Capacity and Decentralized Admission/Congestion Control in the Up and Down-link of the CDMA Networks

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EXTENDED ABSTRACT

This work concerns the evaluation of the global up and down-link *capacity* of CDMA networks under maximal power constraints, with a special emphasis on the limitations of capacity due to inter-cell and own-cell interferences. More precisely we analyze the maximal number of users that such a network can serve at a given bit-rate and/or the maximal bit-rates that such a network can provide to a given user population.

The quantification of these capacity constraints is shown to allow one to define

- *Admission control policies* in the case of predefined user bit-rates (e.g. voice); i.e., schemes allowing one to decide whether a new user can be admitted or should be rejected as its admission could make the *global* power allocation problem infeasible;
- *Congestion control policies* in the case of users with elastic bit-rates (e.g. data); i.e., schemes allowing one to determine the maximal fair user bit-rates that preserve the feasibility of the power control problem at any time, in function of the user population in all cells at this time.

The evaluation part relies on a model which uses planar point processes and stochastic geometry. The model has several key components, the spatial location pattern of base stations (BS's), the spatial location pattern of users, the attenuation (path loss) function and the policy of assignment of users to BS's, which are geometry-dependent, in addition to the non-geometric components such as orthogonality factors, pilot signals and external noise.

We allow both patterns of locations to be countably infinite so as to address the *scalability* questions, and to check the ability of the proposed algorithms to continue to function well as the size of the network goes to infinity.

The basic assignment policy will be that where each mobile is served by the closest BS. It is basically equivalent to the optimal-SIR-choice scheme and to the honeycomb model in the classical hexagonal case.

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Considering the downlink power allocation algebra one finds that the network can handle a population of mobiles on the downlink iff some linear matrix inequality (understood coordinate-wise)

$$\mathbf{S} \geq \mathbb{A}\mathbf{S} + \mathbf{b}\,, \tag{1}$$

has a finite solution in the total powers $\mathbf{S} = (S_j)$ emitted by the BS's on traffic channels. The elements a_{jk} of $\mathbb{A}$ and the elements b_j of $\mathbf{b}$ are given (for the downlink) by

$$\begin{aligned} a_{jj} &= \sum_i \kappa_j H_i^j\,, \\ a_{jk} &= \gamma \sum_i \frac{H_i^j l(Y_k, X_i^j)}{l(Y_j, X_i^j)},\ k \neq j \\ b_j &= \sum_i H_i^j \Bigg(\frac{W_i^j}{l(Y_j, X_i^j)} + \kappa_j P_j \\ &\qquad + \gamma \sum_{k \neq j} \frac{l(Y_k, X_i^j)}{l(Y_j, X_i^j)} P_k \Bigg)\,, \end{aligned}$$

where

$$H_i^j = \frac{C_i^j}{1 + \kappa_j C_i^j}$$

are related to the required SIR C_i^j, $l(\cdot,\cdot)$ are respective path-loss, κ_j, γ are respective orthogonality factors, W_i^j are external noises and P_k are powers emitted on common overhead channels.

Existence of nonnegative solutions of (1) is equivalent to saying that the positive matrix of *relative path-losses* $\mathbb{A}$ has the spectral radius less then 1. Then for the solution $\mathbf{S}$ to be implementable another condition saying that $\mathbf{S}$ is not bigger than the vector of maximal powers of BS's has to be imposed. Similar analysis concerns the uplink.

The evaluation of the spectral radius of $\mathbb{A}$ is not simple and the proposed algorithms for its on-line estimations seem to be non-scalable. Thus, the key idea that we proposed is to use a sufficient condition of *sub-stochasticity* of the matrix $\mathbb{A}$. That matrix $\mathbb{A}$ is sub-stochastic if and only

if for each BS j

$$\sum_{i \in \mathcal{N}_M^j} f_i^j \leq 1, \tag{2}$$

where

$$f_i^j = \kappa_j H_i^j + \gamma \sum_{k \neq j} \frac{H_i^j l(Y_k, X_i^j)}{l(Y_j, X_i^j)}, \ i \in \mathcal{N}_M^j.$$

denotes some (theoretic) cost for the BS j to serve the mobile i with a given bit-rate related to the required SIR C_i^j via H_i^j and subject to given relative other/own-BS path-losses $l(Y_k, X_i^j)/l(Y_j, X_i^j)$, and where the summation is taken over all mobiles $i \in \mathcal{N}_M^j$ served by the BS j.

The last condition is only sufficient and non-necessary for the existence of solutions of the problem and thus in principle it only gives a conservative (lower) bound on the capacity of the system. However, the capacity calculus based on it has a decentralized nature that follows from the fact that for each j, inequality (2) depends on the characteristics of the mobiles in cell j but not on the location or number of the mobiles of $\mathcal{N}_M^k$ for other BS's $k \neq j$.

Moreover, under the Poisson assumption for BS's and mobiles, the mean version of (2) can be calculated explicitly in terms of the intensities of both Poisson processes, respectively λ_{BS} and λ_M. For the simplified path-loss model that depends only on the path-loss exponent α, the mean condition (2) reads

$$\frac{\lambda_M}{\lambda_{BS}} EH(\kappa + 2\gamma/(\alpha - 2)) < 1.$$

In particular, for a given intensity of mobiles we can give explicit formula for the minimal intensity of BS that can serve that population of mobiles, and conversely, for a given intensity of BS's we can express explicitly the maximal density of mobiles acceptable by the network. Explicit formulas can also be derived for the mean model in the case of hexagonal pattern for BS's locations and Poisson assumption for mobiles.

More fine analysis leads to the following *stochastic capacity* of the model:

(Admission-Capacity of the Poisson Voronoi Model): For a given $\lambda_{BS} > 0$ *and* $\epsilon > 0$ *let* $\lambda_M^\epsilon = \lambda_M^\epsilon(\lambda_{BS})$ *be the maximal intensity of* $\mathcal{N}_M$ *s.t.*

$$\Pr\Big(\textit{inequality } (2) \textit{ holds for } j = 0\Big) \geq 1 - \epsilon\,.$$

The above probability can be approximated via (rather crude) Markov inequality. More precise bounds can be obtained via Chernov's inequality.

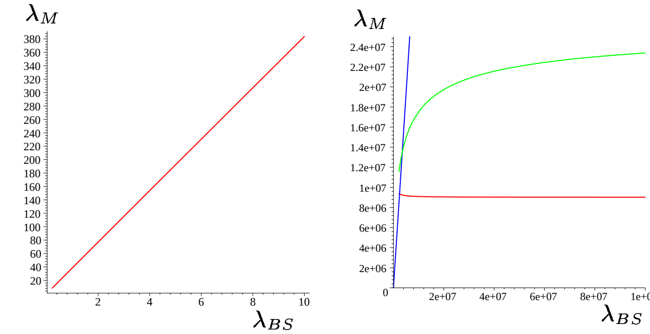

Fig. 1. Maximal intensity λ_M of mobiles per km^2 and $\rho = \lambda_M/\lambda_{BS}$ of mobiles per base-station satisfying (2) for Poisson-Voronoi model; the real and extremely dense case.

The main operational aspects of the presented approach are to propose the following decentralized and scalable admission and congestion control protocols that guarantee that the network remains in a position to solve the power allocation problem at any time.

Decentralized Admission Control Protocol (DACP): In the context of fixed bit-rate demands each BS checks periodically whether Condition (2) is satisfied and if not, enforces it by reducing the population $\mathcal{N}_M^j$ of its mobiles to some subset $\widetilde{\mathcal{N}}_M^j$ such that inequality (2) holds when $\mathcal{N}_M^j$ is replaced by $\widetilde{\mathcal{N}}_M^j$. When a new user arrives to some BS, the BS accepts it if condition (2) is satisfied with this additional user and rejects it otherwise.

Decentralized Congestion Control Protocol (DCCP): In the context of elastic traffic, each BS periodically allocates to each its mobile some fair rate via the same SIR $\mathcal{C}^j$ for all users in the cell that is calculated on the basis of the condition (2) and is equal to

$$\mathcal{C}^j = \frac{1}{\left(\kappa_j(\#\mathcal{N}_M^j - 1) + \gamma \sum_{i \in \mathcal{N}_M^j} \sum_{k \neq j} \frac{l(Y_k, X_i^j)}{l(Y_j, X_i^j)}\right)_+}$$

where x_+ is $\max(x, 0)$. Using a Gaussian channel approximation, one can deduce the maximal fair bit rate offered to users of cell j is

$$\mathcal{B}^j = B \log(1 + \mathcal{C}^j),$$

with B the CDMA channel bandwidth and with $\mathcal{C}^j$ the quantity defined above. This fair rate is also updated at any time when a user joins or leaves the cell. The above bit-rate can be simulated or approximated on average via Markov or Chernov's inequality.

Concluding, we show that the algebraic approach to power control leads to scalable admission and congestion control algorithms for large CDMA networks. In contrast to most studies, we consider a "true" multi-cell model, i.e., we take into account true inter-cell interferences (and not some estimated fractions of own-cell interferences). Stochastic geometry is used to prove that these algorithms yield a positive capacity in infinite networks and to give estimates of the mean values and the fluctuations of capacity within this context. In particular, the analysis using the mean Poisson spatial model offers explicit formulas for the maximal traffic that can be served. Moreover, this approach allows one to address the interplay between several traffic classes (with fixed and elastic bit rates).

In our presentation we will discuss the extension of the above approach that captures both down and uplink and takes into account the maximal power constraints.