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Jean-Claude Bermond, David Coudert

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**The CRC Handbook  
of  
Combinatorial Designs**

*Edited by*

**Charles J. Colbourn**

*Department of Computer Science and Engineering  
Arizona State University*

**Jeffrey H. Dinitz**

*Department of Mathematics and Statistics  
University of Vermont*

AUTHOR PREPARATION VERSION

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# 1 Grooming

JEAN-CLAUDE BERMOND AND DAVID COUDERT

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## 1.1 Definitions and Examples

- 1.1 Remark** Traffic grooming in networks refers to group low rate traffic into higher speed streams (containers) so as to minimize the equipment cost [11, 7, 13, 12, 8, 9]. There are many variants according to the type of network considered, the constraints used and the parameters one wants to optimize which give rise to a lot of interesting design problems (graph decompositions).

To fix ideas, suppose that we have an optical network represented by a directed graph  $G$  (in many cases a symmetric one) on  $n$  vertices, for example a unidirectional ring  $\vec{C}_n$  or a bidirectional ring  $C_n^*$ . We are given also a traffic matrix, that is a family of connection requests represented by a multi-digraph  $I$  (the number of arcs from  $i$  to  $j$  corresponding to the number of requests from  $i$  to  $j$ ). An interesting case is when there is exactly one request from  $i$  to  $j$ ; then  $I = K_n^*$ . Satisfying a request from  $i$  to  $j$  consists in finding a route (dipath) in  $G$  and assigning it a wavelength. The grooming factor,  $g$ , means that a request uses only  $1/g$  of the bandwidth available on a wavelength along its route. Said otherwise, for each arc  $e$  of  $G$  and for each wavelength  $w$ , there are at most  $g$  dipaths with wavelength  $w$  which contain the arc  $e$ .

During the 90's, a lot of research has concentrated in minimizing the number of wavelengths used in the network. In the mean time, the bandwidth of each wavelength ( $> 10$  Gbit/s), the number of wavelengths per fiber ( $> 100$ ) and the number of fibers per cable ( $> 100$ ) have exploded, thus reporting the operational cost of a wavelength into the terminal equipment cost: filters, optical cross-connect, add/drop multiplexer (ADM),.... For example, in a node of an optical network we place an ADM only for those wavelengths carrying a request which has to be added or dropped in this node. So nowadays the objective is to minimize the total number of ADMs in the network, which is a challenging issue.

- 1.2 Definition** [5] Grooming problem: Given a digraph  $G$  (network), a digraph  $I$  (set of requests) and a grooming factor  $g$ , find for each arc  $r \in I$  a path  $P(r)$  in  $G$ , and a partition of the arcs of  $I$  into subgraphs  $I_w$ ,  $1 \leq w \leq W$ , such that  $\forall e \in E(G)$ ,  $\text{load}(I_w, e) = |\{P(r); r \in E(I_w); e \in P(r)\}| \leq g$ . The objective is to minimize  $\sum_{w=1}^W |V(I_w)|$ , and this minimum is denoted by  $A(G, I, g)$ .
- 1.3 Definition**  $TT_n$  is a transitive tournament on  $n$  vertices, that is the digraph with arcs  $\{(i, j) \mid 1 \leq i < j \leq n\}$ . We denote  $\{a, b, c\}$  the  $TT_3$  with arcs  $\{a, b\}$ ,  $\{b, c\}$ , and  $\{a, c\}$ .
- 1.4 Remark** When  $G = P_n^*$ , the shortest path from  $i$  to  $j$  is unique, and we can split the requests into two classes, those with  $i < j$  and those with  $i > j$ . Therefore the grooming problem for  $P_n^*$  can be reduced to two distinct problems on  $\vec{P}_n$ . In particular we have  $A(P_n^*, K_n^*, g) = 2A(\vec{P}_n, TT_n, g)$ .
- 1.5 Example**  $A(\vec{P}_7, TT_7, 2) = 20$ , and the partition consists of 6 subgraphs, the 5  $TT_5$   $\{2,4,5\}$ ,  $\{3,4,6\}$ ,  $\{1,5,6\}$ ,  $\{2,6,7\}$ , and  $\{1,4,7\}$ , plus the union of two  $TT_3$   $\{1,2,3\} + \{3,5,7\}$ .

- 1.6 Theorem** [1] When  $n$  is odd,  $A(\vec{P}_n, TT_n, 2) = \lceil (11n^2 - 8n - 3)/24 \rceil$ ; When  $n$  is even,  $A(\vec{P}_n, K_n, 2) = (11n^2 - 4n)/24 + \varepsilon(n)$ , where  $\varepsilon(n) = 1/2$  when  $n \equiv 2, 6 \pmod{12}$ ,  $\varepsilon(n) = 1/3$  when  $n \equiv 4 \pmod{12}$ ,  $\varepsilon(n) = 5/6$  when  $n \equiv 10 \pmod{12}$ , and  $\varepsilon(n) = 0$  when  $n \equiv 0, 8 \pmod{12}$ .
- 1.7 Remark** In a unidirectional cycle  $\vec{C}_n$ , the path from  $i$  to  $j$  is unique. Wlog we can assign the same wavelength to the two requests  $(i, j)$  and  $(j, i)$ , then the two associated paths contain each arc of  $\vec{C}_n$ . Therefore the load condition becomes  $|E(I_w)| \leq g$  and the grooming problem becomes:
- 1.8 Definition** [5] Grooming problem for  $G = \vec{C}_n$ : given  $n$  and  $g$ , find a partition of  $I$  into subgraphs  $B_w$ ,  $1 \leq w \leq W$ , such that  $|E(B_w)| \leq g$ , which minimizes  $\sum_{w=1}^W |V(B_w)|$ . The minimum value is  $A(\vec{C}_n, I, g)$ .
- 1.9 Remark** The partition of Definition 1.2 is obtained by associating to each  $B_w$  of the partition of Definition 1.8 its symmetric digraph  $B_w^*$  and letting  $I_w = B_w^*$ .
- 1.10 Example**  $A(\vec{C}_4, K_4^*, 3) = 7$ , using a partition of  $K_4$  consisting of the  $K_3$   $\{1, 2, 3\}$  and the  $K_{1,3}$  with edges  $\{1, 4\}$ ,  $\{2, 4\}$ , and  $\{3, 4\}$ .  $A(\vec{C}_7, K_7^*, 3) = 21$  using a  $(7, 3, 1)$  design (steiner triple system) and  $A(\vec{C}_{13}, K_{13}^*, 6) = 52$  using a  $(13, 4, 1)$  design.
- 1.11 Theorem** [2]  $A(\vec{C}_n, K_n^*, 3) = n(n-1)/2 + \varepsilon_3(n)$ , where  $\varepsilon_3(n) = 0$  when  $n \equiv 1, 3 \pmod{6}$ ,  $\varepsilon_3(n) = 2$  when  $n \equiv 5 \pmod{6}$ ,  $\varepsilon_3(n) = \lceil n/4 \rceil + 1$  when  $n \equiv 8 \pmod{12}$ , and  $\varepsilon_3(n) = \lceil n/4 \rceil$  otherwise.
- 1.12 Theorem** [10]  $A(\vec{C}_n, K_n^*, 4) = n(n-1)/2$ .
- 1.13 Theorem** [4]  $A(\vec{C}_n, K_n^*, 5) = 4 \lfloor n(n-1)/10 \rfloor + \varepsilon_5(n)$ , where  $\varepsilon_5(n) = 0$  when  $n \equiv 0, 1 \pmod{5}$ ,  $n \neq 5$ ,  $\varepsilon_5(5) = 1$ ,  $\varepsilon_5(n) = 2$  when  $n \equiv 2, 4 \pmod{5}$ ,  $n \neq 7$ ,  $\varepsilon_5(7) = 3$ ,  $\varepsilon_5(n) = 3$  when  $n \equiv 3 \pmod{5}$ ,  $n \neq 8$ , and  $\varepsilon_5(8) = 4$ .
- 1.14 Theorem** [3]  
 When  $n \equiv 0 \pmod{3}$ , then  $A(\vec{C}_n, K_n^*, 6) = \lceil n(3n-1)/9 \rceil + \varepsilon_6(n)$ , where  $\varepsilon_6(n) = 1$  when  $n \equiv 18, 27 \pmod{36}$ , and  $\varepsilon_6(n) = 0$  otherwise, except for  $n \in \{9, 12\}$  and some possible exceptions when  $n \leq 2580$ .  
 When  $n \equiv 1 \pmod{3}$ ,  $A(\vec{C}_n, K_n^*, 6) = \lceil n(n-1)/3 \rceil + \varepsilon_6(n)$ , where  $\varepsilon_6(n) = 2$  when  $n \equiv 7, 10 \pmod{12}$ , and 0 otherwise, except for  $A(\vec{C}_7, K_7^*, 6) = 17$ ,  $A(\vec{C}_{10}, K_{10}^*, 6) = 34$ , and  $A(\vec{C}_{19}, K_{19}^*, 6) = 119$ .  
 When  $n \equiv 2 \pmod{3}$ , then  $A(\vec{C}_n, K_n^*, 6) = (n^2 + 2)/3$ , except possibly for  $n = 17$ .
- 1.15 Remark** In another grooming problem (see [6]), the requests can be routed via different pipes. Each pipe contains at most  $g$  requests, and the objective is to minimize the total number of pipes (as equipments are placed only at the terminal nodes of the pipe). Thus, given a digraph  $I$  (requests) and a grooming factor  $g$ , the problem consists in finding a virtual multi-digraph  $H$  and, for each arc  $r \in I$ , a path  $P(r)$  in  $H$  such that  $\forall e \in E(H)$ ,  $\text{load}(I, e) \leq g$ . The objective is to minimize  $|E(H)|$ , and the minimum is denoted by  $T(I, g)$ .
- 1.16 Example** When  $I = K_4^*$  and  $C = 2$ , then  $H = C_4^*$ . Requests  $(i, i+1)$  (resp.  $(i, i-1)$ ) are routed via arc  $(i, i+1)$  (resp.  $(i, i-1)$ ), requests  $(1, 3)$  and  $(3, 1)$  are routed clockwise, and  $(2, 4)$  and  $(4, 2)$  counterclockwise.
- 1.17 Remark** For  $C = 2$  the problem can be reduced to a partition of  $K_n^*$  or  $(K_n - e)^*$  in  $TT_3$  (See Directed Design or Mendelsohn's Designs). For  $C = 3$  the result follows

from the existence of a  $PBD(n, \{3, 4, 5\})$  for  $n \neq 6, 8$  (see chapter PBD).

**1.18 Theorem** [6]  $T(K_n^*, 2) = \lceil 2n(n-1)/3 \rceil$  and  $T(K_n^*, 3) = n(n-1)/2$ .

## 1.2 See Also

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§???	Directed designs.
§???	Graph decompositions
§???	Mendelsohn designs

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