Node Placement for Connected Coverage in Sensor Networks

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***Abstract*— We address the problem of optimal node placement for ensuring connected coverage in sensor networks. We consider two different practical scenarios. In the first scenario, a certain region (or a set of regions) are to be provided connected coverage, while in the second case, a given set of n points are to be covered and connected. For the first case, we provide solutions that are within a small factor of the optimum. For the second case, we present an algorithm that runs in polynomial time, and guarantees a constant factor approximation ratio.**

I. Introduction

We address two issues that are quite important in the deployment of wireless sensor networks. They are:

Coverage: Each sensor device typically has a physical sensing range within which it is able to perform its operation. One goal of a sensor network is that each location in the physical space of interest, should be within the sensing range of at least one of the sensors.

Connectivity: It is typically more energy efficient to aggregate all the sensor data at a few specific wireless nodes (gateways) from where the data can be uploaded to the remotely located monitoring station. Therefore the sensors need to organize themselves into a connected ad-hoc network. Since the wireless radio in each sensor node also has a transmission range, the location and placement of the sensors determine the connectivity of the sensor network.

Wireless sensor networks need to meet both these requirements of coverage and connectivity. In general, there are two different ways of deploying a sensor network. We consider the scenario where it is possible to explicitly place a set of sensor nodes at specific locations as desired. Such a deployment mechanism is possible in friendly and accessible environments. We focus on the problem of optimal placement of sensor nodes to guarantee such connected coverage.

In the problem we address, we are given a region, a set of regions, or a set of points that are to be covered sensors, such that the set of sensors form a connected network. The optimization objective is to minimize the number of sensors used. We assume that each sensor is capable of detecting signals within a fixed radius r around it, i.e., each sensor "covers" all points that are within a circle of radius r around it. Moreover, we assume that the communication range of any sensor is also r, i.e., a sensor is "connected" only with sensors that are within a fixed radius r from it. The sensing and the communicating radii are in general different. In this extended abstract, however, we focus only on the case where the sensing and communication radii are equal and identical for all the sensors.

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We address several important and interesting versions of the optimal connected coverage problem. First we consider the case where the object to be covered is a region in a two-dimensional plane. Next we consider the problem where we are required to cover a given set of n points in the two-dimensional plane. In this abstract, we refer to these two problems as the *region-coverage* and the *point-coverage* problems respectively. The region-coverage problem is important when the goal is to gather information about an entire region under study, e.g. environment and weather monitoring. In contrast, the point-coverage problem is useful when it is sufficient is to monitor the state of a set of specific locations in the region, e.g. monitoring leakage of hazardous gaseous materials at ventilation points.

The problem of providing coverage to a given region, or a given set of points, and its variants, has received prior attention in the research community [1], [2], [3], [4]. However, there has not been much significant research on the problem of *connected* coverage which we address in this abstract.

II. The region-coverage problem

We first study the case in which the region to be covered is the entire two-dimensional plane. The solution for this simple special case provides some valuable insights for approaching the more complex region-coverage problems. We then discuss how regions of finite sizes can be provided connected coverage.

We use the following terminology in the rest of this abstract. Sensors with a sensing/transmitting radius r can be modelled as a disk with radius r. We refer to such disk as an r-*disk*. An r-disk covers only those points that fall within it. Two r-disks are connected if the center of each one falls in the other. Clearly, to provide connected coverage to a region, the set of disks used much cover all points in that region, and the connectivity graph of all the r-disks must form a single connected component in a graph theoretic sense.

A. Connected coverage for the two-dimensional plane

In order to cover the entire two-dimensional plane with r-disks, we clearly need an infinite number of disks. The appropriate optimization metric in this case should therefore be the number of disks used per unit area (which we call *density*). Next we outline a simple disk placement pattern which achieves a density that is approximately optimal.

To describe the solution, we first definite a pattern referred here as an r-*strip*. As shown in Panel (a), Figure 1, an r-strip is a string of r-disks placed along a line such that the distance between the centers of any two adjacent r-disks is r. Clearly, the nodes in an r-strip form a single connected component. Now let us tile the entire plane with these r-strips, all oriented along the x-axis. For every even integer k, place an r-strip oriented parallel to the x-axis such that the point $(0, k(\frac{\sqrt{3}}{2}+1)r)$ is the

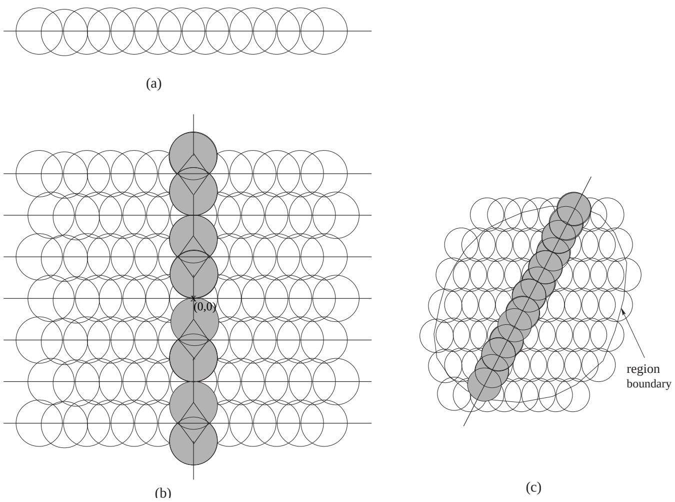

Fig. 1. (a) An r-strip, (b) The pattern P_1, (c) The pattern P_2.

center of an r-disk constituting the strip. Also, for every odd integer k, place an r-strip oriented parallel to the x-axis such that the point $(\frac{r}{2}, k(\frac{\sqrt{3}}{2}+1)r)$ is the center of an r-disk in the strip. This is shown in Panel (b), Figure 1. Next place some r-disks along the y-axis in the following way. For every odd integer k, place two r-disks, at $(0, k(\frac{\sqrt{3}}{2}+1)r \pm \frac{\sqrt{3}}{2}r)$. These disks placed along the y-axis are the shaded disks shown in the figure. Let us denote the entire disk placement pattern (as shown in Figure 1) as P_1. It can be verified that P_1 provides connected coverage to the entire two-dimensional plane. The following result shows that the density of our solution is within 3% of the optimum.

Theorem 1: Let d_{P_1} denote the density of the disk placement pattern P_1, and d_{OPT} denote the optimal density. Then $d_{P_1}/d_{OPT} \leq (\frac{\pi}{3} + \frac{\sqrt{3}}{2})/(1+\frac{\sqrt{3}}{2}) \approx 1.026$.

B. *Connected coverage for a region of finite size*

Assume that the region that has a convex shape. Then this region can be provided connected coverage in the following way. First place the shape (to be covered) on the plane tiled by the r-strips as shown in Figure 1. Now include all the non-shaded disks of the pattern P_1 that intersect the region, and exclude the rest. Then take another r-strip and place it in a such a way that it intersects all the other r-strips intersecting the region. This guarantees connectivity. Panel (c), Figure 1 shows a possible pattern thus generated, for a particular convex shape. The last strip that intersects all the other parallel strips is shaded in the figure. Note that since the region has a convex shape, it is always possible to place the last r-strip in such a way that it intersects all the parallel r-strips covering the region.

Let P_2 denote a solution constructed according to the rules described above. It is easy to see that P_2 covers the region, and all the disks in P_2 are connected.

Lemma 1: Let R be a bounded convex region, with perimeter L and area A $(\geq \pi r^2)$. Let n_{P_2} denote the number of disks used by P_2 to provide connected coverage to R, and n_{OPT} be the optimal solution. Then $n_{P_2}/n_{OPT} \leq 2.693\ (1 + 2.243\ Lr/A)$.

(1) Compute the euclidean minimum spanning tree (T) for the n given points. Choose any point p that corresponds to a leaf node of T, and set $C \leftarrow \{p\}$. Also, set $k \leftarrow 0$.
(2) Set $k \leftarrow k+1$. Choose any point $p' \in C$. The r-disk centered at p' is chosen as D_k.
(3) Update C as follows:
i) Remove any points in C that are covered by D_k.
ii) Let S_k be set of points at which the boundary of D_k intersects T. For each point $p'' \in S_k$, include p'' in C iff $p'' \notin D_1 \cup D_2 \cup ... \cup D_{k-1}$, and the path from p (the center of D_1) to p'' in T is completely covered by $D_1 \cup D_2 \cup ... \cup D_k$.
(4) If C is non-empty, go to (2), else stop.

Fig. 2. The Point-Coverage Algorithm. C consists of the set of points where the center of an r-disk can be placed at each successive step. The r-disk chosen at the kth iteration is denoted by D_k.

The solution described above can be extended to non-convex shapes, and a result similar to Lemma 1 can be shown to hold.

III. Connected coverage for multiple points

We now consider the problem of providing connected coverage to a set of n given points in a two-dimensional Euclidean plane. Our proposed polynomial-time algorithm (Figure 2) approximates the optimal solution within a constant factor.

Let $\bar{L}$ be the maximum distance between the given points. Then running time of the algorithm is $O(\frac{\bar{L}}{r}n^2)$. Therefore, if the the maximum distance remains bounded, the algorithm runs in polynomial time. The following result states the approximation ratio of the algorithm.

Theorem 2: Let n_{P_3} denote the number of disks used by our algorithm to provide connected coverage to the n given points. Let n_{OPT} be the optimal solution. Then $n_{P_3}/n_{OPT} \leq 4\pi/\sqrt{3} \approx 7.256$.

IV. Connected coverage for multiple regions

Finally we summarize our solution to the problem of providing connected coverage to a set of disjoint convex regions. The approach is as follows. First, we provide connected coverage to each of the regions separately using the approach outlined in Section II. Then we "connect" the disjoint convex regions using a minimum spanning tree-based approach, similar to that described in Section III. The exact details on this algorithm will be presented in the full version of this work. Under certain weak assumptions, it can be shown that our solution is guaranteed to be within a factor of $6\max(2/\sqrt{3}, \alpha)$ of the optimum, where α is the worst-case approximation ratio for the problem of providing connected coverage to each region separately.

References

[1] R. Kershner, "The number of circles covering a set", *American Journal of Mathematics*, 61:665–671, 1939.
[2] D. S. Hochbaum and W. Maas, "Approximation schemes for covering and packing problems in image processing and VLSI", *J. ACM*, 32:130–136, 1985.
[3] M. Franceschetti, J. Bruck, M. Cook, "A Geometric Theorem for Wireless Network Design Optimization", http://www.paradise.caltech.edu/~massimo/homepage/papers/covering.ps.
[4] S. Shakkottai, R. Srikant, N. Shroff,"Unreliable Sensor Grids: Coverage, Connectivity and Diameter", To appear in the *Proc. IEEE Infocom*, San Francisco, March 2003.