Benchmarking the (1+1)-ES with One-Fifth Success rule on the BBOB-2009 Noisy Testbed

Anne Auger

Benchmarking the (1+1)-ES with One-Fifth Success rule on the BBOB-2009 Noisy Testbed

Anne Auger
TAO Team, INRIA Saclay Ile-de-France
LRI, Bat 490 Univ. Paris-Sud
91405 Orsay Cedex France
Anne.Auger@inria.fr

ABSTRACT

The (1+1)-ES with one-fifth success rule is one of the first and simplest stochastic algorithm proposed for optimization on a continuous search space in a black-box scenario. In this paper, we benchmark an independent-restart (1+1)-ES with one-fifth success rule on the BBOB-2009 noisy testbed. The maximum number of function evaluations used equals 10^6 times the dimension of the search space. The algorithm could only solve 3 functions with moderate noise in 5-D and 2 functions in 20-D.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation

1. INTRODUCTION

The (1+1)-ES with one-fifth success rule is one of the earliest and simplest adaptive stochastic search algorithm [8, 7, 3]. This paper complements [1] where an independent-restart implementation of the (1+1)-ES with one-fifth success rule is benchmarked on the BBOB-2009 noise-free testbed. Indeed, we test exactly the same algorithm, using the same settings on the BBOB-2009 noisy testbed. For the description of the algorithm and the settings we refer to [1].

GECCO'09, July 8–12, 2009, Montréal Québec, Canada.

2. RESULTS AND DISCUSSION

Results from experiments according to [5] on the benchmark functions given in [4, 6] are presented in Figures 1 and 2 and in Tables 1 and 2.

We observe that globally the algorithm performs poorly. In 5-D, only f_{101}, f_{102}, f_{103} are solved and in 20-D only f_{101} and f_{102} are solved. The functions solved belong to the class of functions with moderate noise.

Acknowledgments

The author would like to acknowledge Nikolaus Hansen, Steffen Finck and Raymond Ros for their great and hard work on the BBOB project.

3. REFERENCES

[1] A. Auger. Benchmarking the (1+1) Evolution Strategy with One-Fifth Success Rule on the BBOB-2009 Function Testbed. In *Workshop Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2009)*. ACM Press, 2009.

[2] A. Auger and N. Hansen. Benchmarking the (1+1)-CMA-ES on the BBOB-2009 Function Testbed. In *Workshop Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2009)*. ACM Press, 2009.

[3] L. Devroye. The compund random search. In *International Symposium on Systems Engineering and Analysis*, pages 195–110. Purdue University, 1972.

[4] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2009.

[5] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2009: Experimental setup. Technical Report RR-6828, INRIA, 2009.

[6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009.

[7] I. Rechenberg. *Evolutionsstrategie*. Friedrich Frommann Verlag (Günther Holzboog KG), Stuttgart, 1973.

[8] M. Schumer and K. Steiglitz. Adaptive step size random search. *Automatic Control, IEEE Transactions on*, 13:270–276, 1968.

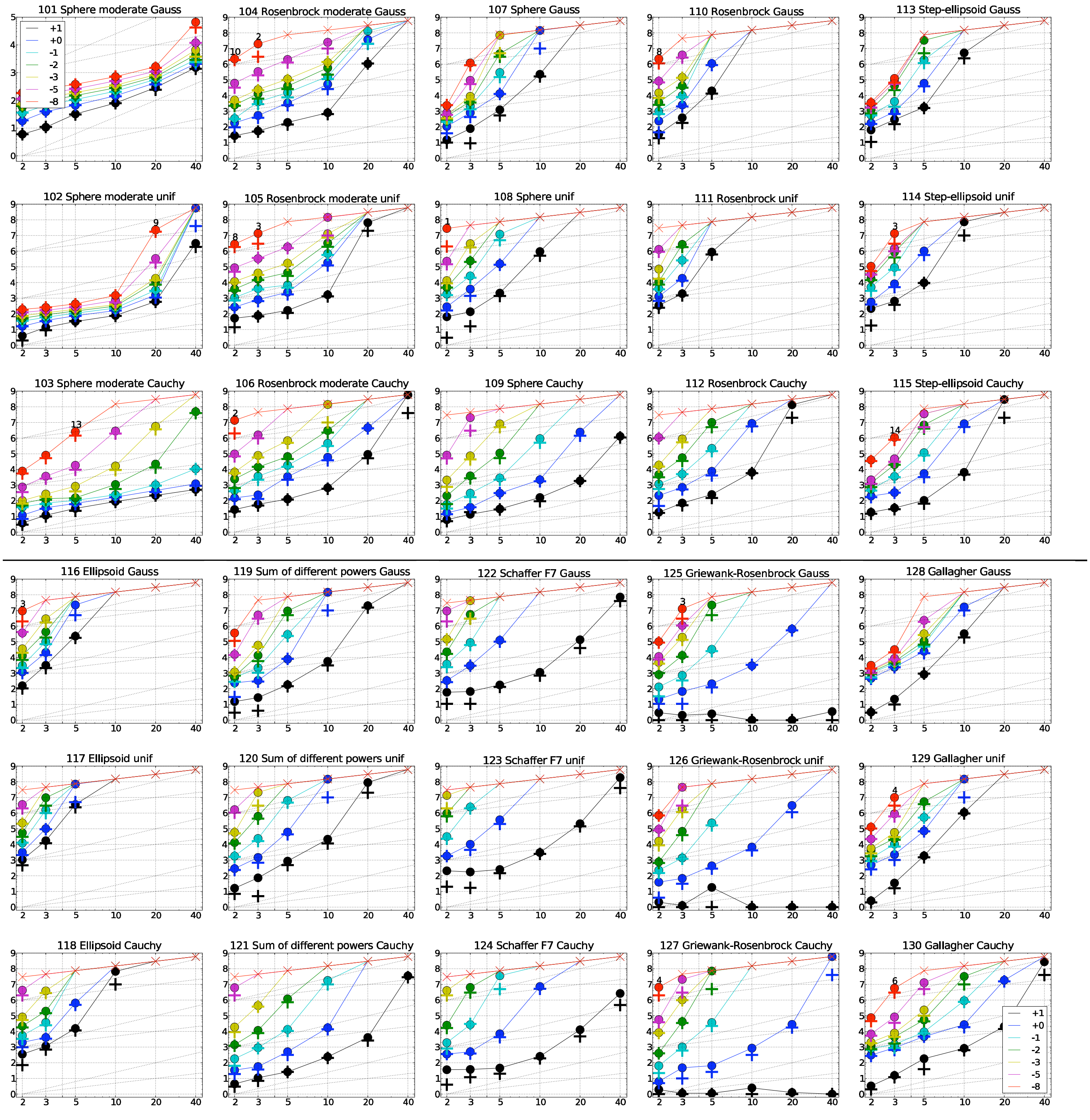

Figure 1: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\rm opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and $f_{\rm opt}$ denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

Δf	*f*101 in 5-D, N=15, mFE=438					*f*101 in 20-D, N=15, mFE=2971				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.2e1	2.7e1	3.8e1	3.2e1	15	2.9e2	2.4e2	3.5e2	2.9e2
1	15	6.7e1	5.9e1	7.4e1	6.7e1	15	4.3e2	3.7e2	4.9e2	4.3e2
1e−1	15	1.1e2	9.9e1	1.2e2	1.1e2	15	5.7e2	5.2e2	6.3e2	5.7e2
1e−3	15	1.9e2	1.8e2	2.0e2	1.9e2	15	8.4e2	7.9e2	9.0e2	8.4e2
1e−5	15	2.6e2	2.5e2	2.7e2	2.6e2	15	1.1e3	1.1e3	1.2e3	1.1e3
1e−8	15	3.8e2	3.7e2	3.9e2	3.8e2	15	1.6e3	1.5e3	1.8e3	1.6e3

Δf	*f*102 in 5-D, N=15, mFE=886					*f*102 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.2e1	2.7e1	3.7e1	3.2e1	15	6.1e2	5.0e2	7.2e2	6.1e2
1	15	7.6e1	6.9e1	8.2e1	7.6e1	15	1.6e3	1.2e3	2.0e3	1.6e3
1e−1	15	1.1e2	1.0e2	1.2e2	1.1e2	15	3.2e3	2.4e3	4.0e3	3.2e3
1e−3	15	2.0e2	1.9e2	2.0e2	2.0e2	15	1.8e4	1.4e4	2.3e4	1.8e4
1e−5	15	2.8e2	2.7e2	2.9e2	2.8e2	15	3.4e5	2.4e5	4.4e5	3.4e5
1e−8	15	4.3e2	4.0e2	4.8e2	4.3e2	9	2.3e7	1.7e7	3.3e7	1.3e7

Δf	*f*103 in 5-D, N=15, mFE=5.00e6					*f*103 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.3e1	2.5e1	4.1e1	3.3e1	15	2.2e2	2.1e2	2.3e2	2.2e2
1	15	6.8e1	6.1e1	7.5e1	6.8e1	15	4.6e2	3.9e2	5.3e2	4.6e2
1e−1	15	1.1e2	9.9e1	1.1e2	1.1e2	15	1.0e3	8.5e2	1.3e3	1.0e3
1e−3	15	8.4e2	5.2e2	1.2e3	8.4e2	14	5.7e6	4.0e6	7.5e6	5.5e6
1e−5	15	1.8e4	9.3e3	2.9e4	1.8e4	0	*42e−5*	*32e−5*	*74e−5*	7.9e6
1e−8	13	2.6e6	1.9e6	3.3e6	2.4e6	.	.	.	.	.

Δf	*f*104 in 5-D, N=15, mFE=5.00e6					*f*104 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e2	1.4e2	2.5e2	1.9e2	15	1.1e6	8.0e5	1.3e6	1.1e6
1	15	3.3e3	2.3e3	4.4e3	3.3e3	6	3.8e7	2.5e7	6.5e7	1.5e7
1e−1	15	1.5e4	1.1e4	2.0e4	1.5e4	2	1.3e8	6.6e7	>3e8	1.2e7
1e−3	15	1.1e5	7.7e4	1.5e5	1.1e5	0	*12e−1*	*97e−3*	*33e−1*	7.9e6
1e−5	14	2.0e6	1.5e6	2.6e6	2.0e6	.	.	.	.	.
1e−8	0	*25e−8*	*15e−9*	*73e−7*	1.8e6	.	.	.	.	.

Δf	*f*105 in 5-D, N=15, mFE=5.00e6					*f*105 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.7e2	1.2e2	2.2e2	1.7e2	4	6.6e7	4.0e7	1.4e8	1.5e7
1	15	2.4e3	1.9e3	2.9e3	2.4e3	0	*13e+0*	*37e−1*	*15e+0*	1.1e7
1e−1	15	6.9e3	5.0e3	8.9e3	6.9e3	.	.	.	.	.
1e−3	15	1.7e5	1.2e5	2.2e5	1.7e5	.	.	.	.	.
1e−5	14	1.9e6	1.4e6	2.4e6	1.8e6	.	.	.	.	.
1e−8	0	*42e−8*	*13e−8*	*38e−7*	2.8e6	.	.	.	.	.

Δf	*f*106 in 5-D, N=15, mFE=5.00e6					*f*106 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e2	1.1e2	1.4e2	1.3e2	15	9.2e4	4.6e4	1.4e5	9.2e4
1	15	3.5e3	2.4e3	4.6e3	3.5e3	15	4.6e6	3.5e6	5.8e6	4.6e6
1e−1	15	1.8e4	1.3e4	2.4e4	1.8e4	0	*42e−2*	*13e−2*	*62e−2*	1.0e7
1e−3	15	6.9e5	4.6e5	9.4e5	6.9e5	.	.	.	.	.
1e−5	0	*10e−5*	*35e−6*	*28e−5*	3.2e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*107 in 5-D, N=15, mFE=5.00e6					*f*107 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.2e3	8.3e2	1.6e3	1.2e3	0	*34e+0*	*28e+0*	*48e+0*	1.0e7
1	15	1.3e4	1.1e4	1.6e4	1.3e4	.	.	.	.	.
1e−1	15	2.9e5	1.9e5	4.0e5	2.9e5	.	.	.	.	.
1e−3	1	7.1e7	3.3e7	>7e7	5.9e5	.	.	.	.	.
1e−5	0	*50e−4*	*13e−4*	*16e−3*	2.8e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*108 in 5-D, N=15, mFE=5.00e6					*f*108 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.1e3	1.4e3	2.8e3	2.1e3	0	*41e+0*	*33e+0*	*49e+0*	1.0e7
1	15	1.6e5	1.1e5	2.0e5	1.6e5	.	.	.	.	.
1e−1	5	1.2e7	7.7e6	2.3e7	3.3e6	.	.	.	.	.
1e−3	0	*12e−2*	*41e−3*	*24e−2*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*109 in 5-D, N=15, mFE=5.00e6					*f*109 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.0e1	2.4e1	3.6e1	3.0e1	15	1.9e3	1.4e3	2.4e3	1.9e3
1	15	3.1e2	2.3e2	3.8e2	3.1e2	15	2.4e6	1.6e6	3.3e6	2.4e6
1e−1	15	2.9e3	2.1e3	3.7e3	2.9e3	0	*72e−2*	*58e−2*	*82e−2*	1.4e7
1e−3	7	8.1e6	5.6e6	1.3e7	3.6e6	.	.	.	.	.
1e−5	0	*13e−4*	*23e−5*	*22e−4*	3.2e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*110 in 5-D, N=15, mFE=5.00e6					*f*110 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.0e4	1.4e4	2.5e4	2.0e4	0	*11e+3*	*77e+2*	*16e+3*	7.1e6
1	15	1.1e6	7.8e5	1.4e6	1.1e6	.	.	.	.	.
1e−1	0	*29e−2*	*15e−2*	*49e−2*	3.2e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*111 in 5-D, N=15, mFE=5.00e6					*f*111 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	8.8e5	6.1e5	1.1e6	8.8e5	0	*15e+3*	*11e+3*	*20e+3*	1.1e7
1	0	*45e−1*	*22e−1*	*66e−1*	2.8e6	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*112 in 5-D, N=15, mFE=5.00e6					*f*112 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.5e2	1.9e2	3.1e2	2.5e2	2	1.4e8	6.8e7	>3e8	1.4e7
1	15	7.6e3	5.3e3	1.0e4	7.6e3	0	*15e+0*	*87e−1*	*17e+0*	7.9e6
1e−1	15	2.2e5	1.5e5	3.0e5	2.2e5	.	.	.	.	.
1e−3	0	*14e−3*	*43e−4*	*29e−3*	2.8e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*113 in 5-D, N=15, mFE=5.00e6					*f*113 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e3	1.4e3	2.2e3	1.8e3	0	*13e+1*	*10e+1*	*17e+1*	1.0e7
1	15	6.1e4	4.1e4	8.0e4	6.1e4	.	.	.	.	.
1e−1	13	1.9e6	1.2e6	2.7e6	1.4e6	.	.	.	.	.
1e−3	0	*23e−3*	*96e−4*	*11e−2*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*114 in 5-D, N=15, mFE=5.00e6					*f*114 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	9.7e3	7.2e3	1.2e4	9.7e3	0	*16e+1*	*12e+1*	*22e+1*	1.0e7
1	15	1.0e6	6.9e5	1.4e6	1.0e6	.	.	.	.	.
1e−1	0	*29e−2*	*20e−2*	*53e−2*	3.2e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*115 in 5-D, N=15, mFE=5.00e6					*f*115 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e2	7.8e1	1.4e2	1.1e2	1	2.9e8	1.4e8	>3e8	2.0e7
1	15	5.6e3	3.7e3	7.6e3	5.6e3	0	*14e+0*	*11e+0*	*17e+0*	1.1e7
1e−1	15	1.2e5	7.5e4	1.6e5	1.2e5	.	.	.	.	.
1e−3	2	3.6e7	1.9e7	>7e7	5.0e6	.	.	.	.	.
1e−5	2	3.6e7	1.9e7	>7e7	5.0e6	.	.	.	.	.
1e−8	0	*96e−4*	*30e−8*	*14e−3*	2.2e6	.	.	.	.	.

Δf	*f*116 in 5-D, N=15, mFE=5.00e6					*f*116 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.3e5	1.6e5	3.1e5	2.3e5	0	*59e+2*	*47e+2*	*72e+2*	7.9e6
1	3	2.2e7	1.3e7	6.6e7	5.0e6	.	.	.	.	.
1e−1	0	*17e−1*	*80e−2*	*33e−1*	2.8e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*117 in 5-D, N=15, mFE=5.00e6					*f*117 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	12	3.5e6	2.7e6	4.4e6	3.0e6	0	*69e+2*	*53e+2*	*97e+2*	1.3e7
1	1	7.1e7	3.3e7	>7e7	5.0e6	.	.	.	.	.
1e−1	0	*67e−1*	*14e−1*	*12e+0*	2.5e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*118 in 5-D, N=15, mFE=5.00e6					*f*118 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.5e4	1.1e4	2.1e4	1.5e4	0	*18e+1*	*13e+1*	*21e+1*	1.0e7
1	15	6.4e5	4.5e5	8.4e5	6.4e5	.	.	.	.	.
1e−1	0	*22e−2*	*12e−2*	*56e−2*	2.8e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*119 in 5-D, N=15, mFE=5.00e6					*f*119 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e2	1.3e2	2.4e2	1.8e2	9	2.0e7	1.6e7	2.6e7	1.6e7
1	15	7.7e3	5.6e3	1.0e4	7.7e3	0	*97e−1*	*84e−1*	*11e+0*	7.9e6
1e−1	15	2.9e5	2.2e5	3.7e5	2.9e5	.	.	.	.	.
1e−3	0	*13e−3*	*53e−4*	*24e−3*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	*f*120 in 5-D, N=15, mFE=5.00e6					*f*120 in 20-D, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	8.6e2	5.3e2	1.2e3	8.6e2	3	8.9e7	5.5e7	2.6e8	2.0e7
1	15	6.2e4	4.6e4	7.8e4	6.2e4	0	*11e+0*	*89e−1*	*13e+0*	8.9e6
1e−1	8	6.4e6	4.5e6	9.6e6	3.3e6	.	.	.	.	.
1e−3	0	*91e−3*	*22e−3*	*17e−2*	2.5e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

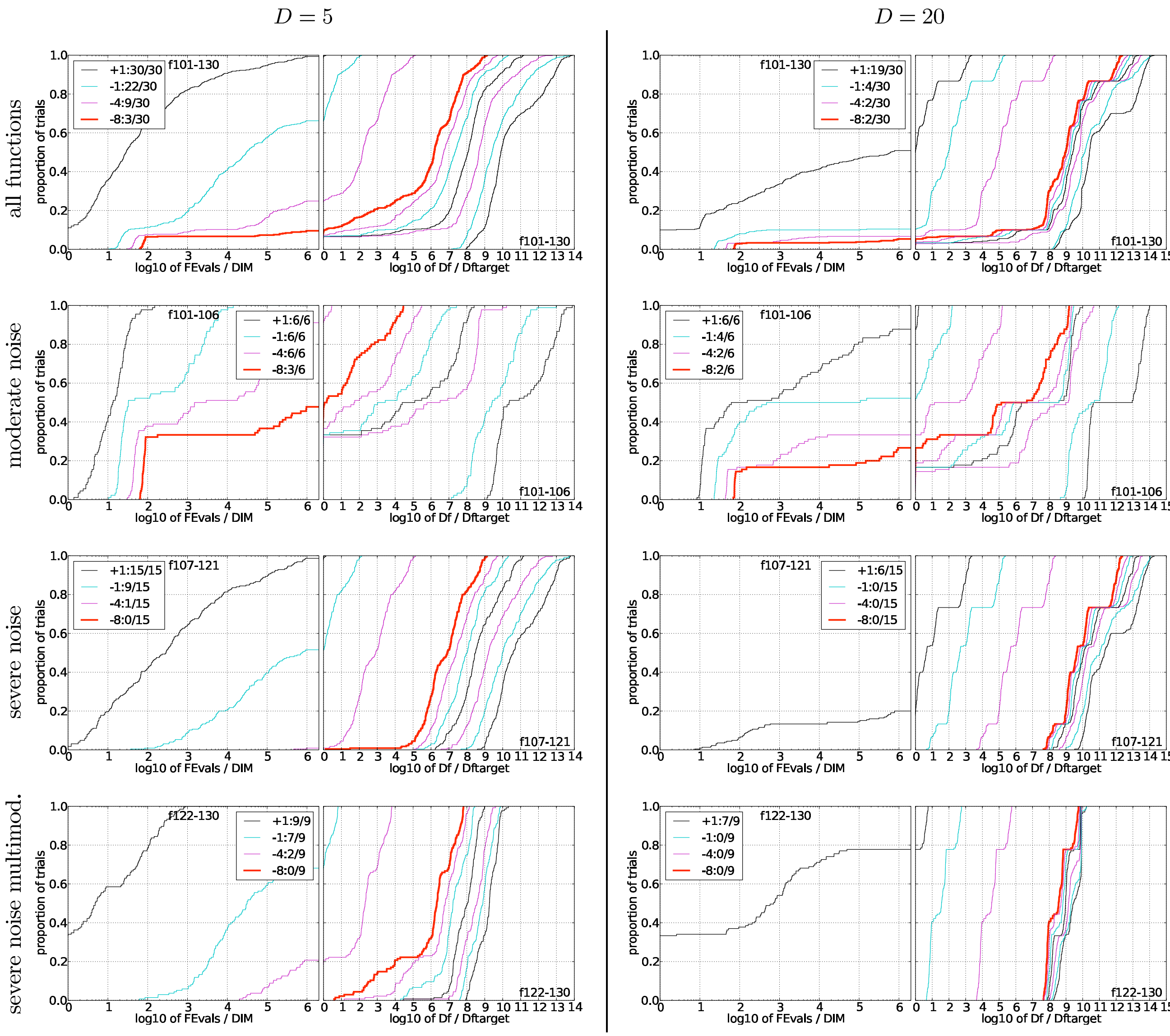

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D\dots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

Δf	***f*121 in 5-D**, N=15, mFE=5.00e6					***f*121 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.6e1	2.1e1	3.2e1	2.6e1	15	4.1e3	3.0e3	5.3e3	4.1e3
1	15	5.0e2	3.6e2	6.5e2	5.0e2	0	*16e−1*	*12e−1*	*21e−1*	7.9e6
1e−1	15	1.3e4	1.0e4	1.7e4	1.3e4	.	.	.	.	.
1e−3	0	*51e−4*	*28e−4*	*80e−4*	2.5e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*122 in 5-D**, N=15, mFE=5.00e6					***f*122 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e2	1.3e2	2.2e2	1.8e2	15	1.4e5	6.1e4	2.2e5	1.4e5
1	15	1.2e5	9.2e4	1.6e5	1.2e5	0	*57e−1*	*39e−1*	*62e−1*	7.9e6
1e−1	0	*24e−2*	*17e−2*	*40e−2*	1.6e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*123 in 5-D**, N=15, mFE=5.00e6					***f*123 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.5e2	1.6e2	3.4e2	2.5e2	15	2.1e5	1.4e5	2.9e5	2.1e5
1	15	3.7e5	2.6e5	4.9e5	3.7e5	0	*64e−1*	*59e−1*	*68e−1*	7.9e6
1e−1	0	*59e−2*	*45e−2*	*63e−2*	2.2e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*124 in 5-D**, N=15, mFE=5.00e6					***f*124 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.6e1	1.8e1	7.5e1	4.6e1	15	1.3e4	8.4e3	1.7e4	1.3e4
1	15	7.0e3	4.5e3	9.8e3	7.0e3	0	*36e−1*	*32e−1*	*43e−1*	7.1e6
1e−1	2	3.6e7	1.9e7	>7e7	5.0e6	.	.	.	.	.
1e−3	0	*12e−2*	*92e−3*	*15e−2*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*125 in 5-D**, N=15, mFE=5.00e6					***f*125 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.5e0	1.1e0	4.0e0	2.5e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	2.1e2	1.3e2	3.0e2	2.1e2	15	6.5e5	5.0e5	8.2e5	6.5e5
1e−1	15	3.3e4	2.3e4	4.3e4	3.3e4	0	*75e−2*	*66e−2*	*77e−2*	1.3e7
1e−3	0	*13e−3*	*71e−4*	*23e−3*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*126 in 5-D**, N=15, mFE=5.00e6					***f*126 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e1	2.9e0	3.4e1	1.8e1	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	4.3e2	2.7e2	6.1e2	4.3e2	15	3.1e6	2.1e6	4.1e6	3.1e6
1e−1	15	2.4e5	1.7e5	3.0e5	2.4e5	0	*85e−2*	*77e−2*	*90e−2*	8.9e6
1e−3	0	*27e−3*	*15e−3*	*41e−3*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*127 in 5-D**, N=15, mFE=5.00e6					***f*127 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e0	1.0e0	1.3e0	1.1e0	15	1.3e0	1.0e0	1.5e0	1.3e0
1	15	6.5e1	3.1e1	1.0e2	6.5e1	15	2.8e4	1.8e4	3.8e4	2.8e4
1e−1	15	3.6e4	2.5e4	4.9e4	3.6e4	0	*62e−2*	*50e−2*	*66e−2*	1.1e7
1e−3	0	*24e−3*	*15e−3*	*30e−3*	2.5e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*128 in 5-D**, N=15, mFE=5.00e6					***f*128 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	8.4e2	6.4e2	1.0e3	8.4e2	0	*34e+0*	*24e+0*	*48e+0*	7.1e6
1	15	2.5e4	1.9e4	3.0e4	2.5e4	.	.	.	.	.
1e−1	15	6.5e4	4.9e4	8.3e4	6.5e4	.	.	.	.	.
1e−3	15	3.2e5	2.2e5	4.3e5	3.2e5	.	.	.	.	.
1e−5	14	2.3e6	1.7e6	3.0e6	2.1e6	.	.	.	.	.
1e−8	0	*17e−7*	*46e−9*	*68e−7*	2.5e6	.	.	.	.	.

Δf	***f*129 in 5-D**, N=15, mFE=5.00e6					***f*129 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e3	1.5e3	2.3e3	1.9e3	0	*48e+0*	*34e+0*	*56e+0*	1.0e7
1	15	7.3e4	5.2e4	9.2e4	7.3e4	.	.	.	.	.
1e−1	15	5.3e5	4.3e5	6.4e5	5.3e5	.	.	.	.	.
1e−3	0	*60e−4*	*24e−4*	*18e−3*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	***f*130 in 5-D**, N=15, mFE=5.00e6					***f*130 in 20-D**, N=15, mFE=2.00e7				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e2	1.1e2	2.5e2	1.8e2	15	2.1e4	1.4e4	2.7e4	2.1e4
1	15	4.7e3	3.4e3	6.0e3	4.7e3	10	1.8e7	1.2e7	2.8e7	9.2e6
1e−1	15	9.3e3	7.3e3	1.1e4	9.3e3	0	*74e−2*	*46e−2*	*16e−1*	7.1e6
1e−3	15	2.3e5	1.4e5	3.3e5	2.3e5	.	.	.	.	.
1e−5	5	1.3e7	8.6e6	2.3e7	4.3e6	.	.	.	.	.
1e−8	0	*57e−6*	*31e−7*	*13e−5*	2.5e6	.	.	.	.	.

Table 2: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.