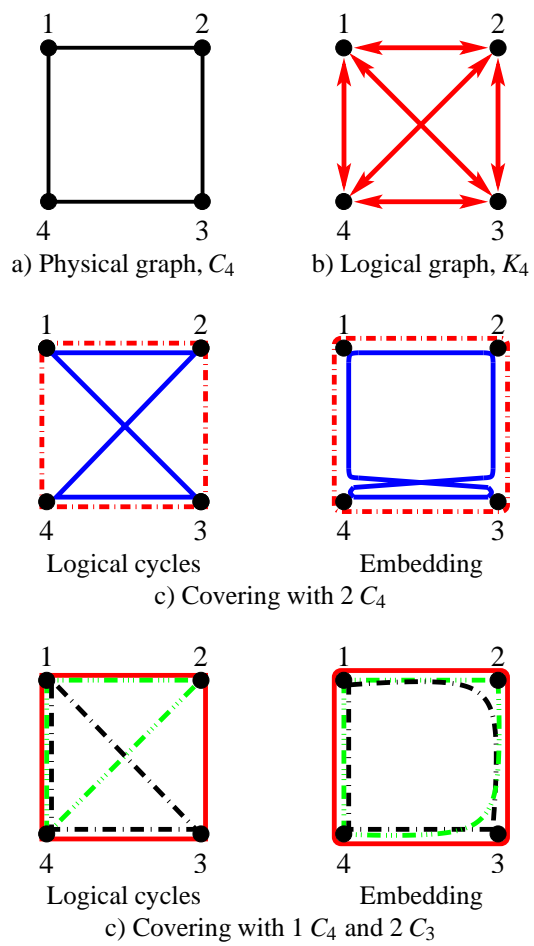


Figure 1: Cycle covering example.

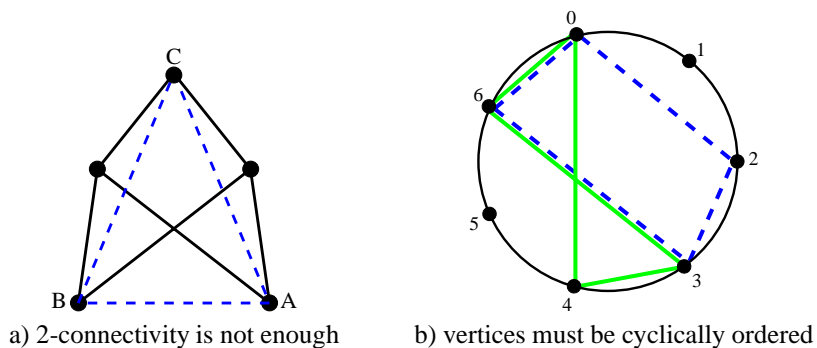


Figure 2: Disjoint Routing Constraint

the network is minimized.

The aim is to minimize the cost of the network; that is a very complex function depending of the size of the ADM (Add and Drop Multiplexer) put in each node, the number of wavelengths (associated to the subnetworks) in transit in each optical node and a cost of regeneration and amplification of the signal. When the physical graph is a ring that corresponds to minimize the number of subgraphs  $I_k$  in the covering (as there is a unique physical path associated to a request). Furthermore it reduce the complexity (in terms of number of sub-networks) of the whole network. Moreover, a ring network is not able to support more than a single failure as it is only 2-connected.

Here we solve the problem when the physical graph  $G$  is the cycle of length  $n$ ,  $C_n$ ,  $I$  is the complete graph  $K_n$  and the  $I_k$  are cycles  $C_k$  of length  $k$ .

### 3 Results without disjoint routing constraint

Our problem has not yet been studied in the literature; however, without the disjoint routing constraint, it is equivalent to find a covering of the edges of a graph  $I$  by subgraphs  $I_k$ . When  $I = K_n$  and  $I_k = K_k$ , this problem is known as the *covering design problem* (See the surveys of Stinson [15] and Mills and Mullin [12]). Moreover, the problem of finding a perfect covering of the edges is related to the existence of an  $(n, k, 1)$ -design. In particular, for the covering by 3-cycles, we have the following results (See [13, 12]):

**Theorem 1** [13, 12] *The minimum number of 3-cycles required to cover the edges of  $K_n$  is  $\lceil \frac{n}{3} \lceil \frac{n-1}{2} \rceil \rceil$ .*

When  $n \equiv 1$  or  $3 \pmod{6}$ , the edges of  $K_n$  can be partitioned into  $\frac{n(n-1)}{6}$  3-cycles, also called Steiner triples.

When  $I_k$  is isomorphic to  $C_k$ , the partition problem of the edges of  $K_n$  in  $C_k$  is known as *cycle designs* (See [11] for a survey). Also, the covering of the edges of  $K_n$  by  $C_k$ ,  $k > 3$ , was considered in [3], where one can find the following result:

**Theorem 2** [3] *The minimum number of 4-cycles required to cover the edges of  $K_n$  is*

$$\left\lceil \frac{n}{4} \left\lceil \frac{n-1}{2} \right\rceil \right\rceil + \varepsilon(n) \quad \text{with } \varepsilon(n) = \begin{cases} 1 & \text{if } n \equiv 3 \pmod{8} \\ 0 & \text{otherwise.} \end{cases}$$

Finally, the partitioning problem of the edges of  $K_n$  by graphs isomorphic to a given graph  $H$  is considered in [4, 9].

## 4 Our results

Here, we consider covering of the edges of  $K_n$  by cycles satisfying the *disjoint routing constraint*. Such a covering will be called a *DRC-covering* of  $K_n$ .

We will denote by  $\rho(n)$  the minimum number of cycles needed in a DRC-covering of  $K_n$ .

### 4.1 Lower bounds

**Theorem 3**  $\rho(2p+1) \geq \frac{p(p+1)}{2}$  with  $p \geq 1$ , and  $\rho(2p) \geq \frac{p^2+1}{2}$ ,  $p \geq 2$ .

**Proof.** Let  $C_k^j$  be a  $k$ -cycle of DRC-covering of  $K_n$ . The disjoint routing property implies that its vertices are cyclically ordered modulo  $n$ . Thus  $C_k^j$  can be written  $(a_1^j, a_2^j, \dots, a_k^j, a_1^j)$ , with  $0 \leq a_1^j \leq a_2^j \leq \dots \leq a_k^j \leq n-1$ .

Let  $\delta_i^j = a_{i+1}^j - a_i^j$ ,  $1 \leq i \leq k-1$ , and  $\delta_k^j = n + a_1^j - a_k^j$ . The disjoint routing property implies  $\sum_i \delta_i^j = n$ .

For an edge  $(x, y)$  of  $K_n$  with  $x < y$ , we call difference of the edge the value  $y-x$  if  $y-x \leq n/2$  or  $x+n-y$  otherwise (it corresponds to the distance between  $x$  and  $y$  on a cycle of length  $n$ ).

In the odd case,  $n = 2p+1$ , the covering must contain the  $n$  edges of difference  $d$  for every  $d$ ,  $1 \leq d \leq p$ . Each difference correspond to an  $\delta_i^j$ , with  $\delta_i^j = d$  or  $n-d$ . Thus,  $\sum_{i,j} \delta_i^j \geq \sum_{d=1}^p nd = n \frac{p(p+1)}{2}$ . Remind that  $\sum_i \delta_i^j = n$ . Consequently, if the covering contains  $\rho(n)$  cycles, we have  $n\rho(n) \geq n \frac{p(p+1)}{2}$  and finally,  $\rho(n) \geq \frac{p(p+1)}{2}$ .

In the even case,  $n = 2p$ , the covering must contain  $n$  edges of difference  $d$ ,  $1 \leq d \leq p-1$  and  $\frac{n}{2} = p$  edges of difference  $p$ . Furthermore, since the degree of the nodes in  $K_n$  is odd (equal to  $n-1$ ) and the degree of the nodes of a cycle is even (equal to 2), the covering must contain extra edges (i.e. in each vertex,



there is an edge covered at least twice). Thus, there are at least  $\frac{n}{2}$  extra edges of difference at least 1 in the covering (corresponding to a perfect matching). Consequently,  $\sum_{i,j} \delta_i^j \geq \left( \sum_{d=1}^{p-1} nd \right) + pp + p = p(p^2 + 1)$  and if the covering contains  $\rho(n)$  cycles, we obtain  $n\rho(n) = 2p\rho(n) \geq p(p^2 + 1)$  and finally  $\rho(n) \geq \frac{p^2+1}{2}$ .  $\square$

Note that for both odd and even cases, the length of the cycles involved in the DRC-covering of  $K_n$  has no influence on the lower bound of  $\rho(n)$ . Thus, as it will be confirmed in the following, small cycles are sufficient to reach the lower bound.

## 4.2 Minimum DRC covering

**Theorem 4** *When  $n = 2p + 1$ ,  $\rho(n) = \frac{p(p+1)}{2}$ . Furthermore, the DRC-covering of  $K_{2p+1}$  consists of  $p$   $C_3$  and  $\frac{p(p-1)}{2}$   $C_4$ .*

**Proof.** (By induction on  $p$ )  $K_3$  is covered using one  $C_3$ . Thus, the theorem is true when  $p = 1$ .

Suppose now that the theorem is true for  $K_{2p+1}$ . We will show that it is also true for  $n = 2p + 3$ . For that, let us arrange the vertices of  $K_{2p+3}$  in the following order:  $A, 0, 1, \dots, p-1, B, p, \dots, 2p$ .

We build a DRC-covering of  $K_{2p+3}$  from a DRC-covering of  $K_{2p+1}$  as follows. The cycles of the DRC-covering of  $K_{2p+3}$  will be

- the  $p(p+1)/2$  cycles of a DRC-covering of the  $K_{2p+1}$  on vertices  $0, 1, \dots, p-1, p, \dots, 2p$ ,
- the  $p$   $C_4$  of a DRC-decomposition of the  $K_{2p,2}$  constructed between vertices  $0, \dots, p-1, p+1, \dots, 2p$  on one side and vertices  $A$  and  $B$  on the other side, namely the  $C_4$   $(A, i, B, p+1+i, A)$ ,  $0 \leq i \leq p-1$ ,
- the  $C_3$   $(A, B, p, A)$ .

One can check that each edge of  $K_{2p+3}$  is covered by exactly one of these cycles and altogether we have  $p(p+1)/2 + p + 1 = (p+1)(p+2)/2$  cycles. Furthermore, there are exactly  $p+1$   $C_3$  and  $p(p+1)/2$   $C_4$ .  $\square$

In Figure 3, we show a covering of  $K_5$  obtained in that way. Let us call the vertices of  $K_5$   $A, 0, B, 1, 2$  in that order. The DRC-covering of  $K_5$  consists of the unique cycle  $(0, 1, 2, 0)$  of the covering of  $K_3$ , plus the  $C_4$   $(A, 0, B, 2, A)$  of a DRC-decomposition of the  $K_{2,2}$  constructed between vertices  $A$  and  $B$  and vertices  $0$  and  $2$ , plus the  $C_3$   $(A, B, 1, A)$ .

**Theorem 5** *When  $n = 2p$ ,  $p \geq 3$ ,  $\rho(n) = \left\lceil \frac{p^2+1}{2} \right\rceil$ . Furthermore, when  $n = 4q$ , the DRC-covering of  $K_{4q}$  consists of  $4$   $C_3$  and  $2q^2 - 3$   $C_4$ , and when  $n = 4q + 2$  the DRC-covering of  $K_{4q+2}$  consists of  $2$   $C_3$  and  $2q^2 + 2q - 1$   $C_4$ .*

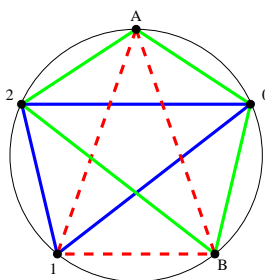


Figure 3: Covering of  $K_5$  obtained from the covering of  $K_3$ .

In order to prove this theorem, we first need to prove some lemmas.

**Lemma 1**  $K_6$  can be covered by 2  $C_3$  and 3  $C_4$ .

**Proof.** The covering is given by the two  $C_3$ :  $(0, 1, 3, 0)$  and  $(0, 1, 4, 0)$ , plus three  $C_4$ :  $(0, 2, 4, 5, 0)$ ,  $(1, 2, 3, 5, 1)$  and  $(2, 3, 4, 5, 2)$ , as shown in Figure 4. Furthermore, there are three edges,  $(0, 1)$ ,  $(2, 3)$  and  $(4, 5)$ , covered exactly twice (they form a perfect matching).  $\square$

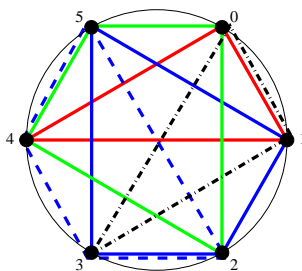


Figure 4:  $K_6$

**Lemma 2** If there exists a DRC-covering of  $K_{4q+2}$  with  $\rho(4q+2) = 2q^2 + 2q + 1$  cycles, then there exists a DRC-covering of  $K_{4q+4}$  with  $\rho(4q+4) = 2q^2 + 4q + 3$  cycles.

**Proof.** Let us call and range the vertices of  $K_{4q+4}$  in the following order:  $A, 0, 1, \dots, 2q, B, 2q+1, \dots, 4q+1$ .

We build a DRC-covering of  $K_{4q+4}$  from a DRC-covering of  $K_{4q+2}$  as follows. The cycles of the DRC-covering of  $K_{4q+4}$  will be

- the  $2q^2 + 2q + 1$  cycles of a DRC-covering of the  $K_{4q+2}$  on vertices  $0, 1, \dots, 2q, 2q+1, \dots, 4q+1$ ,
- the  $2q$   $C_4$  of a DRC-decomposition of the  $K_{4q,2}$  constructed on vertices  $1, \dots, 2q, 2q+1, \dots, 4q$  on one side, and vertices  $A$  and  $B$  on the other side, namely, the  $C_4(A, i, B, 2q+i, A)$ ,  $1 \leq i \leq 2q$ ,
- the 2 triangles  $(A, 0, B, A)$  and  $(A, B, 4q+1, A)$ .

One can check that every edge of  $K_{4q+4}$  is covered by one of these cycles and that altogether we have  $2q^2 + 2q + 1 + 2q + 2 = 2q^2 + 4q + 3 = \left\lceil \frac{(2q+2)^2 + 1}{2} \right\rceil$  cycles. Furthermore, there are exactly 4  $C_3$  in the covering (2 from the DRC-covering of  $K_{4q+2}$  and the 2 extra  $C_3$ ).  $\square$

To illustrate this proof, we indicate in Figure 5 the cycles involved in the covering of  $K_8$ .

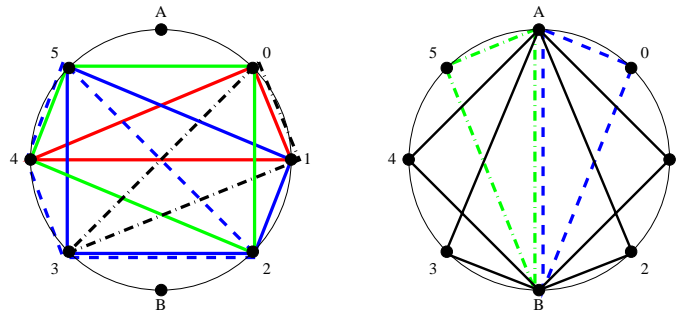


Figure 5: Cycles involved in the covering of  $K_8$ .

**Lemma 3** *If there exists a DRC-covering of  $K_{4q+2}$  with  $\rho(4q+2) = 2q^2 + 2q + 1$  cycles, then there exists a DRC-covering of  $K_{4q+6}$  with  $\rho(4q+6) = 2q^2 + 6q + 5$  cycles.*

**Proof.**

We will prove a little stronger theorem, imposing some extra properties in the decomposition which will be kept in the construction.

Let us suppose that there exists a DRC-covering of  $K_{4q+2}$ , where the nodes  $0, 1, \dots, 4q+1$  are cyclically ordered, with the following properties:

- the edges (of the perfect matching)  $(0, 1), (2, 3), \dots, (4q, 4q+1)$  are covered exactly twice, while other edges are covered exactly once.

- the edge  $(0, 1)$  belongs to the 3-cycle  $(0, 1, x, 0)$ , for some  $x$  different from  $0, 1$ .

We will show that there exists a DRC-covering of  $K_{4q+6}$  with the same properties. Note that these properties are satisfied by the covering of  $K_6$  of Lemma 1 ( $x$  being either 3 or 4).

Let us call and range the vertices of  $K_{4q+6}$  in the following order:  $0, A, B, 1, \dots, 2q+1, C, D, 2q+2, \dots, 4q+1$ . The cycles of the DRC-covering of  $K_{4q+6}$  will be

- the  $2q^2 + 2q$  cycles of the covering of  $K_{4q+2}$  except the 3-cycle  $(0, 1, x, 0)$ ,
- the  $2q$  4-cycles  $(A, i, C, f(i), A)$ , with  $2 \leq i \leq 2q+1$  and where  $f$  is a bijection from  $\{2, 3, \dots, 2q+1\}$  to  $\{2q+2, \dots, 4q+1\}$ ,
- the  $2q+1$  4-cycles  $(B, j, D, g(j), B)$ ,  $1 \leq j \leq 2q+1$ , and where  $g$  is a bijection from  $\{1, 2, \dots, 2q+1\}$  to  $\{2q+2, \dots, 4q+1, 0\}$ .
- the 3  $C_4$   $(A, B, C, D, A)$ ,  $(0, A, 1, x, 0)$ ,  $(B, 1, C, D, B)$  and the  $C_3$   $(0, A, C, 0)$ ,

One can check that each edge of  $K_{4q+6}$  is covered by one of these cycles and that altogether, we have  $2q^2 + 2q + 2q + 2q + 1 + 3 + 1 = 2q^2 + 6q + 5 = \left\lceil \frac{(2q+2)^2 + 1}{2} \right\rceil$  cycles. Furthermore, there are still exactly 2  $C_3$  in the covering. Also, the edges  $(0, A)$ ,  $(B, 1)$ ,  $(2, 3), \dots, (2q, 2q+1), \dots, (C, D)$ ,  $(2q+2, 2q+3), \dots, (4q, 4q+1)$  (corresponding to a perfect matching) are covered twice, while other edges are covered only once. Moreover, the edge  $(0, A)$ , which is covered twice, appears in the 3-cycle  $(0, A, C, 0)$ .  $\square$

Now, we are able to prove Theorem 5.

**Proof.** of Theorem 5 (By induction)

The theorem is true for  $n = 6$  as shown by Lemma 1. Note that the covering of  $K_6$  satisfies the two extra properties needed in the proof of Lemma 3. So using Lemma 3, one can build by induction the DRC-covering of  $K_{4q+2}$ ,  $q \geq 1$ , by  $\rho(4q+2) = 2q^2 + 2q + 1$  cycles. Then, using Lemma 2, one can build the DRC-covering of  $K_{4q+4}$ ,  $q \geq 1$ , by  $\rho(4q+4) = 2q^2 + 4q + 3$  cycles.

So Theorem 5 is proved.  $\square$

## 5 Conclusion

The problem of the design of a survivable WDM network was considered as an extension of the classical edge covering problem by addition of the disjoint routing constraint. In particular, we have studied the case of a physical ring network with the all-to-all ( $K_n$ ) communication instance. For this design problem, we give a solution with the optimal number of sub-network (cycles). As an extension of

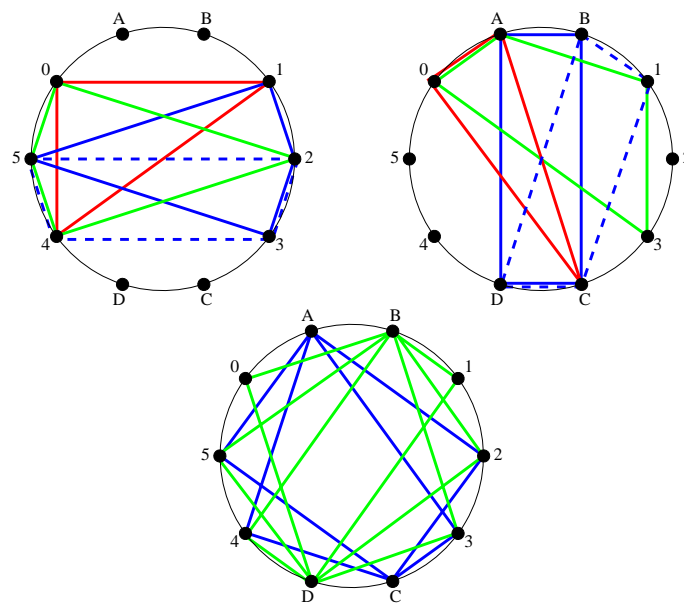


Figure 6: Cycles involved in the covering of  $K_{10}$ .

this problem, we are now investigating cases with other communication instances such as  $\lambda K_n$  (or more general logical graphs). We also think to consider other network topologies for example trees of rings, grids or tori. Moreover it will be interesting to consider in these extensions the real cost function.

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