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We consider the inverse EEG (ElectroEncephaloGraphy) problem that consists in recovering, from measurements on electrodes of the electric potential on the scalp, a distribution of pointwise dipolar current sources located in the brain and modeling e.g. the presence of epileptic foci.

The head $\Omega = \cup_{i=0}^{2} \Omega_i \subset \mathbb{R}^3$ is modeled as a set of nested regions $\Omega_i \subset \mathbb{R}^3$, $i = 0, 1, 2$ (brain, skull, scalp), separated by either spherical or ellipsoidal interfaces S_i (with $S_2 = \partial\Omega$) and with piecewise constant conductivity σ, $\sigma_{|_{\Omega_i}} = \sigma_i > 0$.

Considering a macroscopic physical model of brain activity and using a quasi-static approximation of the Maxwell equations, see [1], the spatial behaviour of the electric potential u in Ω is related to the distribution of m dipolar sources located at $C_k \in \Omega_0$ with moments $p_k \in \mathbb{R}^3$ by Poisson equation:

$$(\mathrm{P}) \quad \begin{cases} \operatorname{div}(\sigma \nabla u) = \sum_{k=1}^{m} p_k . \nabla \delta_{C_k} & \text{in } \Omega\,, \\ u = g \text{ and } \partial_n u = \phi & \text{on } \partial\Omega\,, \end{cases}$$

where g and ϕ denote the given potential and current flux on the scalp (or approximate interpolation of these quantities). The inverse problem of recovering (m, C_k, p_k) from these data can then be divided into 3 steps.

1. Data propagation (cortical mapping step): Since $C_k \in \Omega_0$, the function u is harmonic in the outer layers Ω_1 and Ω_2, where boundary conditions are given by the continuity relations

$$[u]_i = [\sigma\, \partial_n u]_i = 0 \text{ on } S_i\,,$$

if $[\ ]_i$ denotes the jump across the surface S_i. The potential u and flux $\partial_n u$ are propagated from the surface of the scalp S_2 where they are given by g and ϕ to the surface of the brain S_0. This can be achieved by using boundary element methods [2], or by using robust harmonic approximation techniques and expansions into spherical or ellipsoidal harmonics [4].

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2. Anti-harmonic projection (signal space separation): From these data on S_0, the solution u to equation (P) in Ω_0:

$$\begin{cases} \triangle u = \frac{1}{\sigma_0} \sum_{k=1}^{m} p_k . \nabla \delta_{C_k} & \text{in } \Omega_0 \\ u \text{ and } \partial_n u & \text{given on } S_0 \end{cases}$$

assumes the form:

$$u(x) = h(x) + \sum_{k=1}^{m} \frac{< p_k,\ x - C_k >}{4\pi \|x - C_k\|^3} = h(x) + u_a(x) \ , \quad h \text{ harmonic function in } \Omega_0 \, .$$

In order to recover the C_k inside Ω_0, the knowledge of the singular function u_a is required on S_0. This can be deduced from available boundary data by expanding u on appropriated bases of spherical or ellipsoidal harmonics, [4, 5].

3. Best rational approximation on plane sections (source localization): From the knowledge of the anti-harmonic part u_a of u on S_0, we can deduce the C_k as follows, using 2D best approximation schemes on plane sections of the boundary, [3, 4].
We slice Ω_0 along a family of planes, say Π_p, whose intersections with S_0 are circles or ellipses C_p. Ellipses may then preliminary be mapped by a conformal rational transformation onto circles. From u_a on C_p, one can build $f_p = (P_- u_a)^2$, the square of its antianalytic projection there (Fourier coefficients). We then have at our disposal the trace on circles C_p of a function f_p analytic outside the disks D_p and whose singularities $\zeta_{k,p}$ inside D_p are strongly and explicitely linked with the sources C_k. For fixed p, recovering the $\zeta_{k,p}$ in the disk D_p can be done by best L^2 rational approximation on circles C_p: the poles of these approximants accumulate to the (m for a sphere, $2m$ for an ellipsoid) singularities $\zeta_{k,p}$. Varying p, this allows us to approximately locate the m sources C_k in Ω_0.

Numerical illustrations will be discussed, in both spherical and ellipsoidal situations.

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