Theoretical Aspects of the Optical Transpose Interconnecting System Architecture

David Coudert, Afonso Ferreira, Stéphane Pérennes

Theoretical Aspects of the Optical Transpose Interconnecting System Architecture

(extended abstract)

D. Coudert[†‡] A. Ferreira[†] S. Perennes[†]

1 Introduction

An attractive way of implementing efficient local interconnection networks is to use the Optical Transpose Interconnecting System (OTIS) architecture proposed in [8]. This system allows to optically interconnect some set of processors in a Free Space of Optical Interconnections (FSOI). Briefly, it consists of two lenslet arrays allowing a large number of optical interconnections from a set of transmitters to a set of receivers. Note that the OTIS architecture is indeed a three dimension ($3D$) one but it can always be modelized in a $2D$-space.

Two approaches exist, in the first one (All-Optical) the OTIS architecture provides all the interconnections of the network in one-hop, while in the second (Hybrid), some electronic connections are necessary and moreover some connections are directly implemented but may require several hops.

The hybrid approach has been motivated by the results of [5], where it was shown that as soon as an electronic wire is more than 1 cm long, it has more power consumption than its optical conterpart which connects an optical transmitter to an optical receiver through a FSOI. Consequently, the OTIS architecture was used in [9] to realize parts of interconnection networks such as hypercubes, 4-D meshes, mesh-of-trees and butterflies.

The All-Optical approach is enhanced by the opportunity, offered with the OTIS architecture, to easily build a real one-to-one symmetric complete digraph with loops (K_n^*). In fact, using this architecture, it is practically possible to connect 64 processors in a complete graph [7], each processor having 64 transceivers (corresponding to the 64 arcs of one vertex). It has also been shown in [4] how to realize the single-hop multi-OPS POPS network [2], and the multi-hop multi-OPS stack-Kautz network [3] with the OTIS architecture. In this work, we focus on All-Optical networks.

The number of transceivers per processor is technologically limited (the number of transceivers per cm^2 cannot exceed 64 at the moment). Also, whereas a network having the topology of a complete graph with 64 processors is actually feasible and is an important advance for network design, it is not enough as one can wish layouts for large networks having hence bounded degree ($d \ll 64$). Moreover, a low number of transceivers per processor means a reduced price per processor. Consequently, it is important to study the set of network topologies for which it exists an efficient layout with the OTIS architecture and to find which "good" networks admit such a layout, we will call it an $OTIS_{2D}$ ($OTIS_{3D}$).

So we will first try to provide ways of determining if a given general network admits an OTIS layout or not. Then we will study particular case of regular and symmetric networks. Finally we will show that classical topologies like de Bruijn, Kautz and complete digraphs admit an $OTIS_{2D}$-layout. At the end, the results obtained for the $OTIS_{2D}$ model are applied to the $OTIS_{3D}$ case.

[†]Project SLOOP – CNRS-I3S-INRIA – BP 93 – F-06902 Sophia-Antipolis – France.
[‡]Corresponding author. E-mail: David.Coudert@sophia.inria.fr.

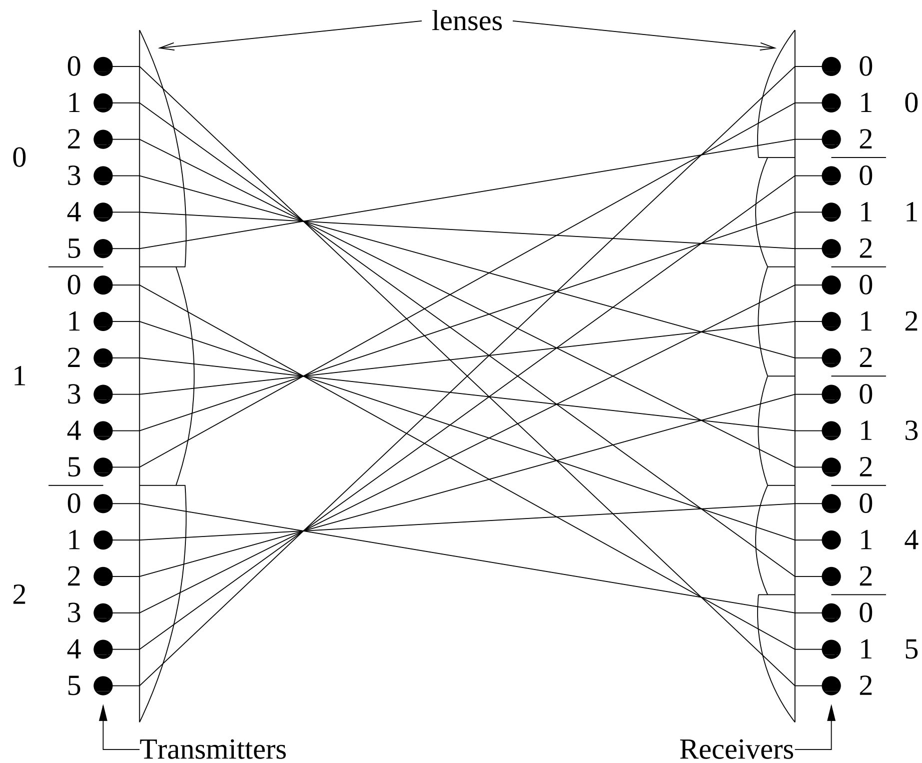

Figure 1: $OTIS(3, 6)$.

2 Preliminaries

2.1 OTIS

The **Optical Transpose Interconnection System** (OTIS) architecture, was first proposed in [8]. $OTIS(p, q)$ is an optical system which allows point-to-point (1-to-1) communications from p groups of size q onto q groups of size p. This architecture connects the transmitter (i, j), $0 \leq i \leq p-1$, $0 \leq j \leq q-1$, to the receiver $(q-1-j, p-1-i)$.

Optical interconnections in the OTIS architecture are realized with two planes of lenses [6] in a free optical space as shown in the figure opposite. The transmitter $t = (i, j)$ and the receiver $r = (i', j')$ are also numbered as integer modulo $m = pq$, $t = ip + j$ and $r = qi' + j'$. Transceivers are associated to Processors in the following way: the transmitters of processor p_i are $di, di+1, \ldots, d(i+1)-1$ and the receivers of p_i are $di, di+1, \ldots, d(i+1)-1$. Hence there is a connection from p_i to p_j if one transmitter of p_i is connected to one receiver of p_j.

Actually, the OTIS architecture which is implemented has three dimensions. $OTIS_{3D}(p, q, p', q')$ connects pp' groups of size qq', to qq' groups of size pp'. The transmitter (i, j, i', j'), $0 \leq i < p$, $0 \leq i' < p'$, $0 \leq j < q$ and $0 \leq j' < q'$, is connected to the receiver $(q-j-1, p-i-1, q'-j'-1, p'-i'-1)$, but the $OTIS_{3D}$ architecture can be modeled by two $OTIS_{2D}$ (See Section 3.5).

2.2 A graph-theoretic model for $OTIS_{2D}$ networks

The $OTIS_{2D}(p, q)$ architecture connects $m = pq$ transmitters to m receivers. Let $m = dn$ and let p_i, $0 \leq i < n$ be the processor corresponding to the transmitters $di, di+1, \ldots, d(i+1)-1$ and the receivers $di, di+1, \ldots, d(i+1)-1$. The $OTIS_{2D}(p, q)$ architecture connects n processors in a network of constant degree d.

Let $H(p,q,d)$ be the directed graph (digraph) corresponding to this network topology. $H(p,q,d)$ has $n = \frac{pq}{d}$ nodes of constant degree d and $m = pq$ arcs. There is an arc from the node u, $0 \le u < n$, to the nodes v_α, $0 \le \alpha < d$, such that

$$v_\alpha = \left\lfloor \frac{(pq-1)\left(\left\lfloor \frac{du+\alpha}{q} \right\rfloor + 1\right) - p(du+\alpha)}{d} \right\rfloor, \quad 0 \le \alpha < d$$

Let $G = (V,E)$ be a digraph, with $|V| = n$ nodes of constant degree d and $|E| = m = dn$ arcs. G has an $OTIS_{2D}$*-layout* if there exist p and q with $pq = m$ and an isomorphism σ from G onto $H(p,q,d)$. This notion can be extended to $OTIS_{3D}$.

Remark 1 *Let us consider the case $p = d$ and $q = n$. We obtain :*

$$\begin{aligned} v_\alpha &= \left\lfloor n\left(\left\lfloor \frac{du+\alpha}{n} \right\rfloor + 1\right) - (du+\alpha) - \frac{\left\lfloor \frac{du+\alpha}{n} \right\rfloor + 1}{d} \right\rfloor, \quad 0 \le \alpha < d \\ &\equiv -(du+\alpha) + \left\lfloor - \frac{\left\lfloor \frac{du+\alpha}{n} \right\rfloor + 1}{d} \right\rfloor \mod n, \qquad 0 \le \alpha < d \end{aligned}$$

As far as $0 \le du + \alpha \le d(n-1) + d - 1 = dn - 1$, it follows that $\left\lfloor - \frac{\left\lfloor \frac{du+\alpha}{n} \right\rfloor + 1}{d} \right\rfloor = -1$.

Consequently,

$$v_\alpha \equiv -du - \alpha \mod n, \quad 1 \le \alpha \le d$$

.

3 Results

Characterizing OTIS-layout graphs appears to be difficult, so we generally restrict ourselves to the case of regular digraphs. Two very simple and useful results are the following ones.

Remark 2 *If there exists an OTIS-layout for a digraph G, then G contains a cycle of length 2.*

Remark 3 *Let G^- be the digraph obtain by reversing all the arcs of the digraph G. If G admit an $OTIS_{2D}(p,q)$-layout then G^- has an $OTIS_{2D}(q,p)$-layout.*

Hence the optical interconnections of the $OTIS_{2D}(p,q)$ architecture are clearly "equivalent" to those of the $OTIS_{2D}(q,p)$ architecture (up to reversing the arcs).

3.1 $OTIS_{2D}(1,m)$

The $OTIS_{2D}(1,m)$ architecture is used to reverse the order of the incoming optical beams, as the transmitter t, $0 \le t < m$ is connected to the receiver $m - t - 1$.

Proposition 1 *Let $G = (V,E)$ be a simple digraph with $|V| = n$ nodes and $|E| = m$ arcs. If there exists an $OTIS_{2D}(1,m)$-layout for G, then:*

1. $\forall u \in V,\ |\{v \in \Gamma^+(u) | d^-(v) > 1\}| \le 2,$
2. $\forall u \in V,\ |\{w \in \Gamma^-(u) | d^+(w) > 1\}| \le 2.$

Using Proposition 1, it is possible to obtain a polynomial algorithm which compute an $OTIS_{2D}(1,m)$-layout for a digraph G when it exists and reject G otherwise.

3.2 Symmetric digraphs

Symmetric digraphs play an important role as interconnection network topologies. We show in this section that only very few of them admit an $OTIS_{2D}$-layout.

Proposition 2 *Let $G = (V, E)$ be a strongly connected symmetric digraph with $|V| = n$ nodes, $|E| = m$ arcs and a constant degree d. Then an OTIS-layout for G can only be realized with an $OTIS_{2D}(p, q)$ architecture in which $p - 1 \leq q \leq p + 1$.*

Proof: Considering such a symetric digraph, a quick study of the adjacences of the node number 0 leads to $m - \alpha q - 1 - (d - 1) \leq m - \alpha p - 1 \leq m - \alpha q - 1 + (d - 1)$, with $0 \leq \alpha < d$. The result is given by $\alpha = d - 1$. □

Proposition 3 *Let $G = (V, E)$ be a strongly connected symmetric digraph with $|V| = n$ nodes, $|E| = m$ arcs and a constant degree d, which has an $OTIS_{2D}$-layout. Then,*

1. *If G is without loop, then $G = K_n^+$ and $d = q = p - 1$ or $d = q - 1 = p$.*
2. *If each node of G has a loop, then $G = K_n^*$ and $d = p = q$.*

Consequently, Propositions 2 and 3 imply that there exists no $OTIS_{2D}$-layout for hypercubes, grids and torus of any sizes and more generally for all symmetric digraphs with constant degree in which either all nodes have a loop or there is no loop (in particular for transitive digraphs), except for complete digraphs K_n^+ and K_n^*.

Note that there exist symmetric digraphs with l loops, $0 < l < n$, admitting an OTIS-layout.

3.3 De Bruijn, Kautz and their generalizations

Two important classes of digraphs with large number of vertices and small diameter are the de Bruijn and the Kautz digraphs which are both particular cases of the *generalized Kautz digraphs* (also called Imase and Itoh digraphs). The de Bruijn digraph is also a particular case of the *generalized de Bruijn digraphs*.

Theorem 1 *The Imase and Itoh digraph, $II(d, n)$, with n nodes of degree d, has an $OTIS_{2D}(d, n)$-layout.*

Proof: Inside the Imase and Itoh digraph, $II(d, n)$, the nodes are integer modulo n and there is and arc from a node u to the nodes v_α, $0 \leq \alpha < d$, such that $v_\alpha \equiv -du - \alpha - 1 \mod n$. Using remark 1, the proof follows. □

Corollary 1 *The de Bruijn and the Kautz digraphs have an $OTIS_{2D}$-layout.*

For both $B(d, k)$ and $K(d, k)$, we have a simple linear algorithm to label the nodes. Notice that the generalized de Bruijn digraph, $GB(d, n)$, with n nodes of degree d, has no $OTIS(d, n)$-layout, and we proposed an $OTIS$-layout using one $OTIS(d, n)$ and one $OTIS(1, dn)$.

3.4 De Bruijn and Kautz bus networks

The de Bruijn and Kautz bus networks were defined in [1] as hypergraphs built from the generalized de Bruijn and the generalized Kautz digraphs. As an Optical Passive Star coupler is equivalent to a directed bus, we propose for both networks an optical implementation using two $OTIS_{2D}$ and one $OTIS_{2D}(1, m)$.

3.5 Extension of the Results to $OTIS_{3D}$

The $OTIS_{3D}$ architecture realizes optical interconnections from a matrix of transmitters to a matrix of receivers. From a topological aspect, those interconnections can be realized by applying an $OTIS_{2D}$ on the matrix columns followed by an $OTIS_{2D}$ on the rows.

Definition 1 *The conjunction $G_1 \otimes G_2$ of two digraphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the digraph with vertex-set $V_1 \times V_2$ and an arc joining (u_1, u_2) to (v_1, v_2) if and only if there is an arc joining u_1 to v_1 in G_1 and an arc joining u_2 to v_2 in G_2.*

Theorem 2 *A digraph H admits an $OTIS_{3D}$-layout if and only if it exist two digraphs G and K, admitting both an $OTIS_{2D}$-layout, and such that $H = G \otimes K$.*

Consequently, $OTIS_{2D} \otimes OTIS_{2D} = OTIS_{3D}$ and questions related to $3D$ case can be reduced to the $2D$ model. The $OTIS_{3D}$ architecture allows more digraphs layout than the $OTIS_{2D}$ architecture. As an example, the digraph $K^+_{n_1} \otimes K^*_{n_2}$ has an $OTIS_{3D}$-layout whereas Proposition 3 forbid its $OTIS_{2D}$-layouts. Otherwise, some digraphs having an $OTIS_{2D}$-layout have also an $OTIS_{3D}$-layout like de Bruijn digraphs, as $B(d,k) \otimes B(d',k) = B(dd',k)$, and K^*_n.

4 Conclusion

We have obtained several results on OTIS networks but the one which is still open is the question to know if finding an OTIS-layout for a given digraph is a polynomial problem or not. Also as OTIS contains several good networks, it would be interesting to study the properties (degree, diameter, routing) of digraphs induced by the $OTIS_{2D}(p,q)$ architecture when processors have degree d.

As the problem of computing the diameter of an OTIS-network seems difficult, we are interested in the problem of finding the largest OTIS-network for given degree d and diameter D. Experimentaly, it appears that Imase and Itoh's digraphs are always the largest. Futhermore, we obtain the same results when we design the OTIS-network of lowest diameter for given number of node and degree. On the other hand, an Imase and Itoh's digraph, $II(d,n)$, has an $OTIS(d,n) - layout$. When n is large compare to d, the OTIS architecture has a large number of small lenses which may be difficult to built. Hence, we are also interested in finding goods OTIS-networks, for given degrees and diameters, in which the values of p and q are close. Table 1 gives the number of nodes and the size of the $OTIS(p,q)$ architecture for networks of degree $d = 4$ and diameter $D = 5$ with $n \geq 400$.

Another interesting issue is to consider combining several OTIS and other optical devices, in order to construct a wide variety of networks, as shown in [4] for the POPS and the stack-Kautz networks.

n	p	q	
400		400	
404	4	404	
408		408	
408	12	136	$\neq$ Imase and Itoh
412		412	
416		416	
$\vdots$	4	$\vdots$	
480		480	
480	8	240	$\neq$ Imase and Itoh
484		484	
$\vdots$	4	$\vdots$	
768		768	
768	16	192	$\neq$ Imase and Itoh
772		772	
$\vdots$	4	$\vdots$	
1020		1024	
1024	4	1024	De Bruijn
1028		1028	
$\vdots$	4	$\vdots$	
1040		1040	
1280	4	1280	Kautz

Table 1: Number of nodes, n, and size of the $OTIS(p, q)$ architecture for networks of degree $d = 4$, diameter $D = 5$ and $n \geq 400$ nodes. All of these networks are isomorphic to Imase and Itoh's digraphs except the ones which are indicated.

References

[1] J-C. Bermond, R. Dawes, and F. Ergincan. De Bruijn and Kautz bus networks. *Networks*, 30:205–218, 1997.

[2] D. Chiarulli, S. Levitan, R. Melhem, J. Teza, and G. Gravenstreter. Partitioned Optical Passive Star (POPS) Topologies for Multiprocessor Interconnection Networks with Distributed Control. *IEEE Journal of Lightwave Technology*, 14(7):1601–1612, 1996.

[3] D. Coudert, A. Ferreira, and X. Muñoz. Multiprocessor Architectures Using Multi-hops Multi-OPS Lightwave Networks and Distributed Control. In *IEEE International Parallel Processing Symposium*, pages 151–155. IEEE Press, 1998.

[4] D. Coudert, A. Ferreira, and X. Muñoz. OTIS-based Multi-Hop Multi-OPS Lightwave Networks. In *IEEE Workshop on Optics and Computer Science*. IEEE Press, 1999.

[5] M. Feldman, S. Esener, C. Guest, and S. Lee. Comparison between electrical and free-space optical interconnects based on power and speed considerations. *Applied Optics*, 27(9):1742–1751, May 1988.

[6] M. Blume and G. Marsen and P. Marchand and S. Esener. Optical Transpose Interconnection System for Vertical Emitters. *OSA Topical Meeting on Optics in Computing, Lake Tahoe*, March 1997.

[7] P. Marchand, F. Zane, R. Paturi, and S. Esener. Parallel Optoelectronic FFT Engine: Comparison to Electronic Implementations. *submitted to Applied Optics*, April 1997.

[8] G. Marsden, P. Marchand, P. Harvey, and S. Esener. Optical transpose interconnection system architectures. *Optics Letters*, 18(13):1083–1085, July 1993.

[9] F. Zane, P. Marchand, R. Paturi, and S. Esener. Scalable Network Architectures Using The Optical Transpose Interconnection System. In *Massively Parallel Processing using Optical Interconnections*, pages 114–121, Maui, Hawaii, October 1996. IEEE Computer Society.