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ABSTRACT

The sequential forms of the spectral MUSIC algorithm, such as the Sequential MUSIC (S-MUSIC) and the Recursively Applied and Projected MUSIC (RAP-MUSIC) algorithms, use the previously estimated DOA (Direction Of Arrival) to form an intermediate array gain matrix and project both the array manifold and the signal subspace estimate into its orthogonal complement. By doing this, these methods avoid the delicate search of multiple maxima and yield a more accurate DOA estimation in difficult scenarios. However, these high-resolution algorithms adapted to a general array geometry suffer from a high computational cost. On the other hand, for linear equispaced sensor array, the root-MUSIC algorithm is a fast and accurate high-resolution scheme which also avoids the delicate search of multiple maxima but a sequential scheme based on the root-MUSIC algorithm does not exist. This paper fills this need. Thus, we present a new sequential high-resolution estimation method, called the Projected Companion Matrix MUSIC (PCM-MUSIC) method, in the context of source localisation in the case of linear equispaced sensor array. Remark that the proposed algorithm can be used without modification in the context of spectral analysis.

1. INTRODUCTION

Localization of sources or Direction Of Arrival (DOA) estimation by a passive sensors array is an important topic in several applications such as radar, sonar, seismology and wireless communications [1]. A variety of high-resolution algorithms are based on the singular/ eigen value decomposition and are referred to as subspace based methods and they split the range space associated with sample covariance matrix into the signal and the noise subspaces.

The MUltiple SIgnal Classification (MUSIC) algorithm introduced by Schmidt in 1979 [2], estimates the DOA by minimizing the orthogonal condition between the noise subspace and the array manifold vector. However for multiple sources, MUSIC needs a peak-picking procedure to find multiple maxima in the pseudo-spectrum. This operation can lead to errors particularly when the sources are closely spaced and/or strongly correlated. To improve the performance of such high-resolution algorithms in difficult scenarios, several authors have proposed sequential versions of the MUSIC algorithm avoiding the delicate search of multiple maxima [3]. These high-resolution algorithms use a sequential procedure in which each DOA is found as the global maximizer of different (but related) cost functions. In [4], Oh *et al.* presented the Sequential MUSIC (S-MUSIC) algorithm where the cost function is obtained by projecting the array manifold onto the orthogonal subspace spanned by the previously estimated DOA. Stoica *et al.* introduced an improved version, called IES-MUSIC for the case of two sources [5]. In [6] and [7], Mosher *et al.* presented the Recursively applied MUSIC (R-MUSIC) and its improved version, called the Recursively Applied and Projected MUSIC (RAP-MUSIC). The RAP-MUSIC algorithm uses a cost function obtained by the projection of both the signal subspace and the array manifold onto the orthogonal subspace spanned by the previously estimated DOA. These sequential methods avoid the delicate search of multiple maxima in the MUSIC pseudo-spectrum and provide improved performance for correlated sources in the context of a general array geometry. But the main drawback is that the computational cost is M times higher than the spectral MUSIC, where M is the number of sources. On the other hand, for linear equispaced sensor array, the root-MUSIC algorithm [9] is based on the diagonalization of the companion matrix associated with a polynomial derived from the noise projector involved in the MUSIC algorithm. This algorithm is fast (with respect to the spectral MUSIC algorithm) and accurate which also avoids the delicate search of multiple maxima but a sequential scheme based on the root-MUSIC algorithm does not exist.

In this paper, we present a fast sequential high-resolution algorithm for source localization called the Projected Companion Matrix MUSIC (PCM-MUSIC) method, which is the sequential version of the root-MUSIC algorithm. This method exploits the confluent Vandermonde structure involved into the Jordan decomposition of the companion matrix [11]. This decomposition is an extension of the well-known Vandermonde decomposition of the companion matrix in case of eigen-values of multiplicity higher or equal to one. More precisely, the PCM-MUSIC algorithm considers the diagonalization of an *ad hoc* projection of the companion matrix exploiting the confluent Vandermonde structure to cancel the previously estimated DOAs. By doing this, this scheme is not based on an exhaustive and costly inspection of the spectral-MUSIC pseudo-spectrum as all the current existing sequential methods (S-MUSIC, IES-MUSIC, R-MUSIC and RAP-MUSIC) but on a fast root-finding technique as for the root-MUSIC algorithm.

2. MATRIX-BASED REPRESENTATION OF THE DOA ESTIMATION PROBLEM

2.1. Parametric model for multiple snapshots

Assume there are M narrowband plane waves simultaneously impinging on an Uniform and Linear Array (ULA) with L sensors.

The complex array response for the t^{th} snapshot is given by

$$\boldsymbol{y}(t) = [y_1(t) \dots y_L(t)]^T = \boldsymbol{x}(t) + \boldsymbol{n}(t)$$

with $y_\ell(t)$ is the noisy observation on the ℓ^{th} sensor, $\boldsymbol{x}(t) = \boldsymbol{Z}\boldsymbol{\alpha}(t)$ where $\boldsymbol{\alpha}(t) = [\alpha_1(t) \dots \alpha_M(t)]^T$ and $\alpha_m(t)$ is the complex amplitude of the m^{th} source. The noise vector is denoted by $\boldsymbol{n}(t) = [n_1(t) \dots n_L(t)]^T$ in which the noise on each sensor, denoted by $n_\ell(t)$, is assumed to be additive complex circular white and Gaussian of parameter $\mathcal{N}(0, \sigma^2)$ and σ is a positive real parameter. The matrix $\boldsymbol{Z}$ is the $L \times M$ Vandermonde array manifold defined by

$$\boldsymbol{Z} = [\boldsymbol{p}(\theta_1)\, \boldsymbol{p}(\theta_2) \dots \boldsymbol{p}(\theta_M)]$$

where

$$\boldsymbol{p}(\theta) = [1\ e^{-2i\pi(\Delta/c)\sin(\theta)} \dots e^{-2i\pi(\Delta/c)\sin(\theta)(L-1)}]^T$$

is the steering vector parameterized by DOA θ in which Δ is the fix inter-sensor distance and c is the wavelength. The parameter M is assumed to be known or previously estimated [8, Appendix C]. Thus, the parametric model for T snapshots can be written as

$$\boldsymbol{Y} = [\boldsymbol{y}(1) \dots \boldsymbol{y}(T)] = \boldsymbol{X} + \boldsymbol{N} \tag{1}$$

where $\boldsymbol{X} = [\boldsymbol{x}(1) \dots \boldsymbol{x}(T)] = \boldsymbol{Z}\boldsymbol{\Lambda}$ with $\boldsymbol{\Lambda} = [\boldsymbol{\alpha}(1) \dots \boldsymbol{\alpha}(T)]$ and $\boldsymbol{N} = [\boldsymbol{n}(1) \dots \boldsymbol{n}(T)]$ is the noise matrix.

2.2. Partitioned steering manifold

Assume that we have already estimated S DOAs, denoted by $\boldsymbol{\theta} = [\theta_1 \dots \theta_S]$, among M. Without loss of generality, the steering manifold $\boldsymbol{Z}$ can be partitioned according to

$$\boldsymbol{Z} = \left[\begin{array}{c|c} \underbrace{\boldsymbol{p}(\theta_1) \dots \boldsymbol{p}(\theta_S)}_{\boldsymbol{A} \text{ (previously estimated)}} & \underbrace{\boldsymbol{p}(\theta_{S+1}) \dots \boldsymbol{p}(\theta_M)}_{\boldsymbol{B} \text{ (unknown)}} \end{array} \right] \tag{2}$$

where the $L \times S$ submanifold $\boldsymbol{A}$ is the matrix composed by the S previously estimated DOAs and submanifold $\boldsymbol{B}$ collects the $M-S$ desired DOAs. We name $\mathcal{R}(\boldsymbol{B})$ the subspace of interest or the deflated signal subspace as its dimension is $M-S$ which is smaller than M, the dimension of the signal subspace $\mathcal{R}(\boldsymbol{Z})$.

3. PROJECTED COMPANION MATRIX TECHNIQUE

3.1. The root-MUSIC principle

Assuming that the sensor noise is uncorrelated with the sources, then the $(L \times L)$ spatial covariance matrix admits the decomposition

$$\boldsymbol{R}_{\boldsymbol{Y}} = \mathbb{E}(\boldsymbol{Y}\boldsymbol{Y}^H) = \boldsymbol{R}_{\boldsymbol{X}} + \sigma^2 \boldsymbol{I}_L$$

where $\mathbb{E}(.)$ is the mathematical expectation and the noise-free spatial covariance is given by $\boldsymbol{R}_{\boldsymbol{X}} = \boldsymbol{Z}\boldsymbol{R}_{\boldsymbol{\Lambda}}\boldsymbol{Z}^H$ where $\boldsymbol{R}_{\boldsymbol{\Lambda}}$ is the source covariance matrix. Let $\hat{\boldsymbol{R}}_{\boldsymbol{Y}} = \frac{1}{T}\boldsymbol{Y}\boldsymbol{Y}^H$ be the sample spatial covariance of the noisy observations. Under ergodicity and stationarity assumptions, we know that $\lim_{T\to\infty} \hat{\boldsymbol{R}}_{\boldsymbol{Y}} = \boldsymbol{R}_{\boldsymbol{Y}}$. $\hat{\boldsymbol{R}}_{\boldsymbol{Y}}$ being Hermitian and nonnegative-definite matrix, its eigenvalues λ_i are real values and we sort them in descending order $\lambda_1 \geq \dots \geq \lambda_M \geq \lambda_{M+1} \geq \dots \geq \lambda_L$. Then the sample spatial covariance can be written as $\hat{\boldsymbol{R}}_{\boldsymbol{Y}} = \sum_{n=1}^{L} \lambda_n \boldsymbol{u}_n \boldsymbol{u}_n^H$, where $\boldsymbol{u}_n$ is the eigenvector associated with the eigenvalue λ_n. The noise-subspace, denoted by $\mathcal{R}(\boldsymbol{Z})^\perp$, is then spanned by $\bar{\boldsymbol{U}} = [\boldsymbol{u}_{M+1} \dots \boldsymbol{u}_L]$ and the noise projector is defined by

$$\boldsymbol{\Pi}^\perp = \bar{\boldsymbol{U}}\bar{\boldsymbol{U}}^H = \sum_{n=M+1}^{L} \boldsymbol{u}_n \boldsymbol{u}_n^H.$$

Consequently, the well-known MUSIC criterion [2] identifies candidate steering vectors as the ones which are the farthest from the noise eigenvectors. The spectrum defined by MUSIC acts like an inverse pseudo-distance measure which is given by the maxima of which yield an estimation of the DOA. The key idea of the root-MUSIC [9] algorithm is to obtain the roots/zeros of the following conjugate centro-symmetric polynomial of degree $2L-2$:

$$d(z) = \tilde{\boldsymbol{p}}_L\left(\frac{1}{z}\right)^T \boldsymbol{\Pi}^\perp \tilde{\boldsymbol{p}}_L(z) = \sum_{\ell=0}^{L-1} \frac{q_\ell^*}{z^\ell} + \sum_{\ell=1}^{L-1} q_\ell z^\ell = 0$$

where $\tilde{\boldsymbol{p}}_L(z) = [1\ z\ z^2 \dots z^{L-1}]^T$, thus $\tilde{\boldsymbol{p}}_L(z) = \boldsymbol{p}(\theta)$ when $z = e^{-2i\pi\Delta\sin(\theta)}$. The explicit computation of the coefficients of $d(z)$ denoted by $\{q_\ell\}_{\ell\in[1-L:L-1]}$ is given by summing along each diagonal of the projector matrix. Polynomial $d(z)$ is equal to its reciprocal polynomial [10] and therefore a zero, denoted by z_m, such as $|z_m| = 1$ occurs in pairs. In presence of noise, the DOAs may be extracted (among $2L-2$ possible roots) based on their proximity to the unit circle.

3.2. Jordan decomposition of the companion matrix

Finding the roots of $d(z)$ is equivalent to solves

$$g(z) = \frac{q_{L-1}^*}{q_{L-1}} + \dots + \frac{q_0}{q_{L-1}} z^{L-1} + \dots + z^{2L-2} = 0, \tag{3}$$

with $q_{L-1} \neq 0$. Let ρ_ℓ denote the coefficient of the new polynomial, i.e., $\rho_\ell = \frac{q_{L-(\ell+1)}^*}{q_{L-1}}$ for $\ell \in [0 : L-2]$ and $\rho_\ell = \frac{q_{\ell+1-L}}{q_{L-1}}$ for $\ell \in [L-1 : 2L-3]$. Then the associated companion matrix is given by

$$\boldsymbol{C} = [\boldsymbol{e}_2 \dots \boldsymbol{e}_{2L-2}, -\boldsymbol{\rho}]^T_{(2L-2)\times(2L-2)}$$

where $\boldsymbol{e}_i$ denotes the i^{th} column of $\boldsymbol{I}_{(2L-2)}$, and $\boldsymbol{\rho} = [\rho_0 \dots \rho_{2L-3}]^T$. It is well-known that the characteristic polynomial of $\boldsymbol{C}$ is

$$g(z) = \det(z\boldsymbol{I} - \boldsymbol{C})$$

where $\det(.)$ denotes the determinant. Thus, $\tilde{\lambda}_1, \dots, \tilde{\lambda}_{2L-2}$ denoting the eigen-values of matrix $\boldsymbol{C}$ coincide with the roots of $g(z)$ and thus $d(z)$. Due to the fact that M roots (correspondig to the true M DOAs) occurs in pairs, their associated M eigenvalues have a multiplicity two. Without loss of generality, suppose that the M desired solutions are associated with the M first eigenvalues (each of multiplicity two), *i.e.*, $\{\tilde{\lambda}_1 = \tilde{\lambda}_2 = z_1, \dots, \tilde{\lambda}_{2M-1} = \tilde{\lambda}_{2M} = z_M\}$ and the other ones are the extraneous roots (each of multiplicity one). In this case, the Jordan decomposition of the companion matrix is as follows [11]

$$\boldsymbol{C} = \boldsymbol{V}\boldsymbol{\Delta}\boldsymbol{V}^{-1}$$

in which the square $(2L-2)$-rank confluent Vandermonde structured matrix given by

$$\boldsymbol{V} = [\boldsymbol{P}(z_1)\, \boldsymbol{P}(z_2) \dots \boldsymbol{P}(z_M)\, \tilde{\boldsymbol{p}}_{2L-2}(\tilde{\lambda}_{2M+1}) \dots \tilde{\boldsymbol{p}}_{2L-2}(\tilde{\lambda}_{2L-2})]$$

where

$$\boldsymbol{P}(z_m) = \left[\tilde{\boldsymbol{p}}_{2L-2}(z_m)\ \tilde{\boldsymbol{p}}'(z_m)\right]$$

and

$$\tilde{\boldsymbol{p}}'(z_m) = [0\ 1\ 2z_m\ \ldots (2L-3)z_m^{2L-4}]^T.$$

Note that $\mathrm{rank}(\boldsymbol{P}(z_m)) = 2 < 2L-2$ since $\tilde{\boldsymbol{p}}_{2L-2}(z_m)$ and $\tilde{\boldsymbol{p}}'(z_m)$ cannot be colinearn which implies that $\mathrm{rank}(\boldsymbol{V}) = 2L-2$ as long as the eigen-values are all distinct, and thus justifies the nonsingularity of $\boldsymbol{V}$. Through the above discussion, there exists M Jordan matrices denoted by $\boldsymbol{J}(z_1), \ldots, \boldsymbol{J}(z_M)$ defined according to

$$\boldsymbol{J}(z_m) = \begin{bmatrix} z_m & 1 \\ 0 & z_m \end{bmatrix}.$$

Then, the block-diagonal Jordan matrix is given by

$$\boldsymbol{\Delta} = \mathrm{bdiag}\left(\mathrm{bdiag}(\boldsymbol{J}(z_1)\ldots\boldsymbol{J}(z_M)), \mathrm{diag}(\tilde{\lambda}_{2M+1}\ldots\tilde{\lambda}_{2L-2})\right)$$

where bdiag (.) and diag (.) denotes the block diagonal and the diagonal operator, respectively. We can remark that this decomposition is a straightforward generalization of the well-known property that the companion matrix can be diagonalized in a Vandermonde structured basis in case of single multiplicity of the eigen-values. The particular structure of the Jordan decomposition of $\boldsymbol{C}$ is a consequence of the fact that the eigen-values corresponding to the desired solutions are of multiplicity two.

3.3. Projected Companion matrix

Having S previously estimated DOAs, we can compute the following projector $\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp} = \boldsymbol{I}_{(2L-2)} - \bar{\boldsymbol{A}}(\bar{\boldsymbol{A}}^H\bar{\boldsymbol{A}})^{-1}\bar{\boldsymbol{A}}^H$ onto $\mathcal{R}(\bar{\boldsymbol{A}})^{\perp}$ where $\bar{\boldsymbol{A}} = \begin{bmatrix}\boldsymbol{P}(\hat{z}_1) & \boldsymbol{P}(\hat{z}_2) & \ldots & \boldsymbol{P}(\hat{z}_S)\end{bmatrix}_{(2L-2)\times 2S}$. The aim is now to solve the following polynomial:

$$\tilde{g}(z) = \det\left(z\boldsymbol{I} - \boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{C}\right). \qquad (4)$$

Under high SNR, $\bar{\boldsymbol{A}} = [\boldsymbol{P}(z_1)\ \boldsymbol{P}(z_2)\ldots\boldsymbol{P}(z_S)]$ and thus

$$\begin{aligned}
\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{C} &= \left(\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{V}\right)\boldsymbol{\Delta}\boldsymbol{V}^{-1} \\
&= \sum_{m=1}^{S} \underbrace{\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{P}(z_m)}_{0}\boldsymbol{J}(z_m)\begin{bmatrix}\tilde{\boldsymbol{v}}_{2m-1}^T \\ \tilde{\boldsymbol{v}}_{2m}^T\end{bmatrix} \\
&+ \sum_{m=S+1}^{M} \boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{P}(z_m)\boldsymbol{J}(z_m)\begin{bmatrix}\tilde{\boldsymbol{v}}_{2m-1}^T \\ \tilde{\boldsymbol{v}}_{2m}^T\end{bmatrix} \\
&+ \sum_{m=2M+1}^{2L-2} \lambda_m\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\tilde{\boldsymbol{p}}(\lambda_m)\tilde{\boldsymbol{v}}_m^T
\end{aligned}$$

where matrix $\boldsymbol{V}^{-1}$ has been partitioned according to $\boldsymbol{V}^{-1} = [\tilde{\boldsymbol{v}}_1\ldots\tilde{\boldsymbol{v}}_{2L-2}]^T$. Since we suppose that the DOA are all distinct, we have $\mathrm{rank}(\bar{\boldsymbol{A}}) = 2S$. It follows that $\mathrm{rank}\left(\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\right) = 2(L-S-1)$. Using the rank property of matrix multiplication [8, Appendix A], we obtain $\mathrm{rank}\left(\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{C}\right) = \min\left(\mathrm{rank}\left(\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\right), \mathrm{rank}\left(\boldsymbol{C}\right)\right) = \min\left(2(L-S-1), 2(L-1)\right) = 2(L-S-1)$.

Thus, the effect of this projection is to decrease the rank of the companion matrix to $2(L-S-1)$ by cancelling the previously estimated solutions.

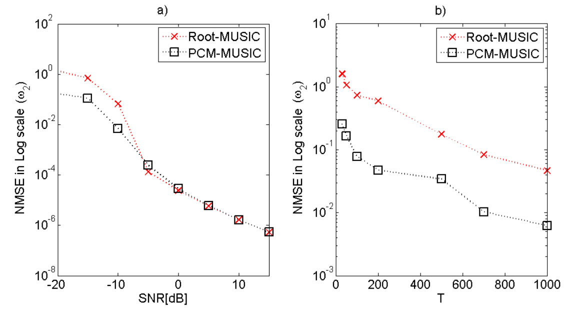

Figure 1: Largely spaced sources $(\omega_1, \omega_2) = (0.5, 0.75)$ rad with 20 sensors and $|\gamma|^2 = 0.95$ (a) NMSE Vs. SNR[dB] for 1000 snapshots (b) NMSE Vs. T for SNR=-10[dB]

3.4. PCM-MUSIC sequential algorithm

The PCM-MUSIC algorithm can be described as follows:

Init. Apply the root-MUSIC algorithm, i.e., $\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp} = \boldsymbol{I}$, and extract $\hat{\theta}_1$ corresponding to the eigen-value whose modulus is the nearest to the unit circle.

Loop. For $m \in [2 : M]$, compute the projector $\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}$ based on $\{\hat{\theta}_1, \ldots, \hat{\theta}_{m-1}\}$. Then, extract $\hat{\theta}_m$ as the eigen-value of $\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}\boldsymbol{C}$ which has the closest modulus to one. A fast computation is possible (see [12]).

4. STATISTICAL AND COMPUTATIONAL EVALUATION

A theoretical derivation of the PCM-MUSIC algorithm variance seems hard to do due to the sequential scheme of the algorithm. Thus, we illustrate its accuracy by means of numerical simulations. In this section, the PCM-MUSIC algorithm is compared to two other sequential algorithms : the S-MUSIC [4] and the RAP-MUSIC [7] algorithms. According to [7], the RAP-MUSIC algorithm yields improved performance over several forms of MUSIC (MUSIC, IES-MUSIC, R-MUSIC).

4.1. Numerical Simulations

The context of these simulations is an ULA of L sensors spaced by a half-wavelength. We consider two sources assumed to be far-field narrowband complex circular Gaussian sequences with zero mean and variance equal to one. Consequently, the SNR and the source covariance matrix are defined, respectively, as SNR[dB] $= 10\log_{10}\left(\frac{1}{\sigma^2}\right)$ and $\boldsymbol{R}_{\boldsymbol{\Lambda}} = \begin{bmatrix} 1 & \frac{\gamma}{\sqrt{2}} \\ \frac{\gamma^*}{\sqrt{2}} & 1 \end{bmatrix}$, where $|\gamma|^2$ determines the degree of correlation between these two sources. In the following, we focus on the Normalized Mean Square Error (NMSE) for ω_2 which is given by NMSE $= \frac{1}{1000}\sum_{i=1}^{1000}\frac{(\hat{\omega}_2(i)-\omega_2)^2}{\omega_2^2}$ where $\hat{\omega}_2(i)$ represents the estimate of ω_2 for the i^{th} trial in which the so-called electrical angle $\omega_2 = 2\pi\frac{\Delta}{c}\sin(\theta_2)$.

Before comparing the PCM-MUSIC algorithm to the S-MUSIC and the RAP-MUSIC sequential algorithms, we notice from Fig. 1.a that the Normalised Mean Square Error (NMSE) of the PCM-MUSIC algorithm can be lower than the one for the root-MUSIC algorithm at low SNR (lower than 0 dB) and for strongly correlated sources. In addition, according to Fig. 1.b, we notice that for a SNR$= -10$dB, the PCM-MUSIC algorithm has a higher accuracy

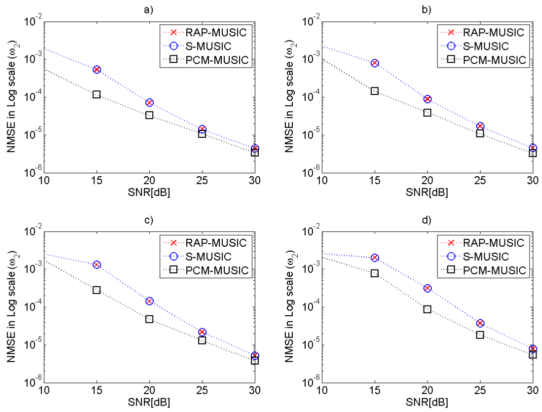

Figure 2: NMSE Vs. SNR[dB], closely spaced DOA with 15 sensors and $T = 1000$ snapshots ; (a) $|\gamma|^2 = 0$, (b) $|\gamma|^2 = 0.2$, (c) $|\gamma|^2 = 0.4$, (d) $|\gamma|^2 = 0.6$.

than the root-MUSIC algorithm whatever the number of snapshots. We now compare the PCM-MUSIC algorithm to the RAP-MUSIC and S-MUSIC algorithms. For largely spaced sources, these algorithms have very close performance and due to the lack of space we have not reported here the NMSE measurements. Next, we consider two closely spaced DOA given by $\omega_1 = 0.4$ rad and $\omega_2 = 0.46$ rad (i.e., $\Delta\theta = 1.10^o$). Fig .2 represents the NMSE of ω_2 for 15 sensors with different values of the degree of correlation. We notice that the PCM-MUSIC algorithm outperforms the S-MUSIC and the RAP-MUSIC algorithms for low and moderate SNR.

4.2. Computational Complexity

The computational evaluation is made by considering the costly operation of each algorithm [12]. For the PCM-MUSIC algorithm, the Moore-Penrose pseudoinverse of matrix $\bar{\boldsymbol{A}}$ was used to compute $(\bar{\boldsymbol{A}}^H\bar{\boldsymbol{A}})^{-1}\bar{\boldsymbol{A}}^H$. Furthermore, for a fast computation and without affecting the performance, we can solve (4) by computing its SVD using the fast recursive orthogonal iteration [12]. Thus, we notice that the costly operation of the PCM-MUSIC algorithm is the computation of the projector $\boldsymbol{P}_{\bar{\boldsymbol{A}}}^{\perp}$ in $O(SL^2)$. Whereas, the complexity of the S-MUSIC and the RAP-MUSIC algorithms is evaluted by the construction of the pseudo-spectrum, which is computed by $O\left(L^2MNr\right)$, where Nr denote the number of samples used to form the pseudo-spectrum. Note that Nr is generally a large number since $Nr = \pi/$accuracy. Consequently, the computational cost of the RAP-MUSIC and the S-MUSIC algorithms is much higher than for the PCM-MUSIC algorithm.

5. CONCLUSION

In this paper, we have presented a new fast sequential high-resolution algorithm, for estimating the DOA of plane waves in the case of linear equispaced sensor array, called the Projected Companion Matrix MUSIC (PCM-MUSIC) method. The proposed scheme is based on the root-MUSIC algorithm and can be used without modification in the context of spectral analysis. The key idea of this scheme is to exploit the knowledge of the previously estimated DOA through the diagonalization of an *ad hoc* projection of the companion matrix based on the confluent Vandermonde structure involved in its Jordan decomposition. Through numerical simulations and computational analysis, we show that : (i) the PCM-MUSIC algorithm outperforms the root-MUSIC algorithm for correlated sources and (ii) for comparable estimation accuracy, the PCM-MUSIC algorithm is less time consuming and less costly than the S-MUSIC and the RAP-MUSIC algorithms. Furthermore, the PCM-MUSIC algorithm outperforms the S-MUSIC and the RAP-MUSIC algorithms for low Signal-to-Noise Ratio (SNR) for closely spaced sources.

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