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Benchmarking the Pure Random Search on the BBOB-2009 Noisy Testbed

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ABSTRACT

We benchmark the Pure-Random-Search algorithm on the BBOB 2009 noisy testbed. Each candidate solution is sampled uniformly in $[-5, 5]^D$, where D denotes the search space dimension. The maximum number of function evaluations chosen is 10^6 times the search space dimension. With this budget the algorithm is not able to solve any single function of the testbed.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization, Global Optimization, Unconstrained Optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation, Pure random search, Monte-Carlo

1. INTRODUCTION

The pure random search proposed by Brooks in 1958 [2] is the most simple stochastic search algorithm. It consists in sampling each search point independently in the search domain and keeping the best solution found.

In this paper, we benchmark the pure random search where each solution is sampled uniformly in $[-5, 5]^D$, where D denotes the dimension of the search space. The maximum number of function evaluations is fixed to $10^6 \times D$. The CPU time experiments and implementation details are given in the companion paper [1].

2. RESULTS AND DISCUSSION

Results from experiments according to [4] on the benchmarks functions given in [3, 5] are presented in Figures 1 and 2 and in Tables 1 and 2.

Not too surprisingly, the pure random search cannot solve any function of the testbed. However, the results provide reference results useful for the investigation of more advanced algorithms.

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3. REFERENCES

- [1] A. Auger and R. Ros. Benchmarking the Pure Random Search on the BBOB-2009 Testbed. In *Workshop Proceedings of the GECCO Genetic and Evolutionary Computation Conference*. ACM, 2009.
- [2] S. H. Brooks. A discussion of random methods for seeking maxima. *Operations Research*, 6:244–251, 1958.
- [3] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2009.
- [4] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2009: Experimental setup. Technical Report RR-6828, INRIA, 2009.
- [5] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009.

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Figure 1: Expected Running Time (ERT, $\bullet$) to reach $f_{\text{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\text{ERT}(\Delta f)$ equals to $\#FEs(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed during the trial. The $\#FEs(\Delta f)$ are the total number of function evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations $\#FEs(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

f_{101} in 5-D, N=15, mFE=5.00e6						f_{101} in 20-D, N=15, mFE=2.00e7						f_{102} in 5-D, N=15, mFE=5.00e6						f_{102} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e2	7.9e1	1.3e2	1.1e2	0	29e+0	20e+0	32e+0	7.1e6	10	15	4.6e1	3.6e1	5.7e1	4.6e1	0	27e+0	20e+0	30e+0	8.9e6		
1	15	1.7e4	1.2e4	2.4e4	1.7e4	.	.	.	.	.	1	15	1.5e4	1.1e4	1.9e4	1.5e4	.	.	.	.	.		
1e-1	7	8.5e6	6.2e6	1.5e7	4.1e6	.	.	.	.	.	1e-1	6	8.5e6	5.9e6	1.7e7	3.7e6	.	.	.	.	.		
1e-3	0	10e-2	41e-3	14e-2	3.2e6	.	.	.	.	.	1e-3	0	11e-2	65e-3	17e-2	1.8e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{103} in 5-D, N=15, mFE=5.00e6						f_{103} in 20-D, N=15, mFE=2.00e7						f_{104} in 5-D, N=15, mFE=5.00e6						f_{104} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	6.8e1	5.4e1	8.7e1	6.8e1	0	28e+0	26e+0	31e+0	1.1e7	10	14	1.6e6	9.9e5	2.2e6	1.4e6	0	91e+2	64e+2	11e+3	1.0e7		
1	15	2.3e4	1.8e4	2.8e4	2.3e4	.	.	.	.	.	1	0	64e-1	44e-1	90e-1	2.0e6	.	.	.	.	.		
1e-1	7	8.7e6	6.6e6	1.5e7	4.5e6	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	0	10e-2	36e-3	16e-2	2.0e6	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{105} in 5-D, N=15, mFE=5.00e6						f_{105} in 20-D, N=15, mFE=2.00e7						f_{106} in 5-D, N=15, mFE=5.00e6						f_{106} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	14	2.3e6	1.7e6	2.9e6	2.1e6	0	88e+2	63e+2	11e+3	1.0e7	10	15	1.2e6	9.1e5	1.7e6	1.2e6	0	98e+2	76e+2	11e+3	1.3e7		
1	0	69e-1	36e-1	93e-1	2.2e6	.	.	.	.	.	1	0	62e-1	53e-1	88e-1	1.4e6	.	.	.	.	.		
1e-1	.	.	.	.	.	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{107} in 5-D, N=15, mFE=5.00e6						f_{107} in 20-D, N=15, mFE=2.00e7						f_{108} in 5-D, N=15, mFE=5.00e6						f_{108} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.8e2	1.0e2	2.7e2	1.8e2	0	27e+0	23e+0	31e+0	1.0e7	10	15	8.7e1	5.9e1	1.2e2	8.7e1	0	28e+0	23e+0	33e+0	8.9e6		
1	15	1.1e4	7.4e3	1.5e4	1.1e4	.	.	.	.	.	1	15	2.1e4	1.6e4	2.6e4	2.1e4	.	.	.	.	.		
1e-1	9	5.7e6	4.2e6	8.4e6	3.1e6	.	.	.	.	.	1e-1	8	6.9e6	5.2e6	9.8e6	3.3e6	.	.	.	.	.		
1e-3	0	96e-3	54e-3	14e-2	2.2e6	.	.	.	.	.	1e-3	0	98e-3	37e-3	17e-2	3.2e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{109} in 5-D, N=15, mFE=5.00e6						f_{109} in 20-D, N=15, mFE=2.00e7						f_{110} in 5-D, N=15, mFE=5.00e6						f_{110} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.2e2	8.9e1	1.5e2	1.2e2	0	28e+0	22e+0	34e+0	1.0e7	10	14	1.8e6	1.3e6	2.3e6	1.4e6	0	80e+2	60e+2	97e+2	7.9e6		
1	15	1.8e4	1.4e4	2.3e4	1.8e4	.	.	.	.	.	1	0	63e-1	36e-1	90e-1	2.8e6	.	.	.	.	.		
1e-1	10	4.8e6	3.7e6	6.8e6	3.3e6	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	0	88e-3	34e-3	16e-2	2.5e6	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{111} in 5-D, N=15, mFE=5.00e6						f_{111} in 20-D, N=15, mFE=2.00e7						f_{112} in 5-D, N=15, mFE=5.00e6						f_{112} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.4e6	1.1e6	1.8e6	1.4e6	0	94e+2	73e+2	12e+3	6.3e6	10	13	2.0e6	1.3e6	2.7e6	1.9e6	0	76e+2	53e+2	95e+2	1.3e7		
1	0	66e-1	46e-1	85e-1	2.2e6	.	.	.	.	.	1	0	71e-1	40e-1	10e+0	1.8e6	.	.	.	.	.		
1e-1	.	.	.	.	.	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{113} in 5-D, N=15, mFE=5.00e6						f_{113} in 20-D, N=15, mFE=2.00e7						f_{114} in 5-D, N=15, mFE=5.00e6						f_{114} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.2e3	8.3e2	1.5e3	1.2e3	0	95e+0	79e+0	11e+1	8.9e6	10	15	2.1e3	1.5e3	2.8e3	2.1e3	0	11e+1	76e+0	14e+1	7.9e6		
1	15	6.6e5	4.6e5	8.6e5	6.6e5	.	.	.	.	.	1	15	5.0e5	2.7e5	7.0e5	5.0e5	.	.	.	.	.		
1e-1	0	31e-2	17e-2	57e-2	2.8e6	.	.	.	.	.	1e-1	1	7.3e7	2.4e7	>7e7	5.0e6	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	0	44e-2	16e-2	73e-2	2.8e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{115} in 5-D, N=15, mFE=5.00e6						f_{115} in 20-D, N=15, mFE=2.00e7						f_{116} in 5-D, N=15, mFE=5.00e6						f_{116} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.4e3	9.8e2	1.7e3	1.4e3	0	11e+1	77e+0	12e+1	1.3e7	10	6	1.0e7	6.6e6	1.7e7	2.9e6	0	44e+2	31e+2	59e+2	8.9e6		
1	15	4.4e5	3.3e5	5.4e5	4.4e5	.	.	.	.	.	1	0	11e+0	80e-1	17e+0	3.2e6	.	.	.	.	.		
1e-1	0	34e-2	16e-2	53e-2	2.2e6	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{117} in 5-D, N=15, mFE=5.00e6						f_{117} in 20-D, N=15, mFE=2.00e7						f_{118} in 5-D, N=15, mFE=5.00e6						f_{118} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	8	6.3e6	4.9e6	8.2e6	4.2e6	0	44e+2	37e+2	66e+2	1.6e7	10	8	6.5e6	4.4e6	8.9e6	3.6e6	0	53e+2	38e+2	63e+2	1.0e7		
1	0	94e-1	28e-1	18e+0	2.8e6	.	.	.	.	.	1	0	10e+0	27e-1	15e+0	2.8e6	.	.	.	.	.		
1e-1	.	.	.	.	.	.	.	.	.	.	1e-1	.	.	.	.	.	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{119} in 5-D, N=15, mFE=5.00e6						f_{119} in 20-D, N=15, mFE=2.00e7						f_{120} in 5-D, N=15, mFE=5.00e6						f_{120} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.7e1	1.4e1	2.0e1	1.7e1	15	4.9e6	3.7e6	6.1e6	4.9e6	10	15	1.8e1	1.4e1	2.2e1	1.8e1	15	4.5e6	3.4e6	5.5e6	4.5e6		
1	15	7.6e3	5.7e3	9.5e3	7.6e3	0	84e-1	68e-1	96e-1	8.9e6	1	15	6.7e3	4.7e3	9.1e3	6.7e3	0	81e-1	69e-1	89e-1	7.9e6		
1e-1	7	7.3e6	4.9e6	1.2e7	3.1e6	.	.	.	.	.	1e-1	6	1.0e7	7.5e6	1.8e7	4.5e6	.	.	.	.	.		
1e-3	0	10e-2	56e-3	14e-2	2.0e6	.	.	.	.	.	1e-3	0	11e-2	36e-3	15e-2	2.5e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10D, 100D \dots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

f_{121} in 5-D, N=15, mFE=5.00e6						f_{121} in 20-D, N=15, mFE=2.00e7						f_{122} in 5-D, N=15, mFE=5.00e6						f_{122} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	2.1e1	1.5e1	2.7e1	2.1e1	14	6.4e6	4.4e6	8.3e6	6.2e6	10	15	2.0e1	1.4e1	2.6e1	2.0e1	15	8.6e3	4.8e3	1.3e4	8.6e3		
1	15	5.9e3	4.0e3	7.8e3	5.9e3	0	82e-1	66e-1	10e+0	1.0e7	1	15	1.8e5	1.4e5	2.3e5	1.8e5	0	49e-1	44e-1	56e-1	1.0e7		
1e-1	9	5.6e6	4.3e6	8.7e6	3.4e6	.	.	.	.	.	1e-1	0	46e-2	31e-2	56e-2	2.2e6	.	.	.	.	.		
1e-3	0	87e-3	36e-3	14e-2	2.0e6	.	.	.	.	.	1e-3	.	.	.	.	.	0	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{123} in 5-D, N=15, mFE=5.00e6						f_{123} in 20-D, N=15, mFE=2.00e7						f_{124} in 5-D, N=15, mFE=5.00e6						f_{124} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e1	8.1e0	1.4e1	1.1e1	15	7.9e3	4.8e3	1.0e4	7.9e3	10	15	1.9e1	1.4e1	2.3e1	1.9e1	15	8.9e3	3.7e3	1.5e4	8.9e3		
1	15	2.4e5	2.0e5	2.8e5	2.4e5	0	53e-1	48e-1	57e-1	1.0e7	1	15	1.9e5	1.2e5	2.7e5	1.9e5	0	51e-1	48e-1	57e-1	1.1e7		
1e-1	0	49e-2	30e-2	58e-2	2.2e6	.	.	.	.	.	1e-1	0	46e-2	25e-2	61e-2	2.5e6	.	.	.	.	.		
1e-3	.	.	.	.	.	.	.	.	.	.	1e-3	.	.	.	.	.	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{125} in 5-D, N=15, mFE=5.00e6						f_{125} in 20-D, N=15, mFE=2.00e7						f_{126} in 5-D, N=15, mFE=5.00e6						f_{126} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e0	1.0e0	1.3e0	1.1e0	15	1.0e0	1.0e0	1.0e0	1.0e0	10	15	1.1e0	1.0e0	1.3e0	1.1e0	15	1.1e0	1.0e0	1.1e0	1.1e0		
1	15	6.0e1	3.1e1	8.3e1	6.0e1	15	5.9e5	4.4e5	7.9e5	5.9e5	1	15	4.3e1	2.8e1	5.4e1	4.3e1	15	3.2e5	2.0e5	4.3e5	3.2e5		
1e-1	15	1.5e5	1.1e5	2.0e5	1.5e5	0	80e-2	70e-2	85e-2	7.9e6	1e-1	15	1.2e5	9.6e4	1.5e5	1.2e5	0	76e-2	72e-2	81e-2	8.9e6		
1e-3	0	36e-3	21e-3	44e-3	3.2e6	.	.	.	.	.	1e-3	0	34e-3	24e-3	51e-3	1.4e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	.	.	.	.	.	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{127} in 5-D, N=15, mFE=5.00e6						f_{127} in 20-D, N=15, mFE=2.00e7						f_{128} in 5-D, N=15, mFE=5.00e6						f_{128} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.3e0	1.1e0	1.4e0	1.3e0	10	15	2.0e2	1.4e2	2.5e2	2.0e2	0	24e+0	15e+0	31e+0	1.0e7		
1	15	4.7e1	3.5e1	6.0e1	4.7e1	15	5.2e5	3.9e5	6.8e5	5.2e5	1	15	1.7e4	1.4e4	2.1e4	1.7e4	.	.	.	.	.		
1e-1	15	1.2e5	8.6e4	1.6e5	1.2e5	0	79e-2	67e-2	83e-2	7.9e6	1e-1	15	3.2e5	2.0e5	4.3e5	3.2e5	.	.	.	.	.		
1e-3	0	33e-3	15e-3	44e-3	2.0e6	.	.	.	.	.	1e-3	1	7.4e7	3.7e7	>7e7	4.1e6	.	.	.	.	.		
1e-5	.	.	.	.	.	.	.	.	.	.	1e-5	0	10e-3	11e-4	22e-3	2.0e6	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		
f_{129} in 5-D, N=15, mFE=5.00e6						f_{129} in 20-D, N=15, mFE=2.00e7						f_{130} in 5-D, N=15, mFE=5.00e6						f_{130} in 20-D, N=15, mFE=2.00e7					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.8e2	1.4e2	2.4e2	1.8e2	0	24e+0	23e+0	29e+0	7.9e6	10	15	1.7e2	1.3e2	2.2e2	1.7e2	0	25e+0	18e+0	29e+0	1.3e7		
1	15	1.4e4	1.1e4	1.7e4	1.4e4	.	.	.	.	.	1	15	2.5e4	1.7e4	3.2e4	2.5e4	.	.	.	.	.		
1e-1	15	3.4e5	2.7e5	4.2e5	3.4e5	.	.	.	.	.	1e-1	15	3.3e5	2.4e5	4.0e5	3.3e5	.	.	.	.	.		
1e-3	1	7.4e7	3.7e7	>7e7	5.0e6	.	.	.	.	.	1e-3	1	7.4e7	2.5e7	>7e7	5.0e6	.	.	.	.	.		
1e-5	0	13e-3	12e-4	31e-3	2.5e6	.	.	.	.	.	1e-5	0	42e-4	16e-4	15e-3	2.5e6	.	.	.	.	.		
1e-8	.	.	.	.	.	.	.	.	.	.	1e-8	.	.	.	.	.	.	.	.	.	.		

Table 2: Shown are, for functions f_{121} - f_{130} and for a given target difference to the optimal function value Δf : the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf -value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.