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# There are arbitrarily large minimal 2-pinning configurations

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Let  $\ell$  be a line and  $\mathcal{C}$  a collection of disjoint convex sets in  $\mathbb{R}^d$ . We say that  $\ell$  is a *transversal* to  $\mathcal{C}$  if it intersects each of its members and that  $\mathcal{C}$  *pins*  $\ell$  if, in addition, no arbitrarily small perturbation of  $\ell$  is a transversal to  $\mathcal{C}$ . We say that  $\mathcal{C}$  *k-pins*  $\ell$  if for any  $k$ -flat  $\Pi$  containing  $\ell$ ,  $\Pi \cap \mathcal{C}$  pins  $\ell$  in  $\Pi$  (so pinning and  $d$ -pinning are the same in  $\mathbb{R}^d$ ). A *minimal k-pinning configuration* is a pair  $(\ell, \mathcal{C})$  where  $\mathcal{C}$   $k$ -pins  $\ell$  but no proper subset of  $\mathcal{C}$  does.

When  $\mathcal{C}$  consists of disjoint balls, all its transversals that meet the balls in the same order make up a connected set in line space [1]. So, by continuity,  $\mathcal{C}$  pins  $\ell$  if and only if no other transversal to  $\mathcal{C}$  realizes the same order as  $\ell$ . Since two lines span a space of dimension at most 3, this implies that  $\mathcal{C}$  pins  $\ell$  if and only if it 3-pins it. As a consequence, all minimal 3-pinning configurations of disjoint balls in  $\mathbb{R}^d$  have size at most  $2d - 1$  [1, 2]. Here, we prove that the situation is different for 2-pinning:

**Theorem 1.** *A minimal 2-pinning configuration of size  $n \geq 6$  in  $\mathbb{R}^3$  exists if and only if  $n$  is even.*

For simplicity we assume that the objects are smooth, e.g. balls. Our proof is essentially combinatorial:

**1. Sign sequence.** Let  $\ell$  be an oriented line in  $\mathbb{R}^2$  tangent to  $O_1, \dots, O_n$  in that order. Its *sign sequence* is a word in  $\{-, +\}^n$  where the  $i^{\text{th}}$  letter is  $+$  if and only if  $O_i$  is on the right of  $\ell$ .

**2. Encoding.** We now return to 3-space, assume that  $\mathcal{C}$  2-pins  $\ell$  and choose some arbitrary orientation on  $\ell$ . The tangent planes to the objects of  $\mathcal{C}$  at their contact point with  $\ell$  partition the space into  $n$  sectors. For all planes containing  $\ell$  lying in the same sector, the sign sequence of the associated planar configuration is the same (up to exchanging  $-$  and  $+$ ). We denote by  $\sigma_0, \dots, \sigma_{n-1}$  the sign sequences obtained successively as we go through all sectors around  $\ell$ .

**3. Translating geometric properties.** The family  $(\sigma_0, \dots, \sigma_{n-1})$  corresponds to a geometric configuration  $(\mathcal{C}, \ell)$  if and only if (1) there exists a permutation  $\pi$  of  $\{1, \dots, n\}$  such that  $\sigma_i$  differs from  $\sigma_{i-1[n]}$  from the inversion of its  $\pi(i)^{\text{th}}$  letter. Then, observe that  $\mathcal{C}$  2-pins  $\ell$  if and only if (2) every  $\sigma_i$  contains an alternating triple. Last, notice that no proper subset of  $\mathcal{C}$  2-pins  $\ell$  if and only if (3) for any  $1 \leq i \leq n$  there is some  $t$  such that  $\sigma_t$  loses all alternating triples if its  $i^{\text{th}}$  letter is deleted.

**4. Wrapping up.** For odd  $n$ , rules (1)–(3) are incompatible. For even  $n$ , the sequence defined by

$$\sigma_0 = + - +^{n-2} \quad \text{and} \quad \pi = (4, 1, 6, 3, 8, 5, \dots, n, n-3, 2, n-1)$$

satisfies rules (1)–(3) and thus corresponds to a minimal 2-pinning configuration.

## References

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