

There are arbitrary large minimal 2-pinning configurations

Xavier Goaoc, Kim Hyo-Sil, Lim Jung-Gun

► **To cite this version:**

Xavier Goaoc, Kim Hyo-Sil, Lim Jung-Gun. There are arbitrary large minimal 2-pinning configurations. The First Asian Association for Algorithms and Computation Annual Meeting - AAAC 08, Apr 2008, Hong-Kong, China. 2008. <inria-00431768>

HAL Id: inria-00431768

<https://hal.inria.fr/inria-00431768>

Submitted on 13 Nov 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

There are arbitrarily large minimal 2-pinning configurations

Xavier Goaoc*

Hyo-Sil Kim†

Jung-Gun Lim‡

Let ℓ be a line and \mathcal{C} a collection of disjoint convex sets in \mathbb{R}^d . We say that ℓ is a *transversal* to \mathcal{C} if it intersects each of its members and that \mathcal{C} *pins* ℓ if, in addition, no arbitrarily small perturbation of ℓ is a transversal to \mathcal{C} . We say that \mathcal{C} *k-pins* ℓ if for any k -flat Π containing ℓ , $\Pi \cap \mathcal{C}$ pins ℓ in Π (so pinning and d -pinning are the same in \mathbb{R}^d). A *minimal k-pinning configuration* is a pair (ℓ, \mathcal{C}) where \mathcal{C} k -pins ℓ but no proper subset of \mathcal{C} does.

When \mathcal{C} consists of disjoint balls, all its transversal that meet the balls in the same order make up a connected set in line space [1]. So, by continuity, \mathcal{C} pins ℓ if and only if no other transversal to \mathcal{C} realizes the same order as ℓ . Since two lines span a space of dimension at most 3, this implies that \mathcal{C} pins ℓ if and only if it 3-pins it. As a consequence, all minimal 3-pinning configurations of disjoint balls in \mathbb{R}^d have size at most $2d - 1$ [1, 2]. Here, we prove that the situation is different for 2-pinning:

Theorem 1. *A minimal 2-pinning configuration of size $n \geq 6$ in \mathbb{R}^3 exists if and only if n is even.*

For simplicity we assume that the objects are smooth, e.g. balls. Our proof is essentially combinatorial:

1. Sign sequence. Let ℓ be an oriented line in \mathbb{R}^2 tangent to O_1, \dots, O_n in that order. Its *sign sequence* is a word in $\{-, +\}^n$ where the i^{th} letter is $+$ if and only if O_i is on the right of ℓ .

2. Encoding. We now return to 3-space, assume that \mathcal{C} 2-pins ℓ and choose some arbitrary orientation on ℓ . The tangent planes to the objects of \mathcal{C} at their contact point with ℓ partition the space into n sectors. For all planes containing ℓ lying in the same sector, the sign sequence of the associated planar configuration is the same (up to exchanging $-$ and $+$). We denote by $\sigma_0, \dots, \sigma_{n-1}$ the sign sequences obtained successively as we go through all sectors around ℓ .

3. Translating geometric properties. The family $(\sigma_0, \dots, \sigma_{n-1})$ corresponds to a geometric configuration (\mathcal{C}, ℓ) if and only if (1) there exists a permutation π of $\{1, \dots, n\}$ such that σ_i differs from $\sigma_{i-1[n]}$ from the inversion of its $\pi(i)^{\text{th}}$ letter. Then, observe that \mathcal{C} 2-pins ℓ if and only if (2) every σ_i contains an alternating triple. Last, notice that no proper subset of \mathcal{C} 2-pins ℓ if and only if (3) for any $1 \leq i \leq n$ there is some t such that σ_t loses all alternating triples if its i^{th} letter is deleted.

4. Wrapping up. For odd n , rules (1)–(3) are incompatible. For even n , the sequence defined by

$$\sigma_0 = + - +^{n-2} \quad \text{and} \quad \pi = (4, 1, 6, 3, 8, 5, \dots, n, n-3, 2, n-1)$$

satisfies rules (1)–(3) and thus corresponds to a minimal 2-pinning configuration.

References

- [1] C. Borcea, X. Goaoc, and S. Petitjean. Line transversals to disjoint balls. *Discrete & Computational Geometry*, 2008. To appear.
- [2] O. Cheong, X. Goaoc, A. Holmsen, and S. Petitjean. Hadwiger and Helly-type theorems for disjoint unit spheres. In J. E. Goodman, J. Pach, and R. Pollack, editors, *Computational Geometry - Twenty Years Later*. AMS. To appear.

*LORIA - INRIA Grand Est, Projet VEGAS. goaoc@loria.fr

†Theory of Computation Lab., Division of Computer Science, KAIST. hyosil@jupiter.kaist.ac.kr

‡NCsoft Corporation. araste@gmail.com