Performance of Repetition Codes and Punctured Codes for Accumulative Broadcast

Ivana Maric, Roy Yates

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Ivana Maric
WINLAB, Rutgers University
ivanam@winlab.rutgers.edu

Roy Yates
WINLAB, Rutgers University
ryates@winlab.rutgers.edu

1 Problem Formulation.

In the problem of minimum-energy broadcasting in a wireless network, the objective is to broadcast data reliably to all network nodes at a fixed rate with minimum transmitted power. In [3], this problem was formulated as a minimum-cost broadcast tree problem and the well-known heuristic for constructing energy-efficient broadcast trees, the Broadcast Incremental Protocol (BIP), was proposed. In this paper, we look to increase the energy efficiency by employing an *accumulative broadcast* strategy [2] that allows nodes to collect the energy of unreliably received signals. As a message is forwarded through the network, a node will have multiple opportunities to reliably receive the message by collecting energy during each retransmission. While we allow nodes to collect unreliably received signals, we impose a *reliable forwarding* constraint that each node can forward a message only after reliably decoding that message. The constraint of reliable forwarding imposes an ordering on the network nodes. The order in which nodes become reliable is referred to as a *reliability schedule*.

Under these assumptions, we formulate the minimum-energy accumulative broadcast problem for two coding schemes: punctured codes with general coding and simple repetition codes. Punctured codes are generated as described in [1] and are used by different nodes to broadcast a message through the network. In particular, for a network of N nodes, a channel code C with 2^{nR} codewords of length nM for some $M \geq N$ is generated and assumed known at all the nodes. Codewords are divided into M subblocks of length n, each transmitted in one slot. We let C_m for $m = 1, \ldots M$ denote the punctured code with codewords of length nm formed from the first m subblocks of each codeword in C. The jth relaying node transmits the jth subblock. An unreliable node, after receiving m signals, attempts to decode C_m for each $m \geq 1$, until the decoding is successful. For repetition coding, only a code C_1 is generated. A codeword of length n transmitted by the source is retransmitted by each relaying node. An unreliable node collects signals received from different nodes until it can reliably decode C_1.

In [2], accumulative broadcast was employed using repetition coding. The network was assumed to be operating in the low-SNR regime where simple repetition coding is optimal. In this work, we examine the benefit of punctured codes with general coding for accumulative broadcast.

2 Approach.

With each transmitting node k we associate an AWGN channel of bandwidth W such that each receiving node m is characterized by a frequency non-selective link gain h_{mk}. We further assume that each node is assigned to transmit in an orthogonal channel thus causing no interference to other transmissions. To formulate the accumulative broadcast problem, we first determine the maximum achievable rate at every node for a given set of transmit powers for both coding schemes. Given the achievable rate at the nodes, we then present a solution to the accumulative broadcast problem employing two subproblems. The first step is finding the best reliability schedule and the second is determining the optimum power levels for that schedule. The solution will specify the nodes that should transmit and their transmission power levels. A reliability schedule can be represented by a matrix $\mathbf{X}$ where

$$x_{ij} = \begin{cases} 1 & \text{if node } i \text{ becomes reliable after node } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Each x_{ij} is an indicator that a node i collects energy from a possible transmission by node j. For repetition codes, the solution to the second subproblem can be found by linear programming as given in [2]. In the case of punctured codes, for a given schedule $\mathbf{X}$, the subproblem can be defined as a convex optimization problem in terms of the vector $\mathbf{p}$ of transmitted powers

$$\rho(\mathbf{X}) = \min \ \mathbf{1}^T \mathbf{p} \tag{2}$$
$$\text{subject to } \frac{1}{2}\sum_{j=1}^{N} \log\left(1 + \frac{h_{ij} x_{ij} p_j}{N_0 W}\right) \geq \frac{\overline{r}}{2W},$$
$$\mathbf{p} \geq \mathbf{0}.$$

where $\overline{r}$ is a required data rate in bits/s.

For the simpler case of repetition coding, the problem of finding the best schedule was shown in [2] to be NP-complete. To find a good schedule, we employ a simple

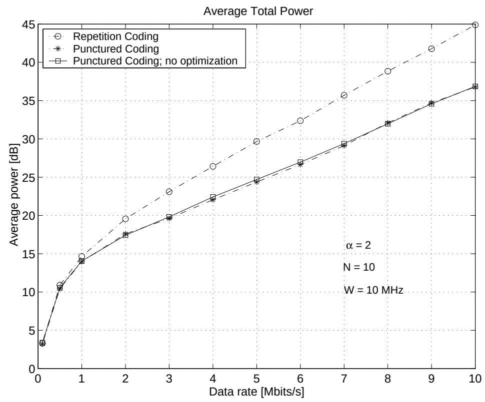

Figure 1: Power used for broadcasting.

heuristic algorithm presented in [2] for repetition coding. We modify the algorithm to be used with punctured coding and evaluate the algorithm performance for both coding schemes. The algorithm starts with a partial reliability schedule that contains only the source. In each step, a reliable node that maximizes the fill rate of the unreliable set is chosen for the next transmission. The node transmits with enough power to make one more node reliable and that node is appended to the schedule. Once the schedule is determined, the optimization routine will find the optimum power levels for that schedule.

In the above algorithm, a reliable node that is chosen to transmit several times, will in fact consolidate its multiple transmissions into a single transmission. The actual power level is found by the optimization routine. As argued in [2], a node using a repetition code cannot gain from successive retransmissions of the same codeword. In the power-limited regime, this property holds for punctured codes as well.

However, when the system is bandlimited and the nodes are operating in higher SNR, the efficient bandwidth utilization becomes important. The above strategy will not, in general, utilize the available network bandwidth if the orthogonal channels are assigned to all the users a priori. In this case, bandwidth will be wasted on the users not chosen by the heuristics to relay the data.

To increase the efficiency of bandwidth utilization in the above scheme, we let the heuristic algorithm allocate the available bandwidth in addition to deciding on the schedule. The bandwidth allocation follows directly when the consecutive transmissions at the nodes are allowed. Therefore, there is no need for further modification of the heuristic. In particular, since the heuristic makes one unreliable node reliable in each step, at most $N-1$ transmissions are needed to broadcast a message. Then the available system bandwidth can be divided into $N-1$ channels and a reliable node that is chosen as the best filling node $t > 1$ times, will be allocated t channels to transmit t times rather then just once. The power levels determined by the heuristic are the actual power levels to be used for broadcasting; no additional optimization is performed. In such a scenario, punctured codes where a node would transmit a new subblock in each new transmission become crucial. If a node would use repetition coding, resending the same codeword in each of t channels would not result in any benefit. To benefit from additional bandwidth, a new repetition code would have to be constructed with codewords of length $n_1 = tn$.

Figure 1 shows the algorithm performance for both coding schemes and for different broadcast data rates. For punctured coding, we evaluated the performance of both strategies described above. The performance was evaluated in a network of $N = 10$ nodes. The transmitted power was attenuated as r^{α} for propagation exponent $\alpha = 2$. Results demonstrate the benefit of using the punctured codes over the repetition coding in the high-SNR regime due to the higher achievable rates. The gain diminishes in the low-SNR regime, as expected.

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Ivana Maric received the B.S. degree in Electrical Engineering in 1995 from the University of Novi Sad, Yugoslavia and the M.S. degree in Electrical Engineering in 2000 from Rutgers University where she is currently pursuing the Ph.D. degree. Her research interests are in the field of wireless communications and, in particular, in mobile ad hoc networks.

Roy Yates received the B.S.E. degree in 1983 from Princeton University, and the S.M. and Ph.D. degrees in 1986 and 1990 from M.I.T., all in Electrical Engineering. Since 1990, he has been with the Wireless Information Networks Laboratory (WINLAB) and the ECE department at Rutgers University. Presently, he serves as an Associate Director of WINLAB and Professor in the ECE Dept.