

Inverse EEG source problems and approximation

F. Ben Hassen, Maureen Clerc, Juliette Leblond, Stéphane Rigat, M. Zghal

► **To cite this version:**

F. Ben Hassen, Maureen Clerc, Juliette Leblond, Stéphane Rigat, M. Zghal. Inverse EEG source problems and approximation. Proceedings of Optimization and Inverse Problems in Electromagnetism (OIPE), Sep 2008, Ilmenau, Germany. <inria-00430198>

HAL Id: inria-00430198

<https://hal.inria.fr/inria-00430198>

Submitted on 6 Nov 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Inverse EEG source problems and approximation

F. Ben Hassen*, M. Clerc†, J. Leblond‡, S. Rigat§, M. Zghal¶

Keywords: mathematical theory and formulation of inverse and optimization problems, identification problems, regularization techniques; reconstruction techniques, robust optimization; applications: non-destructive evaluation, inverse problems in biomedical engineering.

We consider the inverse EEG (ElectroEncephaloGraphy) problem that consists in recovering, from measurements on electrodes of the electric potential on the scalp, a distribution of pointwise dipolar current sources located in the brain and modeling e.g. the presence of epileptic foci.

The head $\Omega = \cup_{i=0}^2 \Omega_i \subset \mathbb{R}^3$ is modeled as a set of nested regions $\Omega_i \subset \mathbb{R}^3$, $i = 0, 1, 2$ (brain, skull, scalp), separated by either spherical or ellipsoidal interfaces S_i (with $S_2 = \partial\Omega$) and with piecewise constant conductivity σ , $\sigma|_{\Omega_i} = \sigma_i > 0$.

Considering a macroscopic physical model of brain activity and using a quasi-static approximation of the Maxwell equations, see [1], the spatial behaviour of the electric potential u in Ω is related to the distribution of m dipolar sources located at $C_k \in \Omega_0$ with moments $p_k \in \mathbb{R}^3$ by Poisson equation:

$$(P) \quad \begin{cases} \operatorname{div}(\sigma \nabla u) = \sum_{k=1}^m p_k \cdot \nabla \delta_{C_k} & \text{in } \Omega, \\ u = g \text{ and } \partial_n u = \phi & \text{on } \partial\Omega, \end{cases}$$

where g and ϕ denote the given potential and current flux on the scalp (or approximate interpolation of these quantities). The inverse problem of recovering (m, C_k, p_k) from these data can then be divided into 3 steps.

1. Data propagation (cortical mapping step): Since $C_k \in \Omega_0$, the function u is harmonic in the outer layers Ω_1 and Ω_2 , where boundary conditions are given by the continuity relations

$$[u]_i = [\sigma \partial_n u]_i = 0 \text{ on } S_i,$$

if $[]_i$ denotes the jump across the surface S_i . The potential u and flux $\partial_n u$ are propagated from the surface of the scalp S_2 where they are given by g and ϕ to the surface of the brain S_0 . This can be achieved by using boundary element methods [2], or by using robust harmonic approximation techniques and expansions into spherical or ellipsoidal harmonics [4].

*LAMSIN-ENIT

†ENPC and INRIA Sophia-Antipolis Méditerranée

‡INRIA Sophia-Antipolis Méditerranée

§LATP-CMI, Univ. Provence, Technopôle Château-Gombert, 39 rue F. Joliot Curie, 13453 Marseille Cedex 13

¶Registered author, LAMSIN-ENIT, BP 37, 1002 Tunis Belvedere, Tunisia

2. Anti-harmonic projection (signal space separation): From these data on S_0 , the solution u to equation (P) in Ω_0 :

$$\begin{cases} \Delta u = \frac{1}{\sigma_0} \sum_{k=1}^m p_k \cdot \nabla \delta_{C_k} & \text{in } \Omega_0 \\ u \text{ and } \partial_n u & \text{given on } S_0 \end{cases}$$

assumes the form:

$$u(x) = h(x) + \sum_{k=1}^m \frac{\langle p_k, x - C_k \rangle}{4\pi \|x - C_k\|^3} = h(x) + u_a(x), \quad h \text{ harmonic function in } \Omega_0.$$

In order to recover the C_k inside Ω_0 , the knowledge of the singular function u_a is required on S_0 . This can be deduced from available boundary data by expanding u on appropriated bases of spherical or ellipsoidal harmonics, [4, 5].

3. Best rational approximation on plane sections (source localization): From the knowledge of the anti-harmonic part u_a of u on S_0 , we can deduce the C_k as follows, using 2D best approximation schemes on plane sections of the boundary, [3, 4].

We slice Ω_0 along a family of planes, say Π_p , whose intersections with S_0 are circles or ellipses C_p . Ellipses may then preliminary be mapped by a conformal rational transformation onto circles. From u_a on C_p , one can build $f_p = (P_- u_a)^2$, the square of its antianalytic projection there (Fourier coefficients). We then have at our disposal the trace on circles C_p of a function f_p analytic outside the disks D_p and whose singularities $\zeta_{k,p}$ inside D_p are strongly and explicitly linked with the sources C_k . For fixed p , recovering the $\zeta_{k,p}$ in the disk D_p can be done by best L^2 rational approximation on circles C_p : the poles of these approximants accumulate to the (m for a sphere, $2m$ for an ellipsoid) singularities $\zeta_{k,p}$. Varying p , this allows us to approximately locate the m sources C_k in Ω_0 .

Numerical illustrations will be discussed, in both spherical and ellipsoidal situations.

References

- [1] M. Hämäläinen, R. Hari, J. Ilmoniemi, J. Knutila, O.V. Lounasmaa, *Magnetoencephalography - theory, instrumentation, and applications to noninvasive studies of the working human brain*, Rev. Modern Phys., 65 , pp. 413-497 (1993).
- [2] J. Kybic, M. Clerc, T. Abboud, O. Faugeras, R. Keriven, T. Papadopoulo: *A common formalism for the integral formulations of the forward EEG problem*, IEEE Trans. Medical Imaging, vol 24, pp. 12-28 (2005).
- [3] L. Baratchart, J. Leblond, J-P. Marmorat, *Inverse source problem in a 3D ball from best meromorphic approximation on 2D slices*, Elec. Trans. Numerical Analysis (ETNA), 25, pp. 41-53 (2006).
- [4] J. Leblond, C. Paduret, S. Rigat, M. Zghal, *Sources localisation in ellipsoids by best meromorphic approximation in planar sections*, subm. for publication.
- [5] S. Taulu, J. Simola, M. Kajola, *Applications of the Signal Space Separation Method*, IEEE Trans. Signal Proces., 53, pp. 3359- 3372 (2005).