On the Coverage and Detectability of Large-scale Wireless Sensor Networks

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I. Introduction

Advances in sensor and communication technologies have made it possible to manufacture small sensors with sensing, processing, and wireless communication capabilities in a cost-effective fashion. A sensor network can be formed by deploying specialized sensors in the region of interest to perform certain sensing and networking tasks. Application scenarios of wireless sensor networks include battlefield surveillance, environment monitoring, and etc. Many of the above application scenarios involve a large number of sensors deployed in a vast geographical area.

Coverage and detectability are two fundamental measures of the performance of a sensor network. In general, sensing coverage represents how well an area is monitored by sensors; and detectability represents the capability that a sensor network detects an object that moves through the network. Formal definitions of the coverage and detectability will be introduced shortly. Characterizations of these two measures present important implications to protocol design and performance of sensor networks.

While most of previous related work on the sensor network coverage and detectability focuses on protocol design [1][2][3][4][5], the goal of our work is to define and characterize the coverage and detectability, and examine the implications of the results. To represent the coverage and detectability of a sensor network, we define the following quantities.

Area coverage (f_a): the fraction of the geographical area that is in the sensing area of one or more sensors. The sensing area of a sensor is the area within which the sensor can provide a valid sensing measurement. This is usually represented by a circle centered at the sensor with a radius of the sensor's sensing range.

Node coverage fraction (f_n): the fraction of sensors whose sensing areas are fully covered by collections of other sensors. This quantity represents the redundancy level of sensors from a coverage perspective. It has a direct effect on the performance of energy-efficient protocols which turn off redundant sensors while preserving area coverage [1].

Detectability (p_d): the probability that no path exists for an object to penetrate a large-scale sensor network from left to right or from top to bottom without being detected. An object is detected by a sensor if it enters the sensing area of the sensor. We consider the worst-case scenario for sensor networks and assume that an object can take arbitrary paths. A penetrating path without being detected should not intersect the sensing areas of any sensors. Detectability of a sensor network is of interest in application scenarios such as soldiers crossing a battlefield in military operations and animal tracking in environment monitoring.

In this work, we consider grid-based sensor networks where sensors are deployed in a square lattice and random sensor networks where sensors are deployed at random locations in a field. We characterize the asymptotic behaviors of the coverage and detectability of sensor networks. Consequently, the requirements of the sensor density to achieve a target area coverage can be derived. While grid-based sensor networks provide efficient area coverage, random networks offer robustness and reliability upon sensor failures at the cost of coverage redundancy. For a random sensor network, we observe that only a small fraction of sensors are covered by other sensors when the density is not high enough. We show that the detectability of a 2-dimensional large scale sensor network exhibits a phase transition at a critical sensor density. Below the critical density, a penetrating path that will not be detected exists almost surely. Above the critical density, the network can detect any crossing object almost surely. Therefore, in order to effectively detect any crossing object, sensors should be deployed at a density higher than the critical value.

II. Grid-based Sensor Networks

In this section, we study the coverage and detectability of grid-based sensor networks. The whole network is divided into a large array of squares, as shown in Fig. 1. Each square has a side length of D. In this model, sensors can only be located at the center of each square. A square is occupied if there is a sensor located at the center. Every sensor has a finite sensing range, within which it can provide reliable sensing measurements. We assume that all the sensors are homogeneous, i.e, they have the same sensing range r. The simple grid-based model has often been used in research [6][7] to obtain closed-form results and provide insights to more general scenarios.

For the deployment of the sensors, we assume that the occupations of the squares follow a spatial Bernoulli process with probability p. An empty square represents one of the following scenarios: (i) no sensor is deployed in the square (ii) the sensor located in the square fails (iii) the sensor located in the square is turned off due to protocol operations. We now present results for $r < D/2$ and $r = D/2$, respectively.

1. $r < D/2$:

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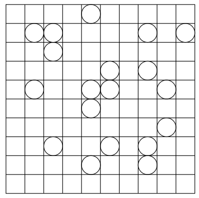

Fig. 1. A gird-bases sensor network

A square is occupied with probability p, with each sensor covering an area of πr^2. We have $f_a = p\pi r^2/D^2$.

In this case, a location is covered by at most one sensor node. There is no overlap between any two sensor nodes. Therefore, $f_n = 0$.

Since $r < D/2$, no grid line is in the sensing area of any sensor. An object moving on grid lines will not be detected by sensors. Therefore, there always exists a path along which an object can cross the sensor network without being detected. Thus, $P_d = 0$.

2. $r = D/2$

In this case, area coverage (f_a) and node coverage (f_n) exhibit the same behavior as in the case $r < D/2$. However, detectability of the network exhibits different behaviors.

Two squares are called nearest neighbors if they have one side in common but not if they only touch at one corner. Since $r = D/2$, an object cannot pass through two nearest occupied neighbors without being detected. A cluster is defined to be a group of neighboring occupied squares. In order not to be detected, an object cannot pass through a cluster but should move around the perimeter of the cluster.

The detectability of a sensor network can be related to the percolation of the sensors in the network. According to percolation theory [8], there exists a critical threshold p_c, where a phase transition occurs with respect to the size of the largest cluster.

More specifically, when the probability is above the threshold ($p > p_c$), there exists a unique unbounded cluster almost surely. The unbounded cluster extends from top to bottom and from left to right. Therefore, an object cannot penetrate the network without being detected.

When the probability is below the threshold ($p < p_c$), all clusters are bounded almost surely. In this case, there always exists a path along which an object can penetrate the network without being detected. Otherwise, there must be a unbounded cluster percolating through the network, contradicting the assumption. Therefore,

$$P_d = \begin{cases} 0 & \text{a.s.} \quad \text{if } p < p_c \\ 1 & \text{a.s.} \quad \text{if } p > p_c \end{cases} \tag{1}$$

where $p_c = 0.5928$ [8].

Note that because of the "zero-one" law of the phase transition, the detectability cannot be anything other than 0 or 1.

III. Random Sensor Networks

In this section, we study the coverage and detectability of large scale random sensor networks, where sensors are randomly distributed in a vast geographical area. In this case, the locations of sensors are described by a stationary Poisson point process. Therefore, a random sensor network can be described by a Poisson Boolean model $B(\lambda, r)$, where r represents the common sensing range of the sensors and λ represents the density of the underlying Poisson point process.

A. One-dimensional case

In the one-dimensional straight-line scenario, sensors are randomly placed on a straight line, each covering a segment of length $2r$. In this case, the *area coverage* refers to the fraction of the line that is covered by one or more sensors. Using the well-known *covered volume fraction* result in stochastic geometry, we can obtain $f_a = 1 - e^{-2\lambda r}$.

In sensor network planning, a designer may want to determine the minimum density required in order to achieve a target area coverage. Using the above area coverage result, we can derive the required sensor density in order to achieve a desired area coverage f_a, $\lambda = -\ln(1 - f_a)/2r$.

A sensor is covered if its sensing area is covered by other sensors; and the sufficient and necessary condition is that the distance between its immediate left and right neighbors is smaller than $2r$. Since the distance follows a 2-nd order Erlang distribution, the node coverage is

$$f_n = 1 - e^{-2\lambda r} - 2\lambda r e^{-2\lambda r} \tag{2}$$

On an one-dimensional line, there is no location that an object can cross without being detected if the distances between adjacent sensors are all smaller than $2r$. There is an infinite number of sensors on an one-dimensional straight line, the detectability can be computed as follows:

$$P_d = \lim_{n \to \infty} (1 - e^{-2\lambda r})^n = 0 \tag{3}$$

B. Two-dimensional infinite plane case

For a random point in the two-dimensional plane, denote the number of sensors covering the location as N. The random variable N follows a Poisson distribution with parameter $\lambda\pi r^2$, i.e., $P(N = n) = e^{-\lambda\pi r^2}(\lambda\pi r^2)^k/k!$; and the area coverage is $f_a = 1 - e^{-\lambda\pi r^2}$. In order to achieve a desired area coverage f_a ($0 < f_a < 1$) almost surely, the node density should be $\lambda = -\ln(1 - f_a)/\pi r^2$.

Figure 2 depicts the analytical (the curve) and simulation results (the points) of the area coverage as a function of node density. In the simulation, we approximated an infinite plane by a domain of 10000 x 10000 pixels. The density of the sensor nodes is measured in pixels^{-2}. The sensing range of each sensor is set to be 10 pixels. From the figure we observe that the simulation results agree with the analytical results very well. Note that in random sensor networks, a location may be covered by several sensors. The expected number of sensors covering any random location is simply $\lambda\pi r^2$. This makes the sensor network more robust and reliable upon sensor failures.

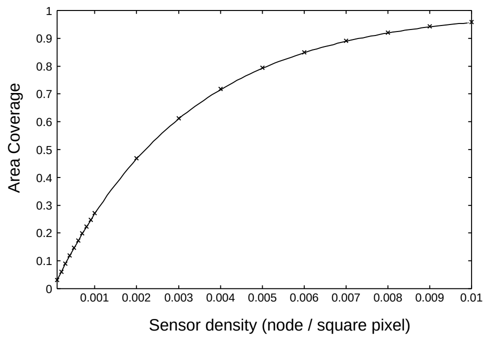

Fig. 2. Area coverage as a function of node density

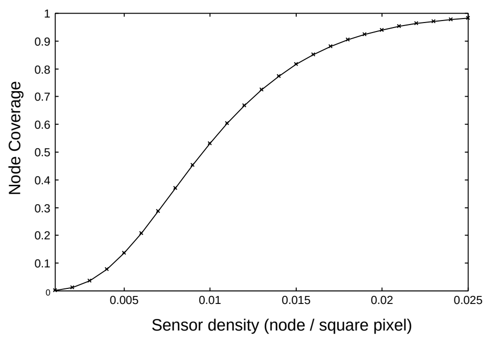

Fig. 3. Node coverage as a function of node density

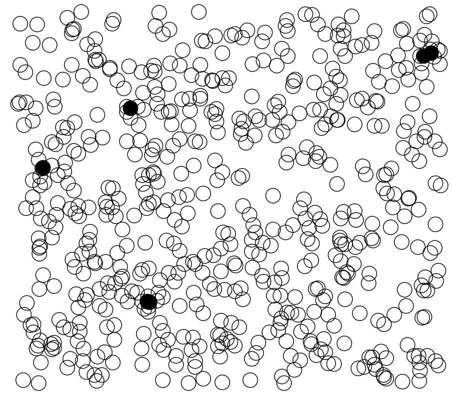

Fig. 4. Part of the sensor network at node density of 2×10^{-3}

Of course, the robustness and reliability are achieved at the cost of the redundancy of the sensors.

Figure 3 shows the simulation results of node coverage (f_n) as a function of node density. We observe that node coverage remains below 1% until the density of the sensor nodes increases to a very high value, 2×10^{-3}. At this density, 47% of the whole area is covered by sensors. To provide a visual illustration of the area coverage and node coverage at this density, we illustrate part of the network in Figure 4. This part is randomly chosen within the domain and contains a large number of sensors, it is representative of the whole network. In the illustration, each sensor is represented by a circle with a radius of the common sensing ranging, while filled circles represent those sensors whose sensing areas are fully covered by other sensors. We observe that the density is quite high and sensors that are covered by other sensors are sparsely distributed in the network. This result suggests that in scenarios where sensors cannot be deployed at a high density due to cost or other reasons, the fraction of nodes that can be turned off without reducing the area coverage is small.

In the same token as for a two-dimensional grid-based sensor network, the detectability of a two-dimensional random sensor network can be related to the continuum percolation [9] of the sensors. There is a critical density λ_c at which an unbounded occupied cluster emerges. Below the critical density λ_c, all clusters are almost surely finite in size. An object can penetrate the network by moving between the boundaries of the clusters. Above the critical density λ_c, an unbounded occupied sensor cluster percolates the whole network, which will detect any object trying to cross the network. Therefore,

$$P_d = \begin{cases} 0 \ \text{ a.s.} & \text{if } \lambda < \lambda_c \\ 1 \ \text{ a.s.} & \text{if } \lambda > \lambda_c \end{cases} \tag{4}$$

The critical density (λ_c) depends on the sensing range of the sensors and can be obtained using simulation.

IV. Conclusions

We characterize the asymptotic behaviors of the coverage and detectability of large-scale sensor networks. We also studied a number of other issues, for example, coverage and detectability of a 2-dimensional finite-width strip sensor network, probability and algorithm to find a path between two random locations without being detected, and etc. The results are not presented here due to space limitations.

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