Selective Hybrid-ARQ turbo schemes with various Combining methods in Fading Channels

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Abstract – **Diversity combining offers a large variety of schemes for improved bit-error-rate (BER) in packet communications. The majority of the schemes are designed to combat noise. The scheme considers metric ratio combining (MRC), Chase combining and code combining. The paper also simulates and derives the theoretical average throughput of a diversity combining schemes employing turbo coding over Rayleigh fading channels. The paper proposes an expression for the selection of hybrid ARQ schemes based on a signal-to-interference-plus-noise-ratio (SINR), in the presence of co-channel interference. The presented SINR expression reduces to an average signal-to-noise ratio (SNR) when no co-channel interference exists.**

Index Terms – **HARQ, turbo coding, diversity combining, fading channels, throughput, selective combining**

I. INTRODUCTION

In wireless fading channels, a wide variety of diversity combining schemes have been proposed to deal with interference in fading channels [1], [2], [3]. Hybrid-ARQ schemes result in a higher effective signal-to-noise (SNR) and a correspondingly lower bit-error-rate (BER). Maximal ratio combining (MRC) with maximum likelihood decoding achieves the largest effective SNR of the diversity combining schemes. However when correlated noise is present from co-channel interference of other users, MRC does not operate optimally and other techniques need to be considered [4].

In this paper, we derive the throughput for various diversity combining techniques with turbo coding. Based on the derivations we comment on their suitability for various applications. In addition we examine a hybrid-ARQ selection technique based on SINR. The paper is organised as follows: Section II briefly explains the different hybrid ARQ schemes, Section III shows the system model Section IV discusses a hybrid ARQ based on SINR estimation and Section V presents a conclusion.

II. HYBRID ARQ TECHNIQUES

Hybrid automatic repeat request (hybrid-ARQ) schemes combine ARQ protocols with forward error correction codes (FEC) to provide increased throughput in packet transmissions [5], [6]. HARQ schemes may be classified as Type-I, Type-II and Type-III Hybrid ARQ schemes depending on the level of complexity employed in there implementation.

- *Type I Hybrid ARQ:* On a decoding error, this ARQ scheme discards erroneous packets and sends a retransmission request to the transmitter. The entire packet is retransmitted on receipt of the NACK. The packets are combined based on either the weighted SNR's of individual bits or soft energy values, in which case the technique is termed Chase combining [2].
- *Type II Hybrid ARQ:* In this ARQ scheme, retransmission requests consist only of parity bits. The receiver combines additional parity bits from retransmission with bits of the first transmission resulting in lower rates, before FEC decoding is attempted [3].
- *Type III Hybrid ARQ:* In Type III ARQ schemes, individually transmitted packets are self-decodable and each packet differs in coded bits from the previous transmission. In Type III ARQ, packets are only combined after decoding has been attempted on the individual packet.

III. SYSTEM MODEL

The system model employed in the paper is shown in Fig. 1. The channel under consideration is a frequency non-selective Rayleigh fading channel. A coherent receiver is employed in this paper

$$y_i = \alpha_i x_i + n_i \tag{1}$$

where y_i is the received signal for the ith bit within a packet, x_i represents the transmitted signal, α_i represents the complex Gaussian fading coefficient and n_i represents the zero mean additive white Gaussian noise (AWGN) of variance σ^2.

Input data is encoded by a turbo code for error correction and by a CRC code for error detection. The CCSDS component codes are employed in the turbo scheme. They are industrial codes capable of adaptation depending on channel conditions [7]. Various code rates {2/3, 1/2, 2/5, 1/3} can be chosen in the system. The rates result in the transmission of 2, 4, 6 or 8 digits for every 4 information bits. The corresponding puncturing matrices P_0, P_1, P_2 and P_3 are given [19],

$$\overbrace{\begin{bmatrix}1&1&1&1\\1&0&0&0\\0&0&0&1\end{bmatrix}}^{P_0},\ \overbrace{\begin{bmatrix}1&1&1&1\\1&0&1&0\\0&1&0&1\end{bmatrix}}^{P_1},\ \overbrace{\begin{bmatrix}1&1&1&1\\1&1&1&0\\0&1&1&1\end{bmatrix}}^{P_2},\ \overbrace{\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}}^{P_3}$$

It is noted that the first puncture matrix permits a parity digit from each recursive systematic convolutional (RSC) encoder, thus soft-input-soft-output (SISO) and iterative decoding can be carried out once a packet is received The initial packet contains 1024 systematic symbols and 256 parity symbols for each component code. Subsequent puncture patterns permit an additional parity digits for each component code, resulting in improved iterative decoding performance.

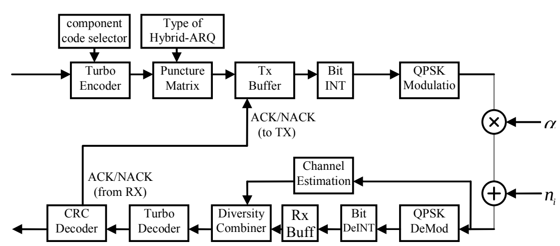

Fig. 1 Communication system model with Rayleigh channel

IV. THROUGHPUT ANALYSIS

The performance of an ARQ is measured by its connection reliability and throughput performance [5]. The throughput in a standard ARQ is given by

$$\eta = \left(\frac{k}{n + K_{CRC}}\right)\left(\frac{1 - FER}{\overline{T}_R}\right) \tag{2}$$

where k is the number of information bits per packet, n is the total coded digits per packet, K_{CRC} are the bits appended to the packet to aid in error detection, FER is the residual frame error rate and T_1 is the average number of transmit attempts.

A. Code Combining

The code combining of J packets results in rate $1/\mathrm{J}n$, where n is the number of coded bits appended per transmit attempt. The decoding of code combined packet results in one of three possible error events; an undetectable error $D_u^{(J)}$, a detectable error $D_d^{(J)}$ and a correctable error $D_c^{(J)}$. The exhaustive set of probability events can be expressed as

$$P(D_u^{(J_c)} \mid \alpha) + P(D_c^{(J_c)} \mid \alpha) + P(D_d^{(J_c)} \mid \alpha) = 1 \tag{3}$$

where α is the Rayleigh fading coefficient. Assuming that the probability of an undetectable error may be considered negligible and that maximum-likelihood trellis decoding is employed, the packet retransmission probability is expressed as

$$P(D_d^{(J_c)} \mid \alpha) \simeq \int_\alpha^\infty 1-(1-P(E^{(J_c)} \mid \alpha))^n \quad f(\beta)\, d\beta \qquad (4)$$

where $P(E^{(J_c)}|\alpha)$ is the error event probability after decoding the Jth transmission based on *a posteriori* detector and $f(\beta)$ is the chi-square distributed random function. The error event probability $P(E^{(J_c)}|\alpha)$ bounded in terms of the pairwise error probability $P_e(x \rightarrow \hat{x}|\alpha)$ is expressed as

$$P(E^{(J_c)} \mid \alpha) \simeq \sum_e^\infty P_e\left(x_i \rightarrow \hat{x}_i \mid \alpha\right) \qquad (5)$$

The pairwise error probability of turbo codes is in turn expressed as,

$$P_e(x_i \rightarrow \hat{x}_i \mid \alpha) = Q\left(\sqrt{2R^{(J)} \frac{E_b}{N_o} \sum_{k=1}^{d} \alpha_{ik}^2}\right) \qquad (6)$$

where $R^{(J)}$ is the rate of the turbo code after code combining J transmissions. In the CCSDS scheme, code combining results in $R^{(J)} = \{2/3, 1/2, 2/5, 1/3\}$. The average number of transmissions attempts in code combining is bounded by a probability of detection [7],

$$1+\sum_{n=1}^{\infty}\prod P\left(D_d^{(J_c)}\right) \leq \bar{J}_c \leq 1+\sum_{n=1}^{\infty} P\left(D_d^{(J_{c,\max})}\right) \qquad (7)$$

Thus the average throughput efficiency of a code combined ARQ protocol assuming identical size of packets in each transmission, is lower bounded by,

$$\eta = \left(\frac{k}{\bar{J}_c\left(n_p + K_{CRC}\right)}\right)\left(1-FER\right) \qquad (8)$$

Consider an ARQ scheme in which systematic bits and trellis termination bits are contained only in the initial packet, such as an incremental redundancy scheme. The throughput of the incremental redundancy scheme is thus represented as

$$\eta = \left(\frac{k}{\left(n_1 + K_{CRC}\right) + \left(\bar{J}_c - 1\right)\left(n_J\right)}\right)\left(1-FER\right) \qquad (9)$$

where n_1 is the number of coded digits in the initial transmission of the ARQ and n_J is the incremental digits with each retransmission.

B. Chase Combining

Let us consider a Chase combining scheme where soft information is employed to aid in decoding of received packets [2]. The average number of transmission attempts of a Chase combined scheme was is lower bounded by [3],

$$\bar{J}_{Ch} \geq 1 + P\left(R_d \mid \alpha\right) + \sum_{i=1}^{\infty} P\left(D_d^{(i)} \mid \alpha\right) \qquad (10)$$

where $P\left(R_d \mid \alpha\right)$ is the probability that the received packet transmitted over a fading channel contains a detectable error and $P\left(D_d^{(J)}|\alpha\right)$ is the probability that J Chase combined packets conditioned on the fading channel. Thus the throughput of the scheme is expressed as,

$$\eta = \left(\frac{k}{\bar{J}_{Ch}\left(n + K_{CRC}\right)}\right)\left(1-FER\right) \qquad (11)$$

C. Metric Ratio Combining

Let us consider the average throughput of MRC scheme. The pairwise error probability of J metric ratio combined packets is considered as the probability of J consecutive error events of identically encoded packets. Therefore,

$$P_e\left(E_1, E_2, ..., E_J\right) = P_e\left(E_1\right) P_e\left(E_2\right) ... P_e\left(E_J\right) \qquad (12)$$

Considering the pairwise error probability in equ. (5) and that the channel is Rayleigh it can be shown that is a chi-square distributed random variable, $\beta = \sum_{k=1}^{d} \alpha_{ik}^2$ [18]. The conditional pairwise error probability of J transmissions is given by

$$P_e(x_i \rightarrow \hat{x}_i \mid \alpha) = \prod_{J=1}^{J_{\max}-1}\left\{\frac{1}{2}\exp\left(-R\frac{E_b}{2N_o}\sum_{k=1}^{d}\beta\right)\right\} \qquad (13)$$

Hence the average number of transmission attempts in a metric ratio combined scheme J, employing turbo block coding is given by

$$\bar{J}_M = 1 + P(D_d^{(J_m)}) \simeq 2 - \int_\alpha \left(1 - \sum_c P(c) \sum_e P_e(x \rightarrow \hat{x})\right)^{k+n} f(\beta)\, d\beta \qquad (14)$$

The throughput of the scheme is expressed as,

$$\eta = \left(\frac{k}{\bar{J}_M\left(n + K_{CRC}\right)}\right)\left(1-FER\right) \qquad (15)$$

V. Selection of Hybrid ARQ Scheme

The signal-to-interference-plus-noise ratio (SINR) estimator is based on a maximum-likelihood non-coherent signal-to-noise-ratio (SNR) estimation. In coherent cases the SINR expression is equivalent to the SNR

$$\hat{\Gamma} = \frac{a^2}{\gamma^2} \qquad (16)$$

where a is the signal energy and γ the power of the noise and residual interference. Each packet has the SINR independently estimated and the scheme selects the hybrid ARQ based on the maximisation of the estimate Γ_{ARQ}, Γ_{Code}, Γ_{Chase} and Γ_{MRC}.

V. Conclusion

In this paper, the performance of hybrid ARQ scheme employing turbo codes in frequency non-selective Rayleigh channels was evaluated. Figure 2. and Figure 3. present simulation results for throughput performance and transmit attempts respectively. Type I MRC was found to be favourable for delay sensitive systems due to self decoding packets. Type II RCPT was suited to bandwidth restricted environments due to a high throughput. We also proposed a selective hybrid diversity combining scheme based on SINR. The SINR criterion is reducible to SNR in the single user channels. The selection criteria results in a bandwidth efficient scheme that also exhibits high throughput. .

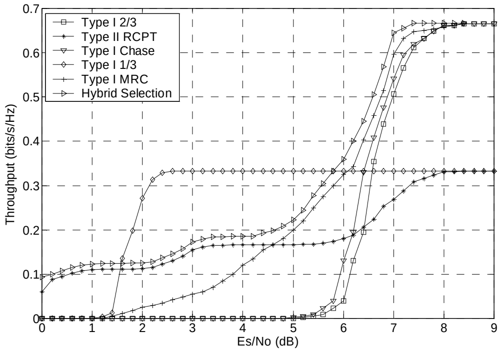

Fig. 2 Throughput performance of hybrid-ARQ schemes employing turbo coding

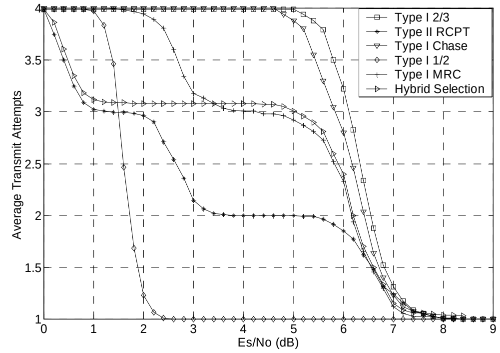

Fig. 3 Average transmit attempts of hybrid-ARQ schemes employing turbo coding

REFERENCES

[1] R. H. Deng "A Hybrid ARQ scheme using TCM and code combining", *Elect. Letters, vol. 27,* pp. 866-868, May 1991

[2] D. Chase, "Code combining – A maximum likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. 33, pp. 385 – 393, May 1985.

[3] S. Kallel, "Analysis of Type II Hybrid ARQ Schemes with code combining", *IEEE Trans. on Commun.*, vol. 38, No. 8, Aug. 1990

[4] Jack H, Winters, "Optimum Combining in Digital Mobile Radio with Co-channel interference," *IEEE Journal on Selected Area in Commun. Vol. Sac-2 No. 4, July 1984*

[5] S. Lin and D. J. Costello, Jr., *Error Control Coding Fundamentals and Applications.* Englewood Cliffs, NJ: Prentice Hall, 1983.

[6] C. Berroux and A. Glavieux, "Near optimum error correcting coding and decoding: turbo-codes", *IEEE Commun. Letter*, vol. 1, no. 3 pp. 77 – 79, May 1997.

[7] S. Lin and P. S. Yu, "A hybrid ARQ scheme with parity retransmission for error control of satellite channels", *IEEE Trans. on Commun.,* vol. 30, pp. 1701-1719, July 1982

[8] 3GPP, TR. 25.848, "Physical Layer Aspects of UTRA High Speed Downlink Packet Access".

[9] Benedetto and G. Montorsi, "Design of parallel concatenated convolutional codes," *IEEE Trans. on Commun.*, vol. 44, pp 591 – 600, May 1996.

[10] Divsalar and F. Pollara, "On the design of Turbo codes," *TDA Progress Report 42-123*, pp 99 – 121, Nov. 1995

[11] D. Divsalar and R. J. McEliece "Effective free distance of turbo codes", *Elect. Letters*, vol. 32, pp.445-446, Feb. 1996

[12] G.M.S. Benedetto, R. Garello, "A search for good convolutional codes to be used in the construction of turbo codes," *IEEE Trans. on Commun., vol. 46, pp. 1101 – 11-5, Sep. 1998*

[13] Rappaport *Wireless Communication: Principles and Practice,* Upple Saddle River NJ: Prentice Hall , 1996

[14] L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate." *IEEE Trans. Inform. Theory,* vol. IT-20 pp. 284 – 287, 1974.

[15] D. Divsalar, S. Dolinar, and F. Pollara "Proposal for CCSDS Turbo Codes: Deep Space and New Earth" NASA JPL, November 7, 1996

[16] D. N. Rowitch and L. B. Milstein "On the Performance of Hybrid FEC/ARW Systems Using Rate Compatible Punctured Turbo (RCPT) Codes" NASA JPL, November 7, 1996

[17] B. Vucetic and J. Yuan *Turbo Codes*, Kluwer Academic Publishers,

[18] J. G. Proakis, Digital Communications, Tokyo, McGraw-Hill International Book Company, 1987

[19] D. N. Rowitch and L. B. Milstein "Rate compatible puncture turbo (RCPT) codes in hybrid FEC/ARQ", Proc. Comm. Theory, Mini-conference of Globecomm '97 pp. 55 – 59, Nov. 1997