Optimal Routing and SINR Target Selection for Power-Controlled CDMA Wireless Networks

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1. INTRODUCTION

Efficient operation of wireless data networks requires not only optimization of the individual layers, but also coordination across layers. For example, the optimal routing problem in the network layer and the resource allocation problem in the radio control layer are coupled through the link capacities, and the optimal performance can only be achieved by simultaneous optimization of routing and resource allocation. In this paper, we formulate the problem of simultaneous routing and power allocation in code-division multiple access (CDMA) wireless data networks. In CDMA systems, the capacity of a link depends on not only the power allocated to itself, but also the powers allocated to other links (due to interferences). Since the capacity constraints are not jointly convex in communication rates and power allocations (a well-known fact, see [1]), a straightforward formulation of the SRRA problem is not a convex optimization problem and thus generally very hard to solve. We suggest an approximate capacity formula for relatively high signal to interference and noise ratios (SINRs) and show how a coordinate transform yields a convex formulation. We note that the optimization is naturally interpreted as the optimal coordination of routing and SINR-target selection for a distributed power control algorithm, and show how the use of approximate capacity constraints in the optimization makes this implementation robust to link gain variations.

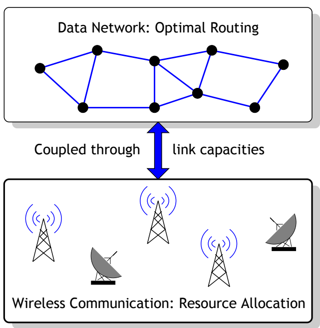

Figure 1: Simultaneous routing and resource allocation

2. SIMULTANEOUS ROUTING AND RESOURCE ALLOCATION

2.1 Network flow model

Consider a connected communication network containing N nodes labeled $n = 1, \dots, N$ and L directed links labeled $l = 1, \dots, L$. The topology of the network is represented by a *node-link incidence matrix* $A \in \mathbb{R}^{N \times L}$ whose entry A_{nl} is associated with node n and link l via

$$A_{nl} = \begin{cases} 1, & \text{if } n \text{ is the start node of link } l \\ -1, & \text{if } n \text{ is the end node of link } l \\ 0, & \text{otherwise.} \end{cases}$$

We use a multicommodity flow model for the routing of data flows, with average data rates in bits per second. We identify the flows by their destinations, labeled $d = 1, \dots, D$, where $D \le N$. For each destination d, we define a *source-sink vector* $s^{(d)} \in \mathbb{R}^N$, whose nth ($n \neq d$) entry $s_n^{(d)}$ denotes the non-negative amount of flow injected into the network at node n (the source) and destined for node d (the sink), where $s_d^{(d)} = -\sum_{n \neq d} s_n^{(d)}$. We also define $x^{(d)} \in \mathbb{R}_+^L$ as the *flow vector* for destination d, whose component $x_l^{(d)}$ is the amount of flow on each link l and destined for node d. Let t_l and c_l be the total traffic load and the capacity of link l respectively. Our network flow model imposes the following group of constraints on the network flow variables x, s, t:

$$\begin{array}{ll} Ax^{(d)} = s^{(d)}, & d = 1, \dots, D \\ x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1, \dots, D \\ t_l = \sum_d x_l^{(d)}, & l = 1, \dots, L \\ t_l \le c_l, & l = 1, \dots, L \end{array} \tag{1}$$

Here, $\succeq$ means component-wise inequality, and $\succeq_d$ means component-wise inequality except for the dth component.

2.2 Communications model

Let $G \in \mathbb{R}_+^{L \times L}$ be the *channel gain matrix* across the network, whose entry G_{ij} is the power gain from the transmitter of link j to the receiver of link i. Only the diagonal terms G_{ii} are desired, and the off-diagonal terms G_{ij} $(i \neq j)$ lead to interferences. In CDMA systems without interference cancellation, the Shannon capacity is

$$t_l \le \phi_l(P) = \log\left(1 + \frac{G_{ll} P_l}{\sigma_l + \sum_{j \neq l} G_{lj} P_j}\right), \quad l = 1, \dots, L. \tag{2}$$

In addition, we assume that the transmit powers are limited,

$$P_l \le P_l^{\max}, \quad l = 1, \dots, L \tag{3}$$

More complex resource limits can easily be included, as was done for the orthogonal channel models in [2]. However, simple power limits are sufficient for the purposes of this paper.

*This work was sponsored in part by the Swedish Research Council, AFOSR grant F49620-01-1-0365, by DARPA contracts F33615-99-C-3014 and MDA972-02-1-0004 and by MARCO contract 2001-CT-888.

2.3 The SRRA problem

Consider the operation of a wireless data network described by the network flow model (1) and the communications model (2)–(3). The optimal operation, as judged by a convex cost function $f(x,s,t,r)$, can be obtained by solving the following SRRA problem

$$\begin{array}{lll} \text{minimize} & f(x,s,t,P) & \\ \text{subject to} & Ax^{(d)} = s^{(d)}, & d = 1,\ldots,D \\ & x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1,\ldots,D \\ & t_l = \sum_d x_l^{(d)}, & l = 1,\ldots,L \\ & t_l \leq \phi_l(P), & l = 1,\ldots,L \\ & 0 \leq P_l \leq P_l^{\max} & l+1,\ldots,L \end{array} \tag{4}$$

The optimization variables are the routing decisions x, the source-sink flows s, the total link traffic t and the transmit powers P. The SRRA problem is very general and includes many important design problems for wireless data networks, such as maximum total utility or minimum total power operation, minimax power among the nodes, and minimax link utilization [2]. However, since $\phi_l(P)$ are not concave in P, this formulation is not convex and the above optimization problem is generally very hard to solve.

2.4 A convex formulation

The signal to interference and noise rations are defined as

$$\gamma_l(P) = \frac{G_{ll}P_l}{\sigma_l + \sum_{j\neq l} G_{lj}P_j}, \quad l = 1,\ldots,L.$$

When the SINRs are relatively high (*e.g.*, $\gamma_l \geq 5$ or 10), we use the approximation $\log(1+\gamma_l) \approx \log\gamma_l$ and re-write

$$\log\gamma_l(P) \quad = \quad -\log\left(\frac{\sigma_l}{G_{ll}}P_l^{-1} + \sum_{j\neq l}\frac{G_{lj}}{G_{ll}}P_jP_l^{-1}\right).$$

The variable transformation $Q_l = \log(P_l)$ gives

$$\psi_l(Q) = \log\gamma_l(P(Q)) = -\log\left(\frac{\sigma_l}{G_{ll}}e^{-Q_l} + \sum_{j\neq l}\frac{G_{lj}}{G_{ll}}e^{Q_j - Q_l}\right).$$

Note that the functions ψ_l are concave in the variable Q since "log-sum-exp" expressions are convex.

With the approximate capacity formula and the change of variables, we can formulate the following SRRA problem

$$\begin{array}{lll} \text{minimize} & f(x,s,t,Q) & \\ \text{subject to} & A^{(d)}x^{(d)} = s^{(d)}, & d = 1,\ldots,D \\ & x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1,\ldots,D \\ & t_l = \sum_{d=1}^D x_l^{(d)}, & l = 1,\ldots,L \\ & t_l \leq \psi_l(Q), & l = 1,\ldots,L \\ & e^{Q_l} \leq P_l^{\max}, & l = 1,\ldots,N \end{array} \tag{5}$$

where the last constraint (which is convex) is (3) in the new variable Q. Here the capacity constraints $t_l \leq \psi_l(Q)$ are jointly convex in t and Q. This implies that (5) is a convex optimization problem, which can be solved globally and efficiently.

Note that $\log\gamma_l \leq \log(1+\gamma_l)$, *i.e.*, we used an underestimate for the link capacity. This means that the solution to the (restricted) convex program (5) is always feasible to the original problem (4).

2.5 A heuristic link-removal procedure

Any solution to (5) must satisfy $\gamma_l \geq 1$ for all links, since we require $0 \leq t_l \leq \log\gamma_l$. The links with $\gamma_l = 1$ have zero capacity, but are allocated nonzero powers. We can safely remove these links, and still guarantee that the solution is feasible for the SRRA problem with the reduced network topology. If we solve the SRRA problem for the new topology, the objective can only be improved (leading to larger total utility or less total power). This procedure usually converges to an improved solution after very few iterations.

3. DISTRIBUTED POWER CONTROL

Solving the SRRA problem yields a simultaneously optimal combination of power allocation and routing. However, due to the time-varying nature of the wireless channel, keeping these powers fixed would lead to time-varying SINR levels and potentially suboptimal performance. On the other hand, re-optimizing the routing to track fast link gain variations is in most cases computationally infeasible.

A more natural interpretation of the SRRA optimization is that it yields the simultaneously optimal combination of routing and SINR targets. More precisely, an optimal solution $Q^\star$ to (5) corresponds to SINR values

$$\gamma_l^\star = \gamma_l(\exp(Q^\star))$$

We can then use the distributed power control law

$$P_l(k+1) = \min\left\{\gamma_l^{\text{tgt}}\frac{P_l(k)}{\gamma_l(k)}, P_l^{\max}\right\} \tag{6}$$

to track these SINR values in presence of time-varying link gains. Since we use restricted capacity constraints in the optimization it is sufficient to aim for a SINR target of

$$\gamma_l^{\text{tgt}} = \gamma_l^\star - 1 = \gamma_l(\exp(Q^\star)) - 1$$

In the nominal case these target values require uniformly lower transmit powers than the choice $\gamma_l^{\text{tgt}} = \gamma_l^\star$ [1].

Since the SRRA formulation uses the link gains, it is important to try to understand how sensitive the optimal solution is to variations in the gain estimates. It turns out that the use of restricted capacity constraints introduces a natural robustness to link gain variations. In particular, it is possible to verify that the iterative power control loop will converge to the SINR targets for any values of the link gains satisfying

$$G_{ij} = (1+\delta_{ij})G_{ij}^{\text{nom}} \text{ with } |\delta_{ij}| \leq 1/(2\gamma_{\max}^{\text{tgt}} + 1)$$

where $\gamma_{\max}^{\text{tgt}} = \max_l \gamma_l^{\text{tgt}}$ and G_{ij}^{nom} denote the values of the link gains used when solving the SRRA problem. We note that this is a worst-case estimate, and the system may tolerate larger perturbations on individual links.

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[2] L. Xiao, M. Johansson, and S. P. Boyd. Simultaneous routing and resource allocation via dual decomposition. Submitted for journal publication, July 2002.