Optimal Power Control in CDMA Networks with Constraints on Outage Probability

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***Abstract*— This paper proposes a new scheme that couples power control with a minimum outage probability multiuser detector. The resultant iterative algorithm is conceptually simple and finds the minimum sum power of all users with a set of outage probability constraints. Bounds on the outage probability expression are found that extend a previous result that did not include receiver noise. These bounds are used to create a sub-optimal scheme coupling power control and a MMSE multiuser detector. This new problem becomes a variant of an existing problem where outage probability constraints are first mapped to average SIR threshold constraints. Using a recent result that transforms complex SIR expressions into a compact and decoupled form, a non-iterative and computationally inexpensive power control algorithm is developed for large systems of users. Simulation results are presented showing the closeness of the optimal and mapped schemes, speed of convergence and performance comparisons with other standard receivers.**

***Index Terms*— CDMA, Decorrelator, large system analysis, MMSE receiver, multiuser detection, outage probability, power control.**

I. Introduction

Power allocation is an effective way to improve the performance of wireless communication systems. It can mitigate *near-far* effects occurring when a nearby interferer disturbs the reception of a remote user, whose desired signal is attenuated to a greater extent. Careful power allocation can also increase utilization in interference-limited systems such as CDMA, or those multiple access systems employing frequency reuse amongst cells, as in FDMA. Finally, by allocating minimum power across all users, battery life of mobile devices will be extended: users only need to expend sufficient power for acceptable reception as determined by their quality of service (QoS) specifications such as FER, BER or outage probability.

The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices [1]–[4], as iterative Power Control Algorithms (PCA) that converge each users' power to the minimum power result [5]–[6], optimization based approaches [7]–[9] and other variations [10]–[12]. A useful framework for uplink cellular power control is given in [13].

Much of this previous work deals with invariant channel models. Any power control scheme that attempts to follow fast fades would need to be highly efficient to be implemented in practice, or incur a power penalty due to intense signal processing and may require frequent communication with its assigned base station.

The authors are with the ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN), Department of Electrical and Electronic Engineering, University of Melbourne, Melbourne VIC 3010, Australia, e-mail:{jpap, jse, s.dey}@ee.mu.oz.au

This work was supported by the Australian Research Council.

Of particular interest is the work presented in [8] and [9]. In [8], a scheme whereby the statistics of the received signal to interference ratio (SIR) are used to allocate power, rather than an instantaneous SIR. The allocation decisions can then be made on a much slower time-scale (following log-normal shadowing variations for instance). In [9], an optimization problem is considered using average SIR which yields locally optimal solutions.

In a multiple access system, multiuser detection (MUD) can be used to further enhance the performance of wireless systems by exploiting the structure of the multiple access interference [14]. The optimal MUD is near-far resistant however has exponential complexity with the number of active users. The decorrelating detector, with polynomial complexity, can eliminate multiuser interference entirely at the expense of enhancing receiver noise power [15]. Furthermore, in a CDMA system, the noise enhancement increases as the utilization increases until the detector becomes unusable. Minimum mean squared error (MMSE) detection [16]–[17] is based on the minimization of the expected squared error between a transmitted symbol and the received signal. It is also near-far resistant and reduces to the decorrelating detector as the AWGN power is zeroed. The MMSE receiver is considered a compromise between the matched filter and decorrelator.

A major difficulty in the analysis of a power controlled environment is the coupling of all users through the interference they cause each other. One simplifying case is the decorrelating detector as noted above, however recent work has shown that in a large system, decoupling of the interfering effects is also possible for several other receiver structures including the MMSE receiver [18]. This work will lead to a fully decentralized PCA that requires little computation.

Traditionally, power allocation and multiuser detection were considered separately. Power allocation assumed a fixed receiver structure and multiuser detection assumed fixed user transmitter powers. By jointly optimizing both power and linear receivers, an increase in user capacity in addition to a reduction of power consumption is possible [18]. Recent work [19]–[21] has focused on this problem in order to obtain the benefits of both power allocation and MUD.

The main contributions of this paper are:

1) The design of a conceptually simple, iterative algorithm that minimizes the sum power of all users subject to outage constraints. The optimization is performed on choice of user powers and linear multiuser receivers. This problem reduces to that considered in [8] if background noise is neglected and linear receivers fixed.
2) A new sub-optimal iterative PCA-MUD where outage constraints are first mapped to average SIR threshold constraints. Such a mapping permits the use of a variant of an

existing combined PCA and MMSE MUD given in [19].

3) A decentralized non-iterative PCA-MUD for medium to large systems of users using a recent result that transforms complex expressions for SIR into a compact and decoupled form.

Simulation results are also provided comparing each method with established receivers such as the matched filter and decorrelating receiver.

II. System Model

In this paper, we consider the uplink in a synchronous direct sequence CDMA communications system with K users and a processing gain of N. We assume a BPSK modulation scheme and a N-dimensional chip matched filter vector for each symbol interval, given by

$$\mathbf{r}_i = \sum_{j=1}^{K} \sqrt{G_{ij} F_{ij} P_j} b_j \mathbf{s}_j + \mathbf{n} \tag{1}$$

where $G_{ij} F_{ij} P_j$ is the received power power from user j, with transmit power P_j. $G_{ij} F_{ij}$ represents the instantaneous channel gain of user j to the assigned Base Station (BS) of user i. The data bits b_j take on values of ± 1 with equal probability, $\mathbf{s}_j$ is the N-dimensional spreading sequence of user j, and $\mathbf{n}$ is AWGN with zero mean and covariance $\sigma^2 \mathbf{I}$. We assume fixed spreading sequences, with elements of $\mathbf{s}_j$ taking values $\pm 1/\sqrt{N}$.

Following the conventions of [8], we assume G_{ij} is the positive *slow-varying* path gain of user j to the assigned Base Station (BS) of user i, excluding any fading. The analysis that follows holds only over a time scale where factors affecting G_{ij} do not change significantly. The terms F_{ij} model fast time-scale Rayleigh fading and are assumed to be unit mean independent exponentially distributed random variables.

Let $\mathbf{c}_i$ denote the receiver filter coefficients for user i at its assigned BS and $\mathbf{c} = [\mathbf{c}_1, \ldots, \mathbf{c}_K]$. The filter output of user i at its assigned BS is given by

$$\mathbf{y}_i = \mathbf{c}_i^\top \mathbf{r}_i = \sum_{j=1}^{K} \sqrt{G_{ij} F_{ij} P_j} (\mathbf{c}_i^\top \mathbf{s}_j) b_j + \tilde{n}_i \tag{2}$$

where $\tilde{n}_i = \mathbf{c}_i^\top \mathbf{n}$ is $N(0, \sigma^2 \mathbf{c}_i^\top \mathbf{c}_i)$.

III. Outage Probability and Certainty-Equivalent Margin with Noise

To simplify notation in this section, we shall drop the receiver filter terms $(\mathbf{c}_i^\top \mathbf{s}_j)^2$ without a loss of generality (since we can absorb them into the G_{ij} terms). They will become important in Section IV.

A. SIR and Outage Probability

The SIR, γ, of the ith mobile is given by

$$\gamma_i = \frac{G_{ii} F_{ii} P_i}{\sum_{j \neq i} G_{ij} F_{ij} P_j + \sigma^2} \tag{3}$$

The outage probability of user i, denoted O_i, is defined as the proportion of time that some SIR threshold γ_i^{th} is not met for sufficient reception at the BS receiver. By careful choice of γ_i^{th}, we can set a QoS for each user. O_i is given by

$$\begin{aligned} O_i &= Pr\left(\gamma_i \leq \gamma_i^{th}\right) \\ &= Pr\left(G_{ii} F_{ii} P_i \leq \gamma_i^{th} \left\{ \sum_{j \neq i} G_{ij} F_{ij} P_j + \sigma^2 \right\}\right). \end{aligned}$$

The outage probability for the ith user is given by (see [8])

$$O_i = 1 - \exp\left(\frac{-\sigma^2 \gamma_i^{th}}{G_{ii} P_i}\right) \prod_{j \neq i} \frac{1}{1 + \frac{\gamma_i^{th} G_{ij} P_j}{G_{ii} P_i}}. \tag{4}$$

B. Certainty-Equivalent Margin with Noise

The Certainty-Equivalent Margin (CEM) was defined in [8] without noise. It represents a margin of error for *average* SIR when representing the system by a certainty-equivalent form (with all statistical variation in signal and noise power ignored and replaced with their expected values.)

We will take average SIR (denoted $\overline{\text{SIR}}$) to mean the expected value of the i^{th} mobile received power over the expected value of the interference from the $K-1$ other mobiles and background noise. This is also the certainty-equivalent SIR and is given by

$$\overline{\text{SIR}}_i = \gamma_i^{ce} = \frac{E[G_{ii} F_{ii} P_i]}{E[\sum_{j \neq i} G_{ij} F_{ij} P_j + \sigma^2]} = \frac{G_{ii} P_i}{\sum_{j \neq i} G_{ij} P_j + \sigma^2}. \tag{5}$$

As with [8], we also define the CEM (with noise, denoted CEM$^\sigma$) as the ratio of the certainty-equivalent $\overline{\text{SIR}}$ to the average SIR threshold,

$$CEM_i^\sigma = \frac{\gamma_i^{ce}}{\gamma_i^{th}} = \frac{G_{ii} P_i}{\gamma_i^{th} \left\{ \sum_{j \neq i} G_{ij} P_j + \sigma^2 \right\}}. \tag{6}$$

C. Relation Between the CEM^σ and Outage Probability

Using the following result, we briefly sketch the derivation of upper and lower bounds on outage probability in terms of the CEM^σ. The full development can be found in [22]. For $z_1, ..., z_n \geq 0$ it can be shown that the inequalities

$$1 + k + \sum_{i}^{n} z_i \leq e^k \prod_{i}^{n} (1 + z_i) \leq e^{k + \sum_i^n z_i} \tag{7}$$

hold, where k is some constant.

From (4), we have

$$O_i = 1 - \frac{1}{\exp\left(\frac{\sigma^2 \gamma_i^{th}}{G_{ii} P_i}\right) \prod_{j \neq i} 1 + \frac{\gamma_i^{th} G_{ij} P_j}{G_{ii} P_i}}$$

Using the left-hand side inequality in (7), it can be shown that

$$O_i \geq \frac{1}{1 + CEM_i^\sigma}.$$

It can also be shown that the right-hand side inequality in (7) leads to

$$O_i \leq 1 - e^{-1/CEM_i^\sigma}.$$

The upper and lower bounds on outage are thus,

$$\frac{1}{1+CEM_i^\sigma} \leq O_i \leq 1-e^{-1/CEM_i^\sigma} \tag{8}$$

which have the same form and tightness in the region of interest as the noiseless case in [8].

Note that CEM_i^σ (or average SIR with a fixed γ_i^{th}) and outage probability are inversely proportional. As we increase the CEM_i^σ (or $\overline{\text{SIR}}$), we get a lower outage probability and vice-versa.

D. Special Case: Decorrelator

Recall that for $K < N$, the decorrelator nulls out all users other than the one of interest, at the expense of enhanced receiver noise power. The resulting expression for the decorrelator SNR is given by

$$\gamma_i = \frac{G_{ii}F_{ii}P_i}{\sigma^2 \mathbf{R}_{ii}^{-1}}$$

where $\mathbf{R}_{ii}^{-1}$ is the $(i,i)^{th}$ element of the inverted $K \times K$ signature sequence cross-correlation matrix $\mathbf{R}$. The corresponding CEM_i^σ is given by

$$CEM_i^\sigma = \frac{G_{ii}P_i}{\gamma_i^{th}\sigma^2\mathbf{R}_{ii}^{-1}}. \tag{9}$$

By definition, the outage expression is given by

$$\begin{aligned} O_i &= Pr\left(\gamma_i \leq \gamma_i^{th}\right) \\ &= Pr\left(F_{ii} \leq \gamma_i^{th}\frac{\sigma^2\mathbf{R}_{ii}^{-1}}{G_{ii}P_i}\right) \\ &= 1-\exp\left(-\gamma_i^{th}\frac{\sigma^2\mathbf{R}_{ii}^{-1}}{G_{ii}P_i}\right) \end{aligned}$$

or by substituting (9),

$$O_i = 1-e^{-1/CEM_i^\sigma}. \tag{10}$$

IV. Power Control Algorithm

A. Problem Definition

The aim of the PCA is to find the powers, P_i, and filter coefficients, $\mathbf{c}_i$ for $i = 1, ..., K$, such that the total power transmitted by all users is minimized while all outage constraints are met. Formulating this as an optimization problem, we have

$$\begin{aligned} &\min_{\mathbf{P},\mathbf{c}} \sum_{i=1}^{K} P_i \\ &\text{s.t. } 1-e^{\frac{-\sigma^2(\mathbf{c}_i^\top\mathbf{c}_i)\gamma_i^{th}}{G_{ii}(\mathbf{c}_i^\top\mathbf{s}_i)^2P_i}} \prod_{j\neq i}\frac{1}{1+\frac{\gamma_i^{th}G_{ij}(\mathbf{c}_i^\top\mathbf{s}_j)^2P_j}{G_{ii}(\mathbf{c}_i^\top\mathbf{s}_i)^2P_i}} \leq O_i^t, \\ &\quad P_i \geq 0, \;\; \mathbf{c}_i \in \mathbb{R}^N \qquad\qquad i=1,...,K \end{aligned}$$

where we used (4) with the receiver filter included and O_i^t are target outage probability constraints. It can be shown that this problem is equivalent to the following, where the inner optimization has been inserted into the constraint set. See [19] for a similar refinement.

$$\begin{aligned} &\min_{\mathbf{P}} \sum_{i=1}^{K} P_i \\ &\text{s.t. } \min_{\mathbf{c}_i\in\mathbb{R}^N}\left\{1-e^{\frac{-\sigma^2(\mathbf{c}_i^\top\mathbf{c}_i)\gamma_i^{th}}{G_{ii}(\mathbf{c}_i^\top\mathbf{s}_i)^2P_i}} \prod_{j\neq i}\frac{1}{1+\frac{\gamma_i^{th}G_{ij}(\mathbf{c}_i^\top\mathbf{s}_j)^2P_j}{G_{ii}(\mathbf{c}_i^\top\mathbf{s}_i)^2P_i}}\right\} \leq O_i^t, \\ &\quad P_i \geq 0 \qquad\qquad i=1,...,K \end{aligned} \tag{11}$$

B. Optimal Power Control

In this section we describe an iterative algorithm to solve the optimization problem (11).

Taking the log of the outage constraint from (11) and rearranging yields

$$\frac{w_i + \frac{\tilde{G}_{ii}P_i}{\gamma_i^{th}}\sum_{j\neq i}\log\left(1+\frac{\tilde{G}_{ij}P_j}{\frac{\tilde{G}_{ii}P_i}{\gamma_i^{th}}}\right)}{\frac{\tilde{G}_{ii}P_i}{\gamma_i^{th}}} \leq \log\left(\frac{1}{1-O_i^t}\right)$$

with $w_i = \sigma^2(\mathbf{c}_i^\top\mathbf{c}_i)$, $\tilde{G}_{ij} = G_{ij}(\mathbf{c}_i^\top\mathbf{s}_j)^2$, and finally

$$P_i \geq \frac{w_i + \widehat{G}_{ii}P_i\sum_{j\neq i}\log\left(1+\frac{\tilde{G}_{ij}P_j}{\widehat{G}_{ii}P_i}\right)}{\widehat{G}_{ii}\log\left(\frac{1}{1-O_i^t}\right)} \tag{12}$$

where $\widehat{G}_{ii} = \frac{\tilde{G}_{ii}}{\gamma_i^{th}}$.

If we view (12) as representing a set of quasi-interference constraints on the power vector $\mathbf{P}$, we can define a new PCA where each user i iteratively attempts to compensate for the interference. At convergence, we would like each of the outage constraints to be met.

We define

$$\begin{aligned} I_i(\mathbf{P},\mathbf{c}_i) &= \frac{w_i + \widehat{G}_{ii}P_i\sum_{j\neq i}\log\left(1+\frac{\tilde{G}_{ij}P_j}{\widehat{G}_{ii}P_i}\right)}{\widehat{G}_{ii}\log\left(\frac{1}{1-O_i^t}\right)} && (13) \\ T_i(\mathbf{P}) &= \min_{\mathbf{c}_i} I_i(\mathbf{P},\mathbf{c}_i) && (14) \end{aligned}$$

where we let $I_i(\mathbf{P},\mathbf{c}_i) = P_i$ be (12) replaced with equality. We shall now refer to $I_i(\mathbf{P},\mathbf{c}_i)$ as the *interference function* to maintain consistency with the framework in [13].

Furthermore, we propose the PCA

$$\mathbf{P}(n+1) = \mathbf{T}(\mathbf{P}(n)) \tag{15}$$

where $\mathbf{T}(\mathbf{P}) = [T_1(\mathbf{P}), \ldots, T_K(\mathbf{P})]^\top$ and is initialized with powers set to the receiver noise level $P_i(0) = \sigma^2, \forall i$ and matched filter coefficients $\mathbf{c}_i(0) = \mathbf{s}_i, \forall i$.

This PCA is similar in form to that in [19], however we are dealing with outage and average quantities on a slower time scale. In Section IV-C, we will see how a sub-optimal variant of (13)-(15) can be reduced to a PCA that is similar in form to [19].

Since our PCA (15) is of the standard form given by [13], Theorem 1 proposes that (14) is a *standard* interference function and thus the PCA converges to a fixed solution. The proof is based on the three properties of a standard interference function given in [13], repeated below:

Definition 1: Interference function $\mathbf{I}(\mathbf{P})$ is *standard* if for all $\mathbf{P} \geq 0$ the following properties are satisfied.

- *Positivity* $\quad \mathbf{I}(\mathbf{P}) > 0$.
- *Monotonicity* $\quad$ If $\mathbf{P} \geq \mathbf{P}'$, then $\mathbf{I}(\mathbf{P}) \geq \mathbf{I}(\mathbf{P}')$.
- *Scalability* $\quad$ For all $\alpha > 1$, $\alpha\mathbf{I}(\mathbf{P}) > \mathbf{I}(\alpha\mathbf{P})$.

Theorem 1: $\mathbf{T}(\mathbf{P})$ is a standard interference function.

The proof of Theorem 1 is given in [22].

Since $\mathbf{T}(\mathbf{P})$ is a standard interference function, the PCA (15) converges to a final solution $\mathbf{P}^* = \mathbf{T}(\mathbf{P}^*)$. This solution is the minimum power required to meet the outage constraints (12). The filter coefficients converge to a new type of MUD, the Minimum Outage Probability (MOP) receiver. As such, we shall refer to the PCA (15) as the optimal MOP-PCA.

C. Average SIR and Outage Probability

To meet our outage probability constraints, we require that $O_i \leq O_i^t$ for all i. Combining this inequality with the upper bound in (8), we can define a new constraint on CEM_i^σ that when met, will guarantee that our original outage constraints are also met:

$$O_i \leq 1 - e^{-1/CEM_i^\sigma} \leq O_i^t \quad i = 1, \ldots, K. \tag{16}$$

Rearranging the right-hand inequality of (16) yields,

$$\begin{aligned} \frac{1}{CEM_i^\sigma} &\leq \log\left(\frac{1}{1-O_i^t}\right) \\ \gamma_i &\geq \frac{\gamma_i^{th}}{\log\left(\frac{1}{1-O_i^t}\right)} \qquad (17) \\ &= \Gamma_i^{th} \qquad (18) \end{aligned}$$

where we have used the definition of CEM_i^σ from (6) and defined a new quantity,

$$\Gamma_i^{th} = \frac{\gamma_i^{th}}{\log\left(\frac{1}{1-O_i^t}\right)} \tag{19}$$

called the outage-mapped average SIR threshold. We can now define a new problem,

$$\begin{array}{lll} \min\limits_{\mathbf{P},\mathbf{c}} & \sum\limits_{i=1}^{K} P_i & \\ \text{s.t.} & P_i \geq \frac{\Gamma_i^{th}}{G_{ii}} \frac{\sum_{j\neq i} G_{ij}(\mathbf{c}_i^\top \mathbf{s}_j)^2 P_j + \sigma^2(\mathbf{c}_i^\top \mathbf{c}_i)}{(\mathbf{c}_i^\top \mathbf{s}_i)^2} & \\ & P_i \geq 0, \;\; \mathbf{c}_i \in \mathbb{R}^N & i = 1, ..., K \end{array}$$

where we have rearranged (5) to form a $\overline{\text{SIR}}$ constraint. This problem is mathematically equivalent to the power control problem in [19], however we consider *average* channel gains and the outage-mapped average SIR threshold Γ_i^{th} as parameters.

In a similar fashion to [19], the problem is equivalent to

$$\begin{array}{lll} \min\limits_{\mathbf{P}} & \sum\limits_{i=1}^{K} P_i & \\ \text{s.t.} & P_i \geq \frac{\Gamma_i^{th}}{G_{ii}} \min\limits_{\mathbf{c}_i \in \mathbb{R}^N} \frac{\sum_{j\neq i} G_{ij}(\mathbf{c}_i^\top \mathbf{s}_j)^2 P_j + \sigma^2(\mathbf{c}_i^\top \mathbf{c}_i)}{(\mathbf{c}_i^\top \mathbf{s}_i)^2} & \\ & P_i \geq 0 & i = 1, ..., K \end{array}$$

with an associated PCA given by,

$$\begin{aligned} \tilde{I}_i(\mathbf{P}, \mathbf{c}_i) &= \frac{\Gamma_i^{th}}{G_{ii}} \frac{\sum_{j\neq i} G_{ij}(\mathbf{c}_i^\top \mathbf{s}_j)^2 P_j + \sigma^2(\mathbf{c}_i^\top \mathbf{c}_i)}{(\mathbf{c}_i^\top \mathbf{s}_i)^2} \\ \tilde{T}_i(\mathbf{P}) &= \min_{\mathbf{c}_i} \tilde{I}_i(\mathbf{P}, \mathbf{c}_i) \qquad (20) \\ \mathbf{P}(n+1) &= \tilde{\mathbf{T}}(\mathbf{P}(n)) \end{aligned}$$

where $\tilde{\mathbf{T}}(\mathbf{P}) = \left[\tilde{T}_1(\mathbf{P}), \ldots, \tilde{T}_K(\mathbf{P})\right]^\top$. We shall refer to this PCA as the outage-mapped MMSE-PCA.

In [19], it was shown that the MMSE filter coefficients $\mathbf{c}_i$ minimize (20) and so we have the following iterative algorithm for the above problem:

$$\hat{\mathbf{c}}_i = \left(\mathbf{S}_i \mathbf{D}_i \mathbf{S}_i^\top + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{s}_i \tag{21}$$

$$P_i(n+1) = \frac{\Gamma_i^{th}}{G_{ii}} \frac{\sum_{j\neq i} P_j(n) G_{ij}(\hat{\mathbf{c}}_i^\top \mathbf{s}_j)^2 + \sigma^2(\hat{\mathbf{c}}_i^\top \hat{\mathbf{c}}_i)}{(\hat{\mathbf{c}}_i^\top \mathbf{s}_i)^2} \tag{22}$$

where $\mathbf{S}_i = [\mathbf{s}_1, \ldots, \mathbf{s}_{i-1}, \mathbf{s}_{i+1}, \ldots, \mathbf{s}_K]$, and $\mathbf{D}_i$ is a $(K-1) \times (K-1)$ diagonal matrix with entries $P_1 G_{11}, \ldots, P_K G_{KK}$, omitting $P_i G_{ii}$. The filter coefficients estimate $\hat{\mathbf{c}}_i$ is updated on each iteration. The PCA is initialized with powers set to the noise level and matched filter coefficients. The convergence proof of this PCA mirrors that in [19].

Since we have used the upper bound on O_i in the derivation above, the solution is sub-optimal, however we guarantee the outage constraints from the right-hand side of (16). Recall that in the region of interest, the outage bounds are tight, and so we expect that this PCA will result in near-optimal performance as compared to the optimal MOP PCA.

V. Large System Analysis

The linear receiver that achieves the minimal power allocation in the previous problem is the linear MMSE receiver. It does this by minimizing the interference function (20), or equivalently, maximizing the average SIR of user i. The corresponding maximal $\overline{\text{SIR}}$ for user i is given by

$$\overline{\text{SIR}}_i = P_i G_{ii} \mathbf{s}_i^\top \left(\mathbf{S}_i \mathbf{D}_i \mathbf{S}_i^\top + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{s}_i, \tag{23}$$

which we can use to restate the optimization problem of section IV-C as follows:

$$\begin{array}{lll} \min\limits_{\mathbf{P}} & \sum_{i=1}^{K} P_i & \\ \text{s.t.} & \overline{\text{SIR}}_i \geq \Gamma_i^{th} & \\ & P_i \geq 0 & i = 1, ..., K. \end{array} \tag{24}$$

The solution to this problem (if it exists) occurs when all users meet their outage-mapped average SIR constraint with equality. This problem is then equivalent to solving

$$P_i G_i \mathbf{s}_i^\top \left(\mathbf{S}_i \mathbf{D}_i \mathbf{S}_i^\top + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{s}_i = \Gamma_i^{th} \tag{25}$$

for all $i = 1, \ldots, K$.

The Large System Analysis (LSA) in [18] derives a closed form expression that the left-hand side of (25) converges to, in probability, as $N \to \infty$, $K \to \infty$ and the number of users per degree of freedom, $\alpha \equiv \frac{K}{N}$, remains fixed. This SIR, in the limit, is a deterministic quantity for each user i denoted

β_i^*. Somewhat surprisingly, the result enables the uncoupling of each user in the SIR expression above, by representing each user with an *effective interference*.

The result allows us to approximate the average SIR for a given user i, denoted β_i, in a finite sized system with MMSE receivers. The average SIR approximation is given by

$$\beta_i \approx \frac{P_i G_{ii}}{\sigma^2 + \frac{1}{N}\sum_{j=1}^{K} I(P_j G_{jj}, P_i G_{ii}, \beta_i)} \tag{26}$$

where $I(P_j G_{jj}, P_i G_{ii}, \beta_i)$ is the effective interference term contributed from the j^{th} user, and is defined as

$$I(P_j G_{jj}, P_i G_{ii}, \beta_i) \equiv \frac{P_j G_{jj} P_i G_{ii}}{P_i G_{ii} + P_j G_{jj} \beta_i}. \tag{27}$$

Combining (26) and (27) yields

$$\beta_i \approx \frac{P_i G_{ii}}{\sigma^2 + \frac{1}{N}\sum_{j=1}^{K} \frac{P_j G_{jj} P_i G_{ii}}{P_i G_{ii} + P_j G_{jj}\beta_i}}. \tag{28}$$

For the remaining analysis, we shall assume (28) holds with equality.

Let $\alpha_i^* = \frac{\beta_i}{P_i G_{ii}}$. Equation (28) becomes

$$\alpha_i^* P_i G_{ii} = \frac{P_i G_{ii}}{\sigma^2 + \frac{1}{N}\sum_{j=1}^{K} \frac{P_j G_{jj}}{1 + P_j G_{jj} \alpha_i^*}}$$

or

$$\alpha^* = \frac{1}{\sigma^2 + \frac{1}{N}\sum_{j=1}^{K} \frac{P_j G_{jj}}{1 + P_j G_{jj} \alpha^*}} \tag{29}$$

since α_i^* does not depend on i.

Substituting $P_j G_j = \frac{\beta_j}{\alpha^*} = \frac{\Gamma_j^{th}}{\alpha^*}$ into (29) gives

$$\begin{aligned} \alpha^* &= \frac{1}{\sigma^2 + \frac{1}{N}\sum_{j=1}^{K} \frac{\frac{\beta_j}{\alpha^*}}{1 + \frac{\beta_j}{\alpha^*}\alpha^*}} \\ &= \frac{1}{\sigma^2 + \frac{1}{\alpha^*}\frac{1}{N}\sum_{j=1}^{K} \frac{\Gamma_j^{th}}{1+\Gamma_j^{th}}} \end{aligned} \tag{30}$$

The non-trivial solution to the fixed point equation (30) is given by

$$\alpha^* = \frac{\left(1 - \frac{1}{N}\sum_{j=1}^{K} \frac{\Gamma_j^{th}}{1+\Gamma_j^{th}}\right)}{\sigma^2} = \frac{\Gamma_i^{th}}{P_i G_{ii}}$$

where again, $P_i G_{ii} = \frac{\Gamma_i^{th}}{\alpha^*}$. Rearranging the left-hand side for P_i gives the power solution of the i^{th} user,

$$P_i = \left(\frac{\Gamma_i^{th}}{G_{ii}}\right) \frac{\sigma^2}{\left(1 - \frac{1}{N}\sum_{j=1}^{K} \frac{\Gamma_j^{th}}{1+\Gamma_j^{th}}\right)} \tag{31}$$

where we have the necessary and sufficient condition

$$\frac{1}{N}\sum_{j=1}^{K} \frac{\Gamma_j^{th}}{1+\Gamma_j^{th}} < 1 \tag{32}$$

for a solution to exist.

The expression (31) allows for a totally decentralized and non-iterative PCA since we have a closed form expression that only requires the gain of the user of interest and the outage-mapped $\overline{\text{SIR}}$ thresholds for all users – all these parameters are known (or can be measured) at the base station receiver.

We shall refer to this PCA as the LSA-PCA.

VI. Simulation Results

Our simulations consider a single circular CDMA cell with radius 1km. We assume uniform location of users within the cell who are subject to distance dependent loss (loss exponent 4) and log-normal (zero mean, 8dB variance) shadowing. Unless otherwise stated, a processing gain of 32 was chosen, corresponding to a chip rate of 1.2288Mcps and an encoder input rate of 38.4kbps under the cdma2000 specification [23]. Furthermore, an AWGN noise power equal $\sigma^2 = 10^{-13}$ was chosen corresponding to approximately a 1MHz bandwidth.

We define three user classes, each having outage probability and SIR threshold pairs as {(5%, 10dB), (10%, 8dB), (20%, 6dB)}. We assign 25% of users to the first class, 50% to the second and the remaining to the third.

In all simulations that follow, user signature sequences are chosen randomly with elements of $\mathbf{s}_j$ taking values $\pm 1/\sqrt{N}$ with equal probability. Initial filter coefficients are set to the matched filter and initial user powers are set to the receiver noise power $\sigma^2 = 10^{-13}$ where appropriate.

A. Optimal MOP and Outage-Mapped MMSE PCAs

In this section, we compare the optimal MOP-PCA with the outage-mapped MMSE-PCA.

For $K = 4, 8, 16, 32$ users, Fig. 1 shows in log scale the sum power of all users as a function of the iteration step. The optimal MOP-PCA and MMSE-PCA are almost indistinguishable, verifying earlier claims on the tightness of the outage probability bounds.

Figure 2 considers a system of 32 users. It shows the convergence of the outage probabilities for each user as a function of the iteration step. We clearly see the three outage probability classes (5%, 10% and 20%) at convergence.

B. MMSE, Matched Filter and Decorrelator PCAs

In this section we compare three standard receivers with outage constraints under power control: the matched filter (MF), decorrelator (DC) and MMSE receivers. Each receiver has an associated outage constraint mapped to an average SIR constraint using (19). We consider 1,000 independent simulation runs for each receiver type. Signature sequences are chosen independently between runs, however sequences are common across simulations for each receiver type and specific value of K.

For $K = 2, 3, 4, 16, 32$ users, Fig. 3 shows the total power allocation, averaged over each simulation run. Simulations where the solution was not feasible are excluded from this figure – the reason for the MF-PCA plot stopping at $K = 4$.

To appreciate the relative difficulty of obtaining a feasible solution, Fig. 4 shows the percentage of users that met their outage constraints, averaged over all simulation runs.

Note the reduction in total power for the MF-PCA as K was increased from three to four users. This can be attributed to the low number of feasible solutions (0.425%) in the scenario for $K = 4$: only those users with "good" signature sequences could attain a feasabile solution. In the scenario where $K = 3$, there is less competition amongst users (as each has a higher degree of freedom) and a greater number of less-efficient signature sequence combinations were possible, in the sense that more feasible solutions were possible at the expense of a higher total power on average.

C. Large Systems

We now consider the large system results for $K = 16, 32, 64, 128, 256, 512$ users and compare the LSA-PCA with the MMSE-PCA.

A fixed $\alpha = 0.75$ was chosen, with other parameters as above. Figure 5 shows in log scale the sum power of all users for each simulation scenario. As the system size is scaled upwards, the LSA-PCA accuracy improves significantly until it becomes indistinguishable from the MMSE-PCA. For medium-sized systems, the approximate result gives excellent performance considering the reduced complexity and decentralized nature of the result.

Figure 6 shows the percentage of users in each simulation scenario that did not meet their outage constraints, using the LSA-PCA. Similar statistics are not shown for the MMSE-PCA, as all users in all scenarios met their outage constraints with equality. Since the closeness of the LSA- and MMSE-PCAs was noted above, a plot is also shown to quantify the average deviation between the outage probability specification and simulated outage probability, for each user not meeting their outage target. For all scenarios, this deviation is small, ranging from 3% to under 1% and improves as the size of the system is scaled upwards. This statistic is important as it validates the LSA method, even for smaller sized systems.

VII. Conclusion

This paper introduced a new power control problem that aims to jointly optimize user powers and linear receiver filters according to outage specifications. An iterative algorithm to solve this problem was developed and convergence proved. A bound on the outage probability enabled a mapping to take place between outage and an average SIR threshold. From this, a sub-optimal PCA-MUD was developed that utilized the well known MMSE receiver. The approximation to the optimal solution was found to be exceptionally close in the CEM^{σ} region of interest.

Large system approximations were used to decouple complex SIR expressions when the number of users and degrees of freedom approach infinity, with a fixed ratio between the two. This approximation enabled a closed form solution to an outage-mapped, joint power control and MMSE MUD optimization problem. In a finite-sized system, the approximate solution was found to be close to the MMSE-PCA solution, giving excellent results for medium to large finite-sized systems.

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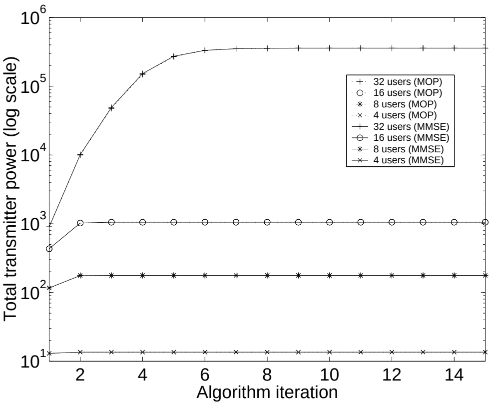

Fig. 1. Total transmitter power with 2–32 users.

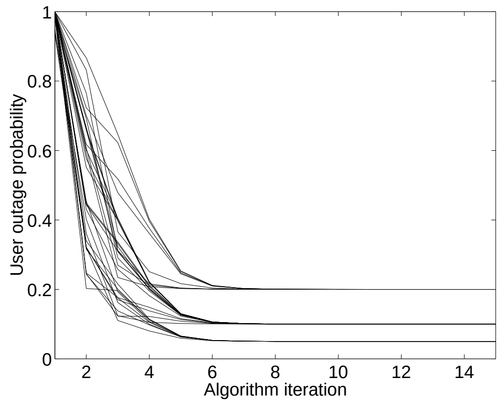

Fig. 2. Outage probability for MMSE-PCA with 32 users.

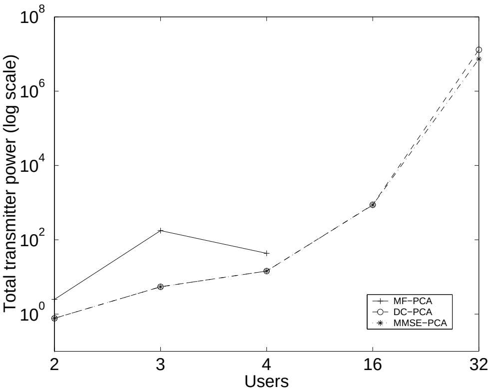

Fig. 3. Total transmitter power with 2–32 users, averaged over 1,000 simulations.

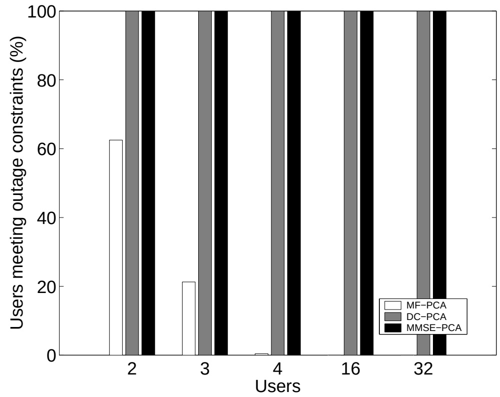

Fig. 4. Percentage of users meeting constraints, averaged over 1,000 simulations.

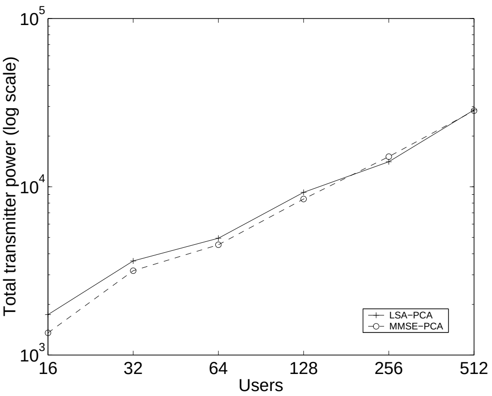

Fig. 5. Comparison between the LSA-PCA and MMSE-PCA with $K = 16, \ldots, 512$ users and $\alpha = 0.75$.

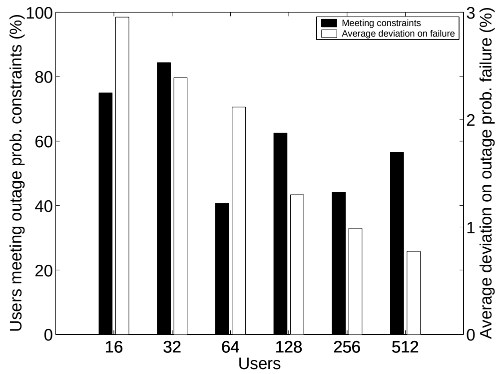

Fig. 6. Performance of the LSA-PCA in meeting user outage probability constraints, and the average deviation between resultant user outage probability and respective targets for those users with an inadequate QoS.